CHAPTER 5: SHEAR IN BEAMS

5.1 Introduction

Loads applied to beams produce bending moments, shearing forces, as shown in Figure 5.1, and in some cases torques. Design for bending has been elaborately discussed in chapter 4 and torsion will be discussed in chapter 6. In this chapter, design of beams for shear will be dealt with.

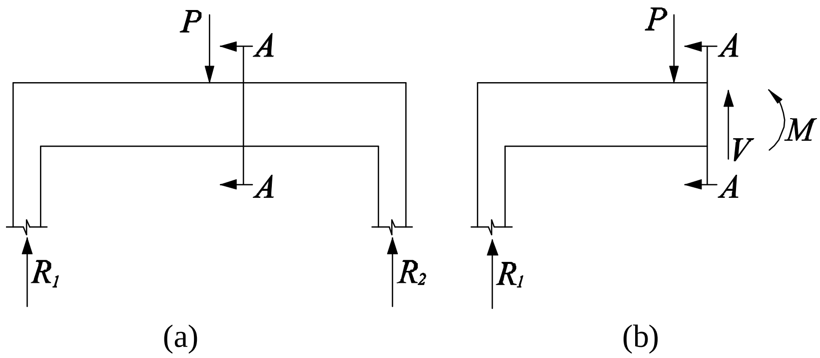


Figure 5.1: Shear in beams: (a) loaded beam; (b) internal forces at section A-A

Beams are usually designed for bending moment first; thus cross sectional dimensions are evaluated along with the required amounts of longitudinal reinforcement. Once this is done, sections should be checked for shear to determine whether shear reinforcement is required or not. This by no means indicates that shear is less significant than bending. On the contrary, shear failure which is usually initiated by diagonal tension is far more dangerous than flexural failure due to its brittle nature.

5.2 Shear in Homogeneous, Elastic Beams

For good understanding of the subject, consider a simply supported beam subjected to a uniformly distributed load as shown in Figure 5.2.a. Furthermore, it is assumed that the beam is made of elastic, homogeneous material.

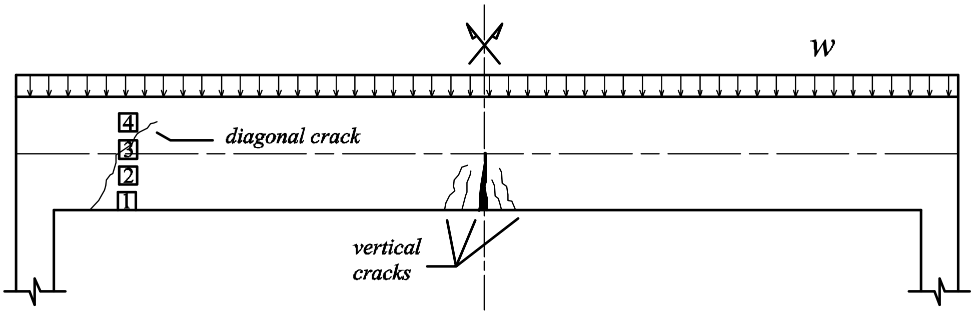


Figure 5.2.a : Loaded beam and orientation of cracks

The normal stresses f_x resulting from bending are given by the following equation as proved by the classical bending theory;

$$f_x = \frac{M_x\ y}{I_x} \qquad (5.1)$$

where M_x is the bending moment at the section under consideration, y is the distance from the point under consideration to the neutral axis, and I_x is the moment of inertia about the neutral axis.

The shearing stresses t_x are given by the following equation proved in the most classical mechanics of materials books:

$$t_x = \frac{V_x\ Q_x}{I_x\ b} \qquad (5.2)$$

where V_x is the shearing force at the considered section, Q_x is the moment of the area of the section located between the point where the shearing stresses are calculated and the extreme fiber of the section about the neutral axis, I_x is the moment of inertia about the neutral axis, and b is the width of the section at the point where shearing stresses are calculated.

In an attempt to establish the cracking pattern, four elements situated at different distances from the neutral axis are studied.

Element (1):

It is subjected only to normal, tensile stresses resulting from bending, as shown in Figure 5.2.b. The principal tensile stress is in the same direction as the normal tensile stress. Thus cracking takes place on a vertical plane due to the weakness of concrete in tension.

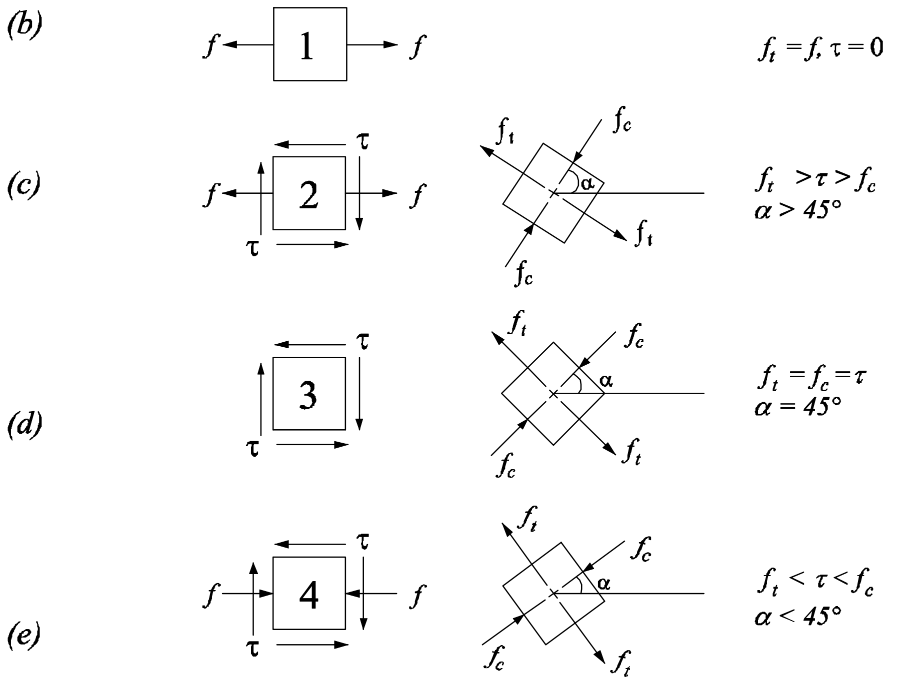


Figure 5.2.b: Principal stresses on elements 1,2,3 and 4

Element (2):

Element (2) that is located below the neutral axis is subjected to normal tensile stresses in addition to shearing stresses. The cracking surface makes an angle larger than 45 degrees with the horizontal axis. Cracking takes place on a plane perpendicular to the plane on which principal tensile stresses occur. See Figure 5.2.b for the orientation of principal tension stresses along a diagonal crack.

Element (3):

Element (3) that is located at the neutral axis is only subjected to pure shearing stresses. The cracking surface makes an angle of 45 degrees with the horizontal axis. Thus cracking takes place on a plane perpendicular to the principal tensile stress, as shown in Figure.5.2.b.

Element (4):

Element (4) that is located above the neutral axis is subjected to normal compressive stresses in addition to shearing stresses. The cracking surface makes an angle smaller than 45 degrees with the horizontal axis. Cracking takes place on a plane perpendicular to the

plane on which principal tensile stresses occur. See Figure 5.2.b for the orientation of principal tension stresses along a diagonal crack.

It is concluded that the shearing force acting on a vertical section in a reinforced concrete beam does not cause direct rupture of that section. Shear by itself or in combination with flexure may cause failure indirectly by producing tensile stresses on inclined planes. If these stresses exceed the relatively low tensile strength of concrete, diagonal cracks develop. If these cracks are not checked, splitting of the beam or what is known as diagonal tension failure will take place.

5.3 Types of Shear Cracks

Two types of inclined cracking occur in beams: flexure-shear cracking and web-shear cracking, shown in Figure 5.3.

A. Flexure-Shear Cracks

The most common type, develops from the tip of a flexural crack at the tension side of the beam and propagates towards mid depth until it is checked on the compression side of the beam. For these cracks to form, the bending moment must exceed the cracking moment of the cross section and a significant shear must exist.

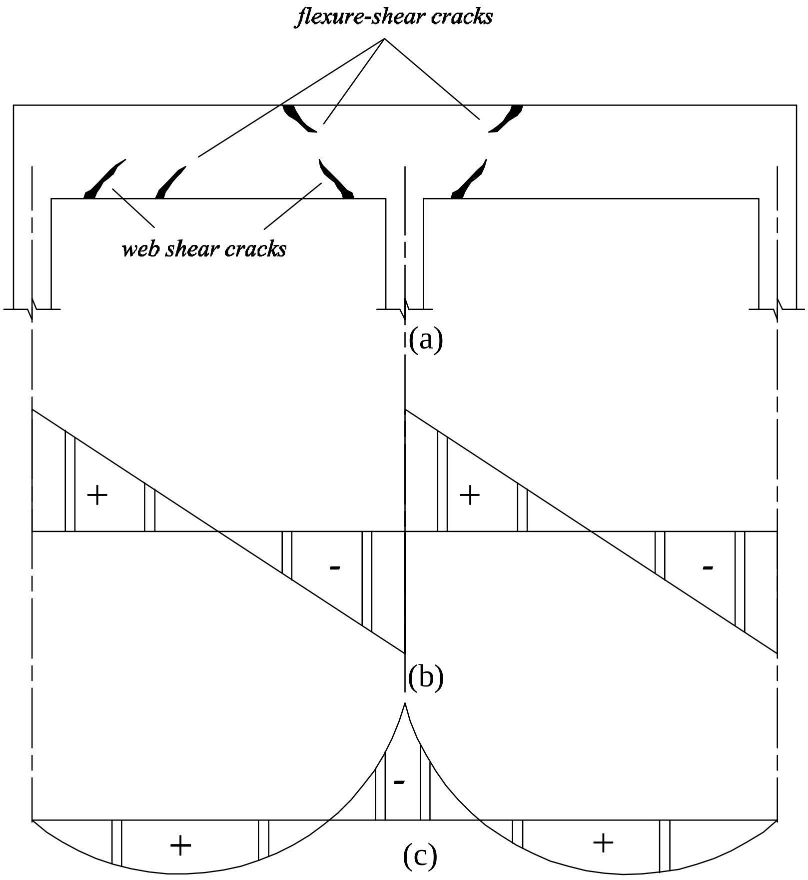


Figure 5.3: Types of cracks and associated internal forces: (a) orientation of cracks; (b) shear force diagram; (c) bending moment diagram

B. Web Shear Cracks

Web shear cracking begins from an interior point in a member at the level of the centroid of uncracked section and moves on a diagonal path to the tension face when the diagonal tensile stresses produced by shear exceed the tensile strength of concrete. This type of cracking is common on beams with thin webs and in regions of high shear and small moment. This combination exists adjacent to simple supports or at points of inflection in continuous beams.

5.4 Nominal Shear Stress

The only equation available to relate shear stress to shearing force is derived for a beam of constant cross section constructed of a homogeneous elastic material. Unfortunately, *Eq.* (5.2) can not be applied to reinforced concrete beams for the following reasons:

§ Reinforced concrete is non-homogeneous material.

- Concrete is not elastic.
- Variable extent of cracking along the length of a beam, making it impossible to determine cross-sectional properties.

Therefore, the *ACI Code* has adopted a simple procedure for establishing the magnitude of shear stress v on a cross section

$$v = \frac{V}{b_w\, d} \qquad (5.3)$$

where

v = nominal shear stress

V = shearing force at specified section

b_w = width of web of cross section

d = effective depth of the section.

5.5 Current Shear Design Philosophy

The current *ACI Code* shear design procedure is based on the assumption that for beams with no shear reinforcement, failure takes place on a vertical plane when the factored shear force on this plane exceeds the fictitious shear strength of concrete. The fictitious shear strength of concrete is evaluated from empirical expressions specified within the *ACI Code*. This simplification is done due to the following reasons:

- Strength of concrete in tension is highly variable, making it hard to evaluate a sustainable diagonal tension.
- Non-homogeneity of reinforced concrete which makes accurate computation of shear stresses on a particular section a tough task.
- Shear failures occur on diagonal planes as they are usually initiated by diagonal tension.

According to *ACI Code 11.1.1*, design of cross sections subject to shear should be based on the following equation.

$$\Phi V_n \geq V_u \qquad (5.4)$$

where

V_u = factored shear force at section considered

V_n = nominal shear strength

Φ = strength reduction factor for shear = 0.75

The nominal shear force is generally resisted by concrete and shear reinforcement or,

$$V_n = V_c + V_s \qquad (5.5)$$

where

V_c = nominal shear force resisted by concrete

V_s = nominal shear force resisted by shear reinforcement

5.6 Internal Forces in a Beam without Shear Reinforcement

The shear strength of a reinforced concrete beam without shear reinforcement is attributed to three main sources, shown in Figure 5.4. V_{cz} is the shear in the uncracked concrete in the compression zone, V_a is the vertical component of the shear transferred across the crack by interlock of the aggregate particles on the two faces of the crack, and V_d is the dowel action of the longitudinal reinforcement. The *ACI Code* considers the three components combined as the shear force resisted by concrete V_c, or

$$V_c = V_{cz} + V_a + V_d \qquad (5.6)$$

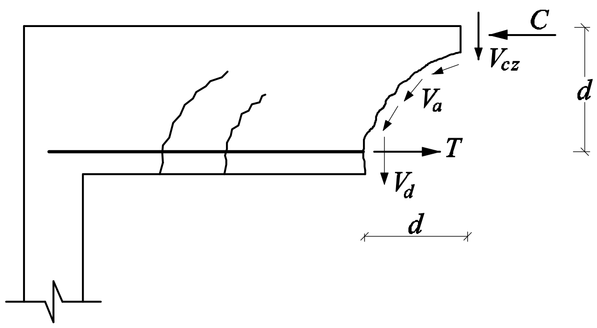


Figure 5.4: Behavior of beams failing in shear

5.7 Strength of Concrete in Shear

Shear strength of concrete V_c is evaluated by loading a plain concrete beam to failure. Shear stresses are computed by dividing the shearing force resisted by concrete V_c by $b_w\, d$. Strength of concrete in shear is directly proportional to the strength of concrete in

tension, inversely proportional to the magnitude of bending moment at the section under consideration, and directly proportional to the reinforcement ratio of flexural reinforcement. For the sake of simplicity V_c is assumed to be the same for beams with or without shear reinforcement.

For members subject to shear and bending only, *ACI Code 11.2.1.1* gives the following equation for evaluating V_c

$$V_c = 0.53\, l\, \sqrt{f'_c}\, b_w\, d \tag{5.7}$$

Eq. (5.7) assumes a constant value of V_c for all sections along the length of the beam.

A more exact formula is specified by *ACI Code 11.2.2.1*, given by *Eq*. (5.8)

$$V_c = \left(0.5\, l\, \sqrt{f'_c} + 176\, r_w \frac{V_u\, d}{M_u} \right) b_w\, d \tag{5.8}$$

where V_u is the factored shearing force, M_u is the factored bending moment occurring simultaneously with V_u at section considered, r_w is the reinforcement ratio of the web, and d is the effective depth of the beam. In *Eq*. (5.8) $\frac{V_u\, d}{M_u}$ should not exceed 1.0, and V_c should not exceed $0.93\, l\, \sqrt{f'_c}\, b_w\, d$.

For members subject to axial compression plus shear, *ACI Code 11.2.1.2* gives the following equation for evaluating V_c

$$V_c = 0.53 \left(1 + \frac{N_u}{140\, A_g} \right) l\, \sqrt{f'_c}\, b_w\, d \tag{5.9}$$

where N_u is the factored axial load normal to the cross section occurring simultaneously with V_u, and A_g is the gross area of the cross section. In *Eq*. (5.9) the shearing force resisted by concrete V_c is increased due to the presence of axial compression since the diagonal tension is decreased.

ACI Code 11.2.2.2 specifies that for members subject to axial compression, it shall be permitted to compute V_c using *Eq*. (5.8) with M_m substituted for M_u and $\frac{V_u\, d}{M_u}$ not then limited to 1.0,

$$M_m = M_u - N_u \frac{(4h - d)}{8} \tag{5.10}$$

where h is overall height of member and V_c shall not be greater than

$$V_c = 0.93\ l\ \sqrt{f'_c}\, b_w\, d\, \sqrt{1 + \frac{N_u}{35\ A_g}} \tag{5.11}$$

When M_m as computed by *Eq*. (5.10) is negative, V_c shall be computed by *Eq*. (5.11).

For members subject to axial tension plus shear, *ACI Code 11.2.1.3* states that V_c shall be taken as zero unless a more detailed analysis is made using *ACI 11.2.2.3*.

ACI 11.2.2.3 gives the following equation for evaluating V_c:

$$V_c = 0.53 \left(1 + \frac{N_u}{35\ A_g}\right) l\ \sqrt{f'_c}\, b_w\, d \tag{5.12}$$

where N_u is the factored axial load normal to the cross section occurring simultaneously with V_u and is taken negative, A_g is the gross area of the cross section and V_c shall not be less than zero.

In *Eq*. (5.12) the shearing force resisted by concrete V_c is decreased due to presence of axial tension which causes widening of cracks.

According to *ACI Code 11.2.3*, for circular members, the area used to compute V_c is taken as the product of the diameter and effective depth of the concrete section. It is permitted to take d as 0.80 times the diameter of the concrete section.

5.8 Strength Provided by Shear Reinforcement

When the nominal shearing force V_n exceeds the shearing force that can be resisted by concrete alone V_c, shear reinforcement, in any of the forms shown in the following section, can be used.

5.8.1 Types of Shear Reinforcement

When shear reinforcement is required, the following types of shear reinforcement are permitted by *ACI Code 11.4.1*, as shown in Figure 5.5.

a. Vertical Stirrups;

b. Inclined stirrups making an angle of 45 degree or more with longitudinal tension reinforcement;
c. Longitudinal reinforcement with bent portion making an angle of 30 degree or more with the tension reinforcement;
d. Spirals, circular ties, or hoops;
e. Combination of stirrups and bent longitudinal reinforcement.
f. Welded wire reinforcement with wires located perpendicular to axis of member.

Before diagonal cracking occurs, the stirrups remain unstressed. After cracking, the stress in the stirrups increases as they pick up a portion of the load formerly carried by the uncracked concrete.

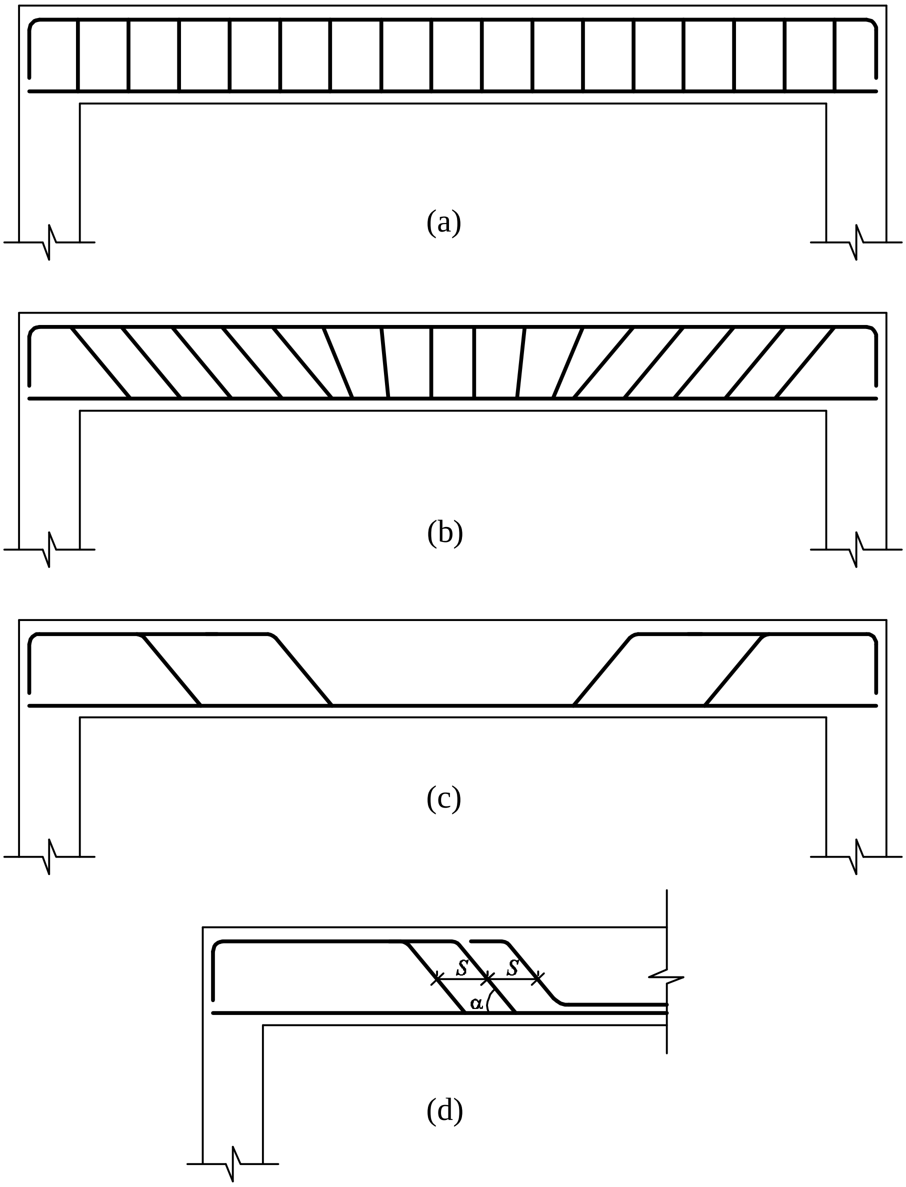


Figure 5.5: Types of shear reinforcement: (a) vertical stirrups; (b) inclined stirrups; (c) bent-up bars (two groups); (d) bent-up bars (three groups)

In Figure 5.6, assume an inclined crack making an angle of 45 degree with the longitudinal reinforcement and extending from the longitudinal reinforcement to the compression zone of the beam. For the shear reinforcement to be effective, it should intersect this diagonal crack. Let n be the number of shear reinforcing bars intersecting such crack. The shearing force resisted by shear reinforcing bars across the crack V_s is given by

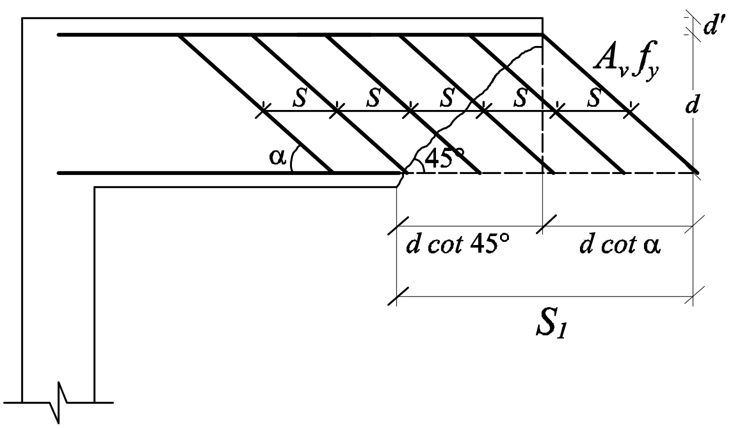


Figure 5.6: Shear resisted by stirrups

$$V_s = T \sin a \tag{5.13}$$

where T is the resultant of the forces in the shear reinforcement across the crack and given by

$$T = n A_v f_{yt} \tag{5.14}$$

where A_v is the total area of shear reinforcement within a distance S ,

and f_{yt} is the yield stress of the shear reinforcement.

n is given by

$$n = \frac{S_1}{S} = \frac{d\left(\cot a + \cot 45^{\circ}\right)}{S} = \frac{d\,(\cot a + 1)}{S} \tag{5.15}$$

Substituting *Eq.* (5.14) and *Eq.* (5.15) into *Eq.* (5.13), one gets

$$V_s = \frac{d\left(\cot a + 1\right) A_v \sin a \; f_{yt}}{S} = \frac{A_v \; f_{yt}\, d\,\left(\sin a + \cos a\right)}{S} \tag{5.16}$$

For vertical stirrups, a is equal to 90 degrees, and *Eq.* (5.16) takes the following form

$$V_s = \frac{A_v \; f_{yt}\, d}{S} \tag{5.17}$$

Eq. (5.18) is valid for inclined stirrups and longitudinal bars bent at more than one point. When longitudinal reinforcement is bent at a single point, V_s is given by

$$V_s = A_v \; f_y \sin a \le 0.8 \sqrt{f'_c}\; b_w\, d \tag{5.18}$$

where a is the angle between bent-up reinforcement and longitudinal axis of the member.

5.8.2 Minimum Amount of Shear Reinforcement

The instant diagonal crack forms, the tension carried by the concrete must be transferred to the stirrups if the beam is not to split into two sections. To ensure that the stirrups will have sufficient strength to absorb the diagonal tension in the concrete, *ACI Code 11.4.6* states that a minimum area of shear reinforcement is to be provided in concrete members where the factored shearing force V_u exceeds half the shear strength provided by concrete $0.50\ \Phi V_c$, except for the following:

- Footings and solid slabs.
- Concrete joist construction.
- Beams with h not greater than 25 cm.
- Beams integral with slabs with h not greater than 60 cm and not greater than the lrger of 2.5 times thickness of flange, and 0.5 times width of web
- Beams with total height not greater than 25 cm, 2.5 times thickness of flange, or 0.50 the width of web, whichever is the greatest.

These exceptions were made because there is a possibility of load sharing between weak and strong areas.

Where shear reinforcement is required by *ACI 11.4.6.*1 or for strength and where *ACI 11.5.1* allows torsion to be neglected, *ACI Code 11.4.6.3* requires that the minimum area of shear reinforcement, $A_{v,min}$ is computed by

$$A_{v,min} = 0.2\sqrt{f'_c}\,\frac{b_w\, S}{f_{yt}} \tag{5.19}$$

but not less than be less than $A_{v,min} = \dfrac{3.5\, b_w\, S}{f_{yt}}$

where $A_{v,min}$ is the minimum area of shear reinforcement within a distance S, b_w is the web width, S is the spacing of shear reinforcement, and f_{yt} is the yield stress of the shear reinforcement.

5.8.3 Maximum Stirrup Spacing

The assumption made in *Eq.* (5.17) is that one or more stirrups cross each potential diagonal crack in order to prevent the beam from splitting into two sections between

stirrups. To ensure that, this requirement is satisfied, *ACI Code 11.4.5.1* through *11.4.5.3* specifies the following limits for maximum spacing of shear reinforcement.

A. Vertical Stirrups:

- When $V_s \le \sqrt{f_c'}\, b_w\, d$, $S_{\max}$ is limited to the smaller of *d/2* or 60 *cm*. This ensures that at least one stirrup with adequate anchorage will be available to hold the upper and lower sections together.
- When $2.10\sqrt{f_c'}\, b_w\, d > V_s > \sqrt{f_c'}\, b_w\, d$, $S_{\max}$ is limited to the smaller of d/4 or 30 *cm*, to ensure that each potential diagonal crack will be crossed by approximately three stirrups. See Figure 5.7 for schematic representation.

B. Inclines stirrups and Bent-up Bars:

When $V_s \le \sqrt{f_c'}\, b_w\, d$, inclined stirrups and bent-up longitudinal bars are spaced such that every 45-degree line, extending toward the reaction from mid depth of member *d*/2 to longitudinal tension reinforcement, is to be crossed by at least one stirrup or bent-up bar. If $2.10\sqrt{f_c'}\, b_w\, d > V_s > \sqrt{f_c'}\, b_w\, d$, the line is to be crossed by at least two stirrups or two groups of bent up bars.

The limit on maximum stirrup spacing serves to limit diagonal crack widths and to provide a better anchorage for the longitudinal reinforcement against breaking through the concrete cover or loss of bond with the concrete around the bars.

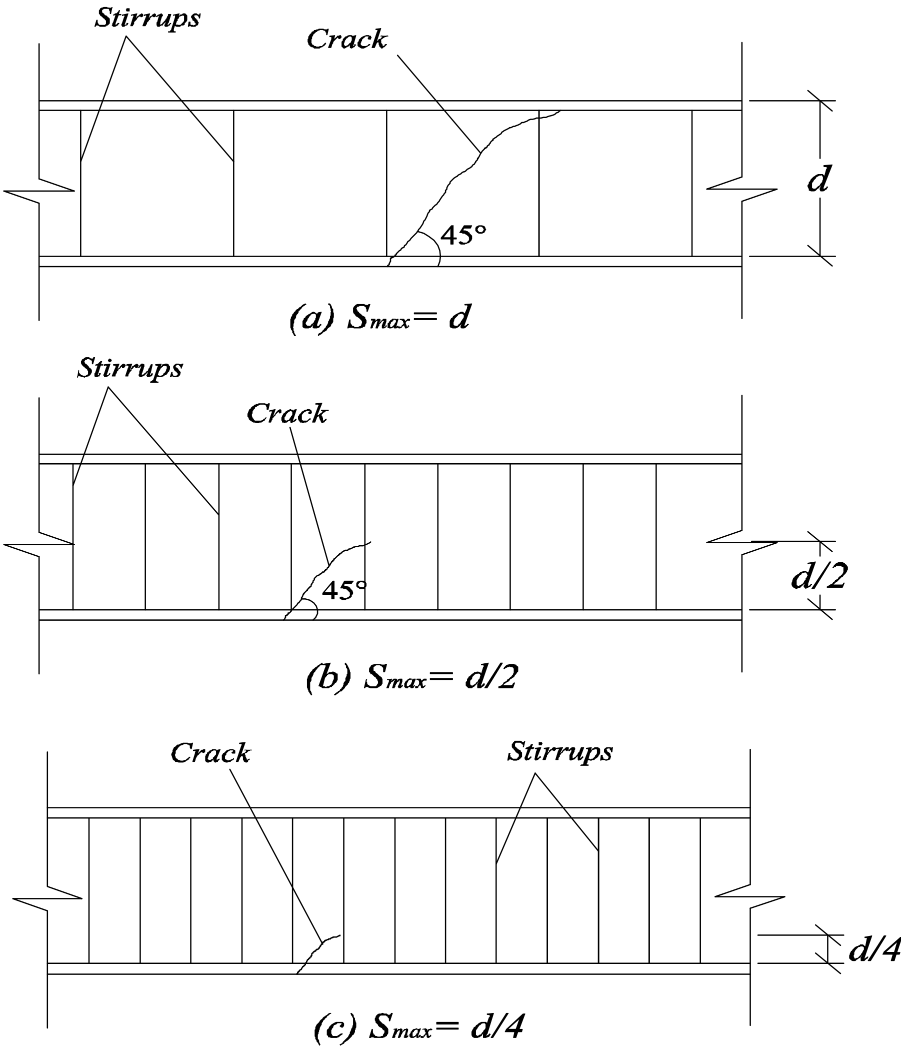


Figure 5.7: Maximum stirrup spacing

5.8.4 Ensuring Ductile Behavior

To prevent a shear-compression failure caused by diagonal compression stresses in the compression zone above the tip of a diagonal crack, *ACI Code 11.4.7.9* requires that the maximum force resisted by shear reinforcement V_s is not to exceed $2.2\sqrt{f'_c}\, b_w\, d$. Since diagonal tensile stresses develop in the direction perpendicular to the compressive stresses, the compressive strength of the concrete will be less than that based on the

Uniaxial test. This is done to ensure a ductile mode of failure by forcing the shear reinforcement to yield before the concrete starts to crush.

5.9 Critical Section for Shear

According to *ACI Code 11.1.3.1*, sections located less than a distance *d* from face of support are permitted to be designed for the same shear as that calculated at a distance *d*. Conditions for the validity of *ACI Code 11.1.3.1* are listed in *ACI Code 11.1.3* and given here:

- Loads applied near the top of the member, shown in 5.8.a and 5.8.b.
- No concentrated loads are applied between the face of support and a distance *d* from it, shown in 5.8.c. Otherwise the critical section for shear is taken at the face of the support.
- Support reaction in direction of applied shear, introducing compression into the end region of the member, shown in 5.8.d. If tension is introduced, the critical section is taken at the face of the support.

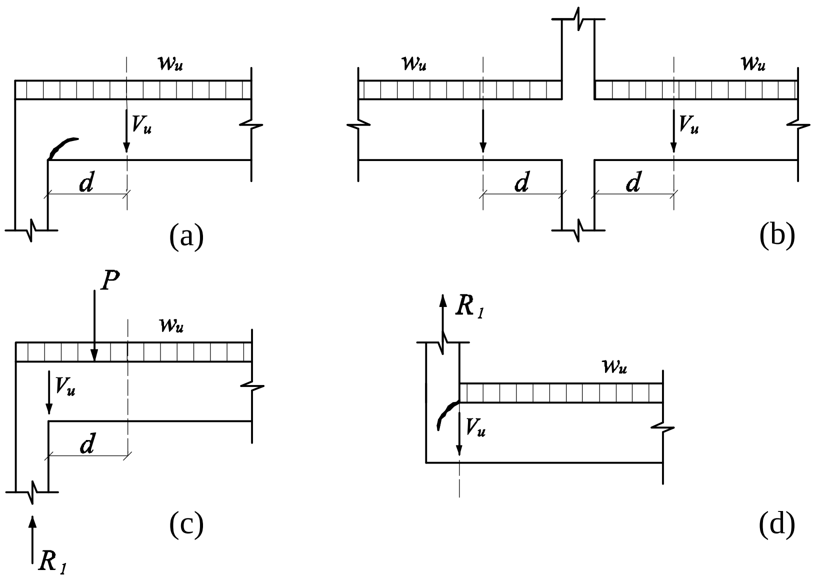


Figure 5.8: Location of critical section for shear

This provision recognizes that a crack adjacent to a support whose reaction induces compression into a beam will have a horizontal projection of at least *d*; therefore the maximum shear force that must be transmitted across the potential failure plane closest to the support will be equal to the reaction *R* reduced by any external forces applied to the beam with a distance *d* from the support.

5-10 Punching (Two-way) Shear

When loads are applied over small areas to slabs and footings with no beams, punching failure may occur. The sloping failure surface takes the shape of a truncated pyramid in case of rectangular or square columns and a truncated cone in case of circular columns, as shown in Figure 5.9.

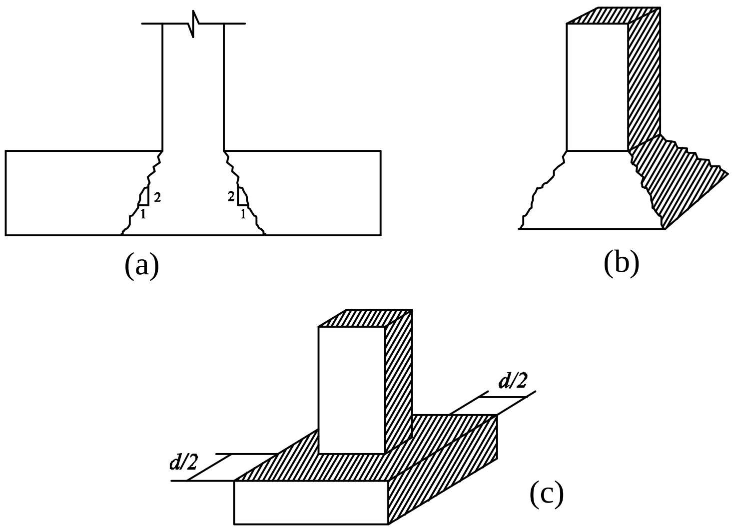


Figure 5.9: Punching shear: (a) punching shear failure of an isolated footing; (b) actual failure surface; (c) assumed failure surface

The *ACI Code R11.11.1.2* assumes that failure takes place on vertical planes located at distance *d/*2 from faces of column. In *ACI Code 11.11.2.1*, the punching shear strength V_c is given by the smallest of:

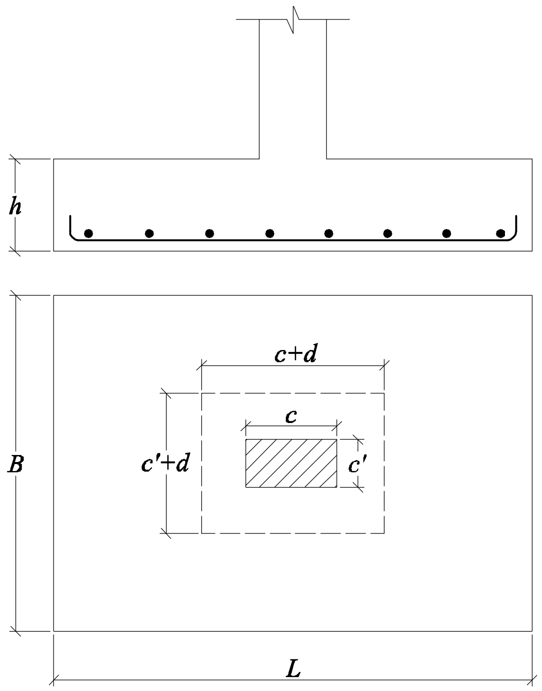
$$V_c = 0.53\sqrt{f'_c}\left(1 + \frac{2}{b}\right) l \; b_0 \; d \qquad (5.20)$$

$$V_c = l \sqrt{f'_c} \; b_0 \; d \qquad (5.21)$$

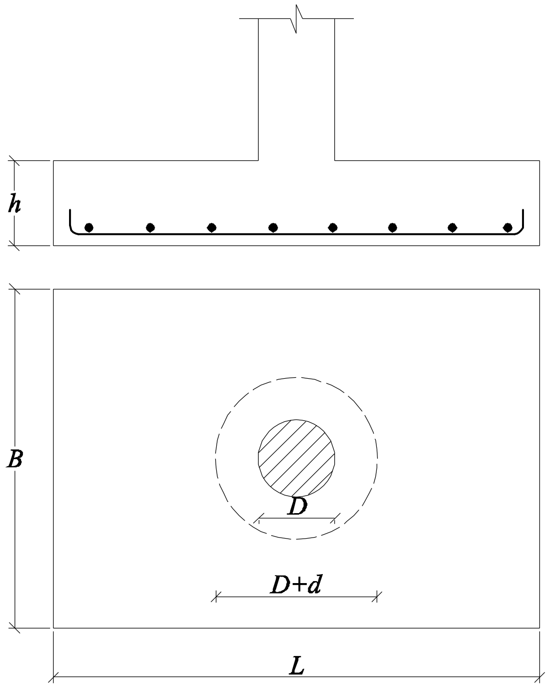
$$V_c = 0.27\left(\frac{a_s \, d}{b_0} + 2\right) l \sqrt{f'_c} \; b_0 \; d \qquad (5.22)$$

where b_o is the perimeter of the critical punching shear section for slabs and footings, d is the effective depth, b_c is the ratio of long side to short side of column, and a_s is a constant dependent on the location of the column relative to the slab or footing; is equal to 40 for interior columns, 30 for side columns, and 20 for corner columns.

For interior square columns, the perimeter of the critical section $b_o = 4(c+d)$, for rectangular columns, $b_o = 2(c+c'+2d)$, and for circular columns, $b_o = p(D+d)$, where c, c', and D are column cross sectional dimensions for rectangular and circular columns respectively. (see Figure 5.10).



(a)



(b)

Figure 5.10: Punching shear surface for isolated footings: (a) rectangular column; (b) circular column

The design equation is given here

$$V_u \le \Phi V_c \tag{5.23}$$

where V_u is the factored shearing force, Φ is the strength reduction factor for shear, and V_c is the nominal punching shear provided by concrete.

5.11 Summary of ACI Shear Design Procedure for Beams

Once the beam is designed for moment, thus establishing the concrete dimensions and the required longitudinal reinforcement, the beam is designed for shear as explained in the next steps.

1. Draw the shearing force diagram and establish the critical section for shear.
2. Calculate the nominal capacity of concrete in shear, V_c.
3. Check whether the chosen concrete dimensions are adequate for ensuring a ductile mode of failure. If not satisfied, the concrete dimensions should be increased.
4. Classify the factored shearing forces acting on the beam according to the following categories:

- For $V_u \le 0.50\,\Phi\,V_c$, no shear reinforcement is required.
- For $\Phi\,V_c > V_u > 0.50\,\Phi\,V_c$, minimum shear reinforcement is required.
- For $V_u \ge \Phi\,V_c$ shear reinforcement is required. When more than one type of shear reinforcement is used to reinforce the same portion of a beam, shear strength V_s is calculated as the sum of the V_s values evaluated from each type.

5. Check the spacing between shear reinforcement according to *ACI Code* limits discussed earlier in this chapter.
6. Sketch the shear reinforcement along the length of the beam.