6.1 Introduction

In addition to creating stresses in the reinforcement and the concrete, flexural deformations of a beam also create stresses between the reinforcement and concrete called bond stresses. If the intensity of these stresses is not restricted, they may produce crushing or splitting of the concrete surrounding the reinforcement, especially if bars are closely spaced or located near the surface of the concrete. Failure of the concrete permits the reinforcement to slip. As slipping occurs, the stress in the reinforcement drops to zero, and the beam which behaves as if it was made of plain concrete is subject to immediate failure as soon as the concrete cracks.

In this chapter, details of reinforcement to reduce the potential of local bond failures will be discussed.

6.2 Mechanics of Bond Strength

A smooth bar embedded in concrete develops bond by adhesion and friction. Since these bond components will be lost at low stress levels, smooth bars are not currently used as main reinforcement bars. In cases where reinforcement bars are used, hooks are required to provide resistance against slipping out of the concrete. On the other hand, a deformed bar generates bond by friction and by bearing on the deformations of the bar against the concrete. At low bar stress, bond resistance is mainly attributed to bar adhesion which has a limited contribution to the bond resistance. Once adhesion is lost at higher bar stress and some slight movement between the reinforcement and the concrete occurs, bond is then provided by friction and bearing on the deformations of the bar. At much higher bar stress, bearing on the deformations of the bar will be the only component contributing to bond strength.

Figure 6.1.a shows forces acting on the bar, while Figure 6.1.b shows forces equal and opposite bearing forces on the concrete. These forces on the concrete have longitudinal and radial components as shown in Figure 6.1.c. Experimental studies indicate that bearing stresses are affected by the slope of the bar deformations. They are inclined to the
longitudinal axis of the bar at an angle $\alpha$ that varies from 45 degrees to 180 degrees. Thus, the radial component is equal to or larger than the longitudinal component. Radial stresses cause circumferential tensile stresses around the bar, shown in Figure 6.1.d. Ultimately, the concrete will split parallel to the bar and the resulting crack will propagate out of the surface of the concrete element. Once these cracks develop, bond transfer drops rapidly unless reinforcement is provided to restrain the opening of the splitting crack. The load at which splitting failure develops is a function of:

a. The minimum distance from the bar to the surface of the concrete or to the next bar. The smaller the distance, the smaller is the splitting load.

b. The tensile strength of the concrete. The higher the tensile strength, the higher is the splitting resistance.

c. The average bond stress. The higher the average bond stress, the higher is the splitting resistance.

Figure 6.1: (a) Forces on bar; (b) Forces on concrete; (c) Components of forces on concrete; (d) Splitting stresses
Splitting failure surfaces tend to develop along the shortest distance between a bar and the concrete surface or between two adjacent bars as shown in Figure 6.2. If the concrete cover and bar spacing are large compared to the bar diameter, a pullout failure can occur, where the bar and the ring of concrete between successive deformations pullout along a cylindrical failure surface joining the tips of the deformations.

Figure 6.2: (a) Side cover and half the bar spacing both less than bottom cover; (b) Side cover is equal to bottom cover, both less than half the bar spacing; (c) Bottom cover less than side cover and half the bar spacing

6.3 Bond Strength

Bond stresses are existent whenever the stress or force in a reinforcing bar changes from point to point along the length of the bar in order to maintain equilibrium.

For the free body diagram shown in Figure 6.3.d, the tensile force at one of the bar ends, $T_1$, is given as

$$T_1 = f_{s1} A_b$$  \hspace{1cm} (6.1)

where $f_{s1}$ is the bar stress at the specified end, and $A_b$ is the cross-sectional area of the bar.

The tensile force at the other end of the bar, $T_2$, is given as

$$T_2 = f_{s2} A_b$$  \hspace{1cm} (6.2)

where $f_{s2}$ is the bar stress at this bar end.

The resultant of bond stresses on the surface of the bar is given by

$$F_b = \mu_{avg} (\pi d_b) l$$  \hspace{1cm} (6.3)

where $\mu_{avg}$ is the average bond stress on the bar surface, $d_b$ is bar diameter, and $l$ is bar length.
Figure 6.3: Bond stresses: (a) beam and loads; (b) internal forces in concrete and reinforcement; (c) free body diagram of reinforcement bar; (d) bond stresses on bar surface

Summing forces in the direction of the bar, and assuming that $f_{s2}$ is larger than $f_{s1}$, one gets

$$A_b (f_{s2} - f_{s1}) = \mu_{avg} \left( \pi d_b \right) l$$

and

$$\left( \frac{\pi}{4} \right) d_b^2 (f_{s2} - f_{s1}) = \mu_{avg} \left( \pi d_b \right) l$$

The average bond stress, $\mu_{avg}$ is given by

$$\mu_{avg} = \frac{(f_{s2} - f_{s1}) d_b}{4 l} \quad (6.4)$$
6.4 Development Length

The development length $l_d$ is the shortest length of a reinforcement bar, within which the bar stress can increase from zero to the yield strength, $f_y$. If the distance from a point where the bar stress equals $f_y$ to the end of the bar is less than the development length, the bar will pullout of the concrete.

The development length can be expressed in terms of the ultimate value of the average bond stress by setting $(f_{r2} - f_{st})$ in Eq. (6.4) equal to $f_y$, or

$$l_d = \frac{f_y d_b}{4 \mu_{avg,u}} \quad (6.5)$$

where $\mu_{avg,u}$ is the value of $\mu_{avg}$ at bond failure.

6.5 ACI Code Current Design Philosophy

The ACI Code 318-08 uses the concept of development length rather than the concept of bond stress in its current design philosophy because the actual bond stress varies along the length of a bar embedded in a concrete tension zone. The development length concept is based on the attainable average bond stress over the length of embedment of the reinforcement.

In application, the development length concept requires minimum lengths or extensions of the reinforcement beyond all points of peak stress in the reinforcement. This development length is necessary on both sides of such peak stress point.

6.6 Development of Deformed Bars in Tension

According to ACI Code 12.2.3, the development length to bar diameter ratio $l_d/d_b$ is given by

$$l_d = \left\{ \frac{f_y \Psi_l \Psi_e \Psi_s}{3.5 \lambda \left( \frac{c_b + K_{ne}}{d_b} \right) \sqrt{f'_c}} \right\} d_b \quad (6.6)$$
in which the term \( (c_b + K_{tr})/d_b \) is not to be greater than 2.5. This limit is included to safeguard against pullout type of failure. It is permitted to use \( K_u = 0 \) as design simplification even if transverse reinforcement is present.

Much simpler formulas are presented in ACI 12.2.2 and shown in Table 1.

In no case shall the development length be smaller than 30 cm.

**Table 1: Simplified formulas for evaluation of \( l_d \) in tension**

<table>
<thead>
<tr>
<th>Spacing and cover</th>
<th>( \Phi \ 19 \text{ mm and smaller bars} )</th>
<th>( \Phi \ 20 \text{ mm and larger bars} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clear spacing of bars being developed or spliced not less than ( d_b ), clear cover not less than ( d_b ), and stirrups or ties throughout ( l_d ) not less than the Code minimum or Clear spacing of bars or wires being developed or spliced not less than 2( d_b ) and clear cover not less than ( d_b )</td>
<td>( \left( \frac{f_{y} \Psi_{t} \Psi_{e}}{6.6 \lambda \sqrt{f'<em>{c}}} \right) d</em>{p} )</td>
<td>( \left( \frac{f_{y} \Psi_{t} \Psi_{e}}{5.3 \lambda \sqrt{f'<em>{c}}} \right) d</em>{p} )</td>
</tr>
</tbody>
</table>

where

\( l_d \) = development length, cm

\( d_b \) = nominal diameter of bar, cm

\( f_{y} \) = specified yield strength of reinforcement, kg/cm\(^2\)

\( \sqrt{f'_{c}} \) = square root of specified compressive strength of concrete, kg/cm\(^2\)

\( c_b \) = spacing or cover dimension, cm. It is the smaller of either the distance from the center of the bar to the nearest concrete surface or one-half the center-to-center spacing of bars being developed.

\( K_{tr} \) = transverse reinforcement index, which represents the contribution of confining reinforcement across potential splitting planes.

\( \Psi_{t} \) = reinforcement location factor

\( \Psi_{e} \) = coating factor

\( \Psi_{s} \) = reinforcement size factor
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\( \lambda = \) lightweight aggregate concrete factor reflecting the reduced mechanical properties of lightweight concrete

*ACI Code 12.2.4* provides the factors for use in the expression for development of deformed bars in tension as follows:

\( \psi_t \) reflect the adverse effects of the top reinforcement casting position, such as bleeding and segregation. This factor is given for two cases:

\( \beta \) Horizontal reinforcement so placed that more than 30 cm of fresh concrete is cast in the member below the development length or splice
\( \beta \) Other reinforcement

\( \psi_e \) reflects the adverse effects of epoxy coating. It is given for three cases:

\( \beta \) Epoxy-coated bars with cover less than \( 3d_e \), or clear spacing less than \( 6d_b \)
\( \beta \) All other epoxy-coated bars
\( \beta \) Uncoated reinforcement

However, the product \( \psi_t \psi_e \) is not to be greater than 1.7.

\( \psi_s \) reflects better performance of the smaller diameter reinforcement. This factor is given for two cases:

\( \beta \) \( \phi 19 \ mm \) and smaller bars
\( \beta \) \( \phi 22 \ mm \) and larger bars

\( \lambda \) is a lightweight concrete factor that reflects the reduction in splitting resistance of lightweight concrete. It takes on one of the following values:

\( \beta \) When lightweight concrete is used
\( \beta \) When normal weight concrete is used

\( K_{tr} \) represents the contribution of confining reinforcement, given by

\[
K_{tr} = \frac{40 A_{tr}}{sn} \tag{6.7}
\]

where:

\( A_{tr} = \) total cross sectional area of all transverse reinforcement within the spacing \( s, \ cm^2 \)
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\[ s = \text{maximum center-to-center spacing of transverse reinforcement within development length} \ l_d, \ cm. \]

\[ n = \text{number of bars being developed along the plane of splitting}. \]

### 6.6.1 Excessive Reinforcement

According to *ACI Code* 12.2.5, reduction in development length is allowed where reinforcement in a flexural member exceeds what is required by analysis. The reduction factor is equal to the area of required reinforcement divided by the area of provided reinforcement.

**Example (6.1):**

A simply supported beam, shown in Figure 6.4, is reinforced with $4 \phi 20 \ mm$ uncoated bars. The ends of these bars terminate 4 cm from the ends of the beam. Determine whether the reinforcement satisfies *ACI Code* requirements for development length.

Use $f'_c = 250 \ kg/cm^2$ normal weight concrete, and $f_s = 4200 \ kg/cm^2$.

**Solution:**

\[ \psi_f = 1, \ \psi_e = 1, \ \psi_s = 0.8, \ \text{and} \ \lambda = 1 \]

\[ c_b = 4.0 + 0.80 + 1.0 = 5.8 \ cm \]

or

\[ c_b = [(30 - 4 \ (2) - 2 \ (0.8) - 2.00)] / (3)(2) = 3.07 \ cm \]

i.e., \( c_b \) is taken as 3.07 cm.
Figure 6.4: (a) Beam and loads; (b) bending moment diagram; (c) section A-A

\[ K_{tr} = \frac{40 A_{tr}}{s n} = \frac{40 \times 2 \times 0.5}{20 \times 4} = 0.5 \text{ cm} \]

\[ \frac{c_b + K_{tr}}{d_b} = \frac{3.07 + 0.5}{2.0} = 1.785 \]

\[ l_d = \frac{f_y \psi_t \psi_e \psi_s}{3.5 \lambda \left( \frac{c_b + K_{tr}}{d_b} \right) \sqrt{f'c}} \]

\[ d_b = \left( \frac{4200 \times 0.80}{3.5 \times 1.785 \sqrt{250}} \right) (2.0) = 68.03 \text{ cm} \]

Available development length = 200 + 15 – 4 = 211 cm > 68.03 cm O.K.

i.e., reinforcement satisfies ACI Code requirements for development length.
**Example (6.2):**

In the isolated footing shown in Figure 6.5, it is required to check whether the flexural reinforcement satisfies *ACI Code* anchorage requirement.

Use $f'_c = 250 \text{ kg/cm}^2$ normal weight concrete, and $f_y = 4200 \text{ kg/cm}^2$, and concrete cover $= 7.50 \text{ cm}$.

**Solution:**

For development of reinforcement in tension

$\psi_t = 1$, $\psi_e = 1$, $\psi_s = 0.8$, and $\lambda = 1$

$c_b = 7.5 + 0.80 = 8.3 \text{ cm}$

or

$c_b = 15/2 = 7.5 \text{ cm}$

i.e., $c_b$ is taken as the smaller of the two values, is equal to 7.5 cm.

$K_{tr} = 0.0 \text{ cm}$, since no stirrups are used.

$$\frac{c_b + K_{tr}}{d_b} = \frac{7.5 + 0.0}{1.6} = 4.68 > 2.5$$

i.e. use $\frac{c_b + K_{tr}}{d_b} = 2.5$

$$l_d = \left( \frac{f_y \psi_t \psi_e \psi_s}{3.5 \lambda \left( \frac{c_b + K_{tr}}{d_b} \right) \sqrt{f'_c}} \right) d_b = \left( \frac{4200 (0.80)}{3.5 (2.5) \sqrt{250}} \right)(1.6) = 38.86 \text{ cm}$$

Available length $= \frac{285 - 40}{2} - 7.5 = 115 \text{ cm} > 38.86 \text{ cm}$ O.K.

Flexural reinforcement satisfies *ACI Code* development requirement, without using standard hooks at bar ends.
6.7 Development of Deformed Bars in Compression

Shorter development lengths are required for compression than for tension since flexural tension cracks are not present for bars in compression.

According to ACI Code 12.3, development length $l_{dc}$, in cm, for deformed bars in compression is computed as the product of the development length $l_{dc}$ and applicable modification factors, but $l_{dc}$ is not to be less than 20 cm.

The development length $l_{dc}$ for deformed bars in compression is given as

$$\frac{0.075 f_y}{\lambda \sqrt{f'_c}} d_b \geq 0.0044 f_y d_b$$

(6.8)

6.7.1 Applicable Modification Factors

1. Excessive reinforcement:

   Based on ACI Code 12.3.3, the modification factor is equal to the area of required reinforcement divided by the area of provided reinforcement.

2. Spirals or Ties:
Based on ACI Code 12.3.3, the modification factor for reinforcement, enclosed within spiral reinforcement not less than 6 mm in diameter and not more than 10 cm pitch, or within $\phi 12$ mm ties spaced at not more than 10 cm center-to-center is given as 0.75.

6.8 Development of Deformed Bundled Bars

Based on ACI Code 12.4, development length of individual bars within a bundle, in tension or compression, is taken as that for individual bar, increased 20 percent for three-bar bundle, and 33 percent for four-bar bundle. The extra extension is required since the grouping makes it more difficult to secure bond resistance from concrete between the bars. For determining the appropriate modification factors, a unit of bundled bars is treated as a single bar of a diameter derived from the equivalent total area of bars.

6.9 Development of Standard Hooks in Tension

Hooks are used to provide additional anchorage when there is insufficient length available to develop a bar. According to ACI Code 12.5, development length $l_{dh}$, for deformed bars in tension terminating in a standard hook is computed as the product of the development length $l_{dh}$ and applicable modification factors, but $l_{dh}$ is not to be less than $8d_b$, nor less than 15 cm.

The development length $l_{dh}$ for hooked bars is given as

$$
\left( \frac{0.075\psi_y f_y}{\lambda \sqrt{f_{c'}}} \right) d_b
$$

(6.9)

where $\psi_e$ is taken as 1.2 for epoxy-coated reinforcement, and $\lambda$ is taken as 0.75 for lightweight concrete. For other cases, $\psi_e$ and $\lambda$ shall be taken as 1.0.

Development length $l_{dh}$ is the distance between the critical section and the start of the hook added to the radius of the bend of hook and one-bar diameter.

Either a 90 or a 180-degree hook, shown in Figure 6.6, may be used. For shallow elements, the 180-degree hook is suitable, while the 90-degree hook is used when the horizontal reinforcement in one element is to be made continuous with the vertical reinforcement in a second element.
Example (6.3):

In the beam shown in Figure 6.7, the top reinforcement is designed for a flexural stress of $f_y$ at the face of the column. Determine whether a standard 90-degree hook is required for anchorage.

Use $f'_c = 250 \, \text{kg/cm}^2$ normal weigh concrete, and $f_y = 4200 \, \text{kg/cm}^2$. 
Solution:
For bars in tension, $\psi_t = 1.3, \psi_e = 1, \psi_s = 0.8, \text{ and } \lambda = 1$

$c_b = 4.0 + 1.0 + 0.90 = 5.9 \text{ cm}$

or

$c_b = [30 - 2(4) - 2(1) - 1.8]/6 = 3.03 \text{ cm}$

i.e., $c_b$ is taken as the smaller of the two values, is equal to 3.03 cm.

\[
K_{tr} = \frac{40 A_{tr}}{s n} = \frac{40 (2)(0.785)}{(20)(4)} = 0.785 \text{ cm}
\]

\[
\frac{c_b + K_{tr}}{d_b} = \frac{3.03 + 0.785}{1.8} = 2.12 < 2.5 \quad O.K
\]

\[
l_d = \left( \frac{f_y \psi_t \psi_e \psi_s}{3.5 \lambda \left( \frac{c_b + K_{tr}}{d_b} \right) \sqrt{f'_c}} \right) d_b = \left( \frac{4200 (1.3)(1.0)(0.80)}{3.5 (2.12)\sqrt{250}} \right) (1.8) = 67.02 \text{ cm}
\]

Available development length at free end = 250 – 4 = 246 cm > 67.02 cm
Therefore, a standard hook is not required at free end. Available development length at column side = 60 – 4 = 56 cm < 67.02 cm. Hence, a standard hook is required at column side.

The development length $l_{dh}$, for deformed bars in tension terminating in a standard hook is computed as the product of the development length $l_{hb}$ and applicable modification factors, but $l_{dh}$ is not to be less than $8d_b$, nor less than 15 cm.

$$l_{dh} = \left( \frac{0.075 \psi_e f_y}{\lambda \sqrt{f'_c}} \right) d_b = \left( \frac{0.075 (1)(4200)}{\sqrt{250}} \right)(1.8) = 35.86 \text{ cm} > 8d_b \text{ and } 15 \text{ cm}$$

The available development length of 56 cm is adequate for providing anchorage using a standard 90-degree hook.

### 6.9.1 Development of Standard Hooks in Compression

*ACI Code 12.5.5* notes that hooks are not considered effective in compression and may not be used as anchorage.

### 6.10 Development of headed deformed bars in tension:

The development length $l_{dt}$ of headed deformed bar, measured from the critical section to the bearing face of the head is given by,

$$\left( \frac{0.06 \psi_e f_y}{\sqrt{f'_c}} \right) d_b$$

(6.10)

where $\psi_e$ is taken as 1.2 for epoxy-coated reinforcement and 1.0 for other cases.

The development length shall not be less than the larger of $8d_b$ and 150 mm.

---

![Figure 6.8: Development of headed deformed bars](image)
Use of heads to develop deformed bars in tension shall be limited to conditions satisfying (a) through (f):
(a) Bar $f_y$ shall not exceed 420 MPa;
(b) Bar size shall not exceed $\phi 36 \text{ mm}$;
(c) Concrete shall be normalweight;
(d) Net bearing area of head $A_{beg}$ shall not be less than $4A_b$;
(e) Clear cover for bar shall not be less than $2d_b$; and
(f) Clear spacing between bars shall not be less than $4d_b$.

Note that heads shall not be considered effective in developing bars in compression.

### 6.11 Splices of Reinforcement

Splicing of reinforcement bars is necessary, either because the available bars are not long enough, or to ease construction, in order to guarantee continuity of the reinforcement according to design requirements. It can be achieved by welding, mechanical connectors such as using screw threads and sleeves, or simply by lapping the reinforcement bars. A lap splice is formed by extending bars past each other for enough distance to allow the force in one bar to be transferred by bond stress through the concrete and into the second bar. Although lap splices are the simplest and most economical method of joining bars, they also have a number of disadvantages, including congestion of reinforcement at the lap splice and development of transverse cracks due to stress concentrations. It is recommended to locate splices at sections where stresses are low and to stagger the location of lap splices for individual bars. The two bars that form the lap splice may be in direct contact or spaced, as shown in Figure 6.9. Transverse reinforcement in the splice region delays the opening of the splitting cracks and hence improves the splice capacity.

![Figure 6.9](image.png)

**Figure 6.9:**

(a) Bars in direct contact; (b) Bars are spaced
6.11.1 Splices of Deformed Bars in Tension

Based on *ACI Code 12.14.2.3*, center-to-center distance between two bars in a lap splice cannot be greater than one-fifth of the splice length, nor 15 cm.

According to *ACI Code 12.15.2*, lap splices of deformed bars in tension are class B splices except that class A splices are allowed where: (a) the area of reinforcement provided is at least twice that required by analysis over the entire length of the splice, and (b) one-half or less of the total reinforcement is spliced within the required lap length.

*ACI Code 12.15.1* specifies that minimum length of lap for tension lap splices is as required for class A or B splice, but the 30 cm limit and the modification factor requirements are waived, where:

Class A splice  

\[ 1.00 \times l_d \]

Class B splice  

\[ 1.30 \times l_d \]

Where \( l_d \) is the tensile development length for the specified yield strength \( f_y \).

When bars of different size are lap spliced in tension, splice length shall be the larger of \( l_d \) of larger bar and tension lap splice length of smaller bar.

**Example (6.4):**

To facilitate construction of a retaining wall, the vertical reinforcement shown in Figure 6.10, is to be spliced to dowels extending from the foundation. If the flexural steel is stressed to its yield point at the bottom of the wall, determine the required splice length when all reinforcement bars are spliced at the same location.

Use \( f'_c = 300 \text{ kg/cm}^2 \) normal weight concrete, and \( f_y = 4200 \text{ kg/cm}^2 \).
Solution:

Class B splice is required where the splice length is taken as $1.3 l_d$.

$\psi_t = 1$, $\psi_e = 1$, $\psi_s = 0.8$, and $\lambda = 1$

$c_b = 7.5 + 0.80 = 8.3 \text{ cm}$

or

$c_b = 25/2 = 12.5 \text{ cm}$

i.e., $c_b$ is taken as the smaller of the two values, is equal to $8.3 \text{ cm}$.

$K_v = 0.0$, since no stirrups are used.

$$\frac{c_b + K_{tr}}{d_b} = \frac{8.3 + 0.0}{1.6} = 5.18 > 2.5$$

i.e. $\frac{c_b + K_{tr}}{d_b} = 2.5$

$$l_d = \frac{f_y \psi_t \psi_e \psi_s}{3.5 \lambda \left( \frac{c_b + K_{tr}}{d_b} \right) \sqrt{f'_c}}$$

$$d_b = \left( \frac{4200 (0.80)}{3.5 (2.5) \sqrt{300}} \right) (1.6) = 35.47 \text{ cm}$$
6.11.2 Splices of deformed Bars in Compression

Bond behavior of compression bars is not complicated by the problem of transverse tension cracking and thus compression splices do not require provisions as strict as those specified for tension splices. According to ACI Code 12.16.2, when bars of different size are lap spliced in compression, splice length shall be the larger of either development length of larger bar, or splice length of smaller bar. Based on ACI Code 12.16.1, the minimum splice length of deformed bars in compression is equal to $0.0073 f_y d_b$ but not less than 30 cm. The computed splice length should be increased by 33% if $f'_c$ is less than $210 \text{kg/cm}^2$.

6.11.3 Lap Splices in Columns

According to ACI 12.17.2.4, in tied reinforced compression members, where ties throughout the lap splice length have an effective area not less than $0.0015 hs$ in both directions, lap splice length is permitted to be multiplied by 0.83, but lap length shall not be less than 30 cm, where $h$ is the dimension of member perpendicular to the direction of the tie legs and $s$ is the spacing of ties. Furthermore ACI Code 12.17.2.5 specifies that in spirally reinforced compression members, lap splice length of bars within a spiral is permitted to be multiplied by 0.75, but lap length shall not be less than 30 cm. The specified reduction factors account for the increase in strength produced by the confinement of concrete.

Example (6.5):
Design a compression lap splice for a tied column whose cross section is shown in Figure 6.11, when:

a. $\phi 16 \text{ mm}$ bars are used on both sides of the splice.

b. $\phi 16 \text{ mm}$ bars are lap spliced with $\phi 18 \text{ mm}$ bars.

Use $f'_c = 300 \text{ kg/cm}^2$ and $f_y = 4200 \text{ kg/cm}^2$. 

Required splice length $l_s = 35.47 (1.3) = 46.11 \text{ cm}$, taken as 50 cm.
Solution:

a. For bars of similar diameter, lap spliced in compression, splice length is equal to

\[ 0.0073 \times (4200) (1.6) = 49.06 \text{ cm} \approx 50 \text{ cm} > 30 \text{ cm} \text{ O.K.} \]

b. For bars of different diameters, lap spliced in compression, splice length is the larger of either development length of the larger diameter bar, or splice length of smaller diameter bar.

The development length of the larger diameter bar is given by

\[ \frac{0.075 \times (4200)}{\sqrt{300}} \times (1.8) = 32.74 \text{ cm} \]

but not less than

\[ 0.0044 \times (1.8) \times (4200) = 33.26 \text{ cm} \]

Splice length of smaller diameter bar is evaluated in part (a) as 50 cm. Thus, the splice length is taken as 50 cm.

Check whether provided column ties qualify the lap splice length to be multiplied by 0.83.

Effective area of ties 2 (0.5) = 1.0 cm²

\[ 0.0015 \times h \times s = 0.0015 \times (40) \times (25) = 1.5 \text{ cm}^2 > 1.0 \text{ cm}^2 \]

i.e., the reduction factor does not apply, and the splice length will be kept unchanged in both parts of the problem.
**Example (6.6):**

In the isolated footing shown in Figure 6.5, 4 φ 16 mm bars are required to transfer the axial compression force in a column into the footing. Determine the minimum extensions of the dowels into the footing.

Use $f'_{c} = 250 \text{ kg/cm}^2$ normal weight concrete, and $f'_{y} = 4200 \text{ kg/cm}^2$.

**Solution:**

Extension of bars into footing is given by

$$\frac{0.075 \times 4200}{\sqrt{250}} \times (1.6) = 31.88 \text{ cm}$$

but not less than

$$0.0044 \times 1.6 \times 4200 = 29.57 \text{ cm}$$

Minimum extension of dowel bars into footing is equal to 31.88 cm $> 20$ cm.

Available length = 55 – 7.5 – 1.6 – 1.6 = 44.3 cm $> 31.88$ cm  O.K.

In this case, hooks are not effective in compression, but made to prevent pushing of the dowel bars into the soil below the footing.

**Splicing of 4 φ 16 mm bars:**

For bars of similar diameter, lap spliced in compression, splice length is equal to

$$\frac{0.0073 \times 4200 \times (1.6)}{1.6} = 49.06 \text{ cm} = 50 \text{ cm} > 30 \text{ cm} \text{ O.K.}$$

### 6.11.4 Practical Considerations

1. For lap splices of slab and wall reinforcement, effective clear spacing of bars being spliced at the same location is taken as the clear spacing between the spliced bars, as shown in Figure 6.12.a.

2. For lap splices of column and beam bars, effective clear spacing between bars being spliced will depend on the orientation of the lapped bars, as shown in Figure 6.12.b.
Figure 6.12: (a) Lap splice of slab and wall reinforcement; (b) lap splices of column and beam reinforcement

3. In Figure 6.13, two types of lap splices are illustrates. In (a), one bar is bent to lap with the other so that the centerlines of the continuing bars coincide. This type of splice is frequently used in columns. In (b), the bars are lap spliced out of line. This type of splice is frequently used in beams. However, when column sizes change at floor level, this type of splice is often used.
6.11.5 Welded, Mechanical, and Butt Splices:

In addition to lap splices, bars stressed in tension or compression may be spliced by welding, or by various mechanical devices, such as threaded sleeves. The use of such splices is governed by ACI Code 12.14.3 and ACI Code 12.16.3.

6.12 Bar Cutoffs And Development Of Flexural Reinforcement

Some of the flexural reinforcement bars can be cutoff where they are no longer needed to resist tensile forces or where the remaining bars are adequate to do so. In a continuous beam of constant cross section, if the areas of steel required at the sections of maximum moment are made continuous throughout each region of positive or negative moment, the beam will be over-designed at most sections. It is often desirable to terminate a portion of the steel when the moment decreases significantly. Reducing the area of reinforcement in regions of low bending moment in a concrete element lowers the cost of the element. Furthermore, for heavily reinforced elements, the reduction in a number of reinforcement bars improves concrete casting and compaction operations.

There must be sufficient extension of each bar, on each side of every critical moment section to develop the force in that bar at that section.

Tension bars, cutoff in a region of moderate shear force, cause a major stress concentration which can lead to major inclined cracks at the bar cutoff. Thus, bar cutoffs should be kept to a minimum, particularly, in zones of tension for ease of design and fabrication.
6.12.1 Development of Flexural Reinforcement - General

According to *ACI Code 12.10.2*, critical sections for development of reinforcement in flexural members are at points of maximum stress and at points within the span where adjacent reinforcement terminates, or is bent.

![Diagram of flexural reinforcement development](image)

Figure 6.14: (a) Location of theoretical cutoff points; (b) section A-A; (c) section B-B

To account for the possibility of higher than anticipated moment at cutoff point due to possible variations in the position of live load, settlements of support, lateral loads, or other causes, *ACI Code 12.10.3* requires the reinforcement to be extended beyond the point at which it is no longer required to resist flexure for a distance equal to the effective depth of the member \(d\) or \(12d_b\), whichever is greater, except at supports of simple spans and at free ends of cantilevers. When bars of different sizes are used, the extension should be in accordance with the diameter of bar being terminated. See Figure 6.14 for reinforcement layout of a simply supported beam.
Based on *ACI Code 12.10.4*, continuing reinforcement is to have an anchorage length not less than the development length $l_d$ beyond the point where bent or cutoff reinforcement is no longer required to resist flexure.

### 6.12.1.1 Development of Positive Moment Reinforcement

Failure of a beam will occur suddenly if the ends of the positive steel extending into a point of inflection are not properly anchored and slip out. Although the moment is zero at these points and the stress in the steel is low, the bond stresses are related to the shear which is maximum at a simple support and often high at a point of inflection.

As specified by *ACI Code 12.11.1*, at least one-third of the positive moment reinforcement in simply supported elements and one-fourth of the positive moment reinforcement in continuously supported elements shall be extended along the same face of member into the support. In beams such reinforcement is extended into the support at least 15 cm. Positive moment reinforcement is carried out into the support to provide for some shifting of the moments due to changes in loading, settlement of supports, and lateral loads.

### 6.12.1.2 Development Of Negative Moment Reinforcement

According to *ACI Code 12.12.1*, negative moment reinforcement in a continuous, restrained cantilever member, or in any member of rigid frame, is to be anchored in or through the supporting member by development length, hooks, or mechanical anchorage, as shown in Figure 6.15.

```
\begin{center}
\includegraphics[width=0.3\textwidth]{figure6.15.png}
\end{center}
```

**Figure 6.15: Anchorage into exterior column**

Based on *ACI Code 12.12.3*, at least one-third of the total tension reinforcement provided for negative moment at a support shall have a development length beyond the point of
inflection not less than the effective depth of member $d$, $12d_b$, or one-sixteenth the clear span, whichever is greatest to provide for possible shifting of the moment diagram at a point of inflection. Inflection point locations for a continuous beam are shown in Figure 6.16.

**Example (6.7):**

In the simply supported beam subjected to factored loads shown in Figure 6.17, 4 φ 22 mm bars are to be cutoff between the supports. Determine bar cutoff location and development requirements of the rest of the reinforcement bars, according to *ACI Code* provisions.

Use $f_c' = 250 \text{ kg/cm}^2$ normal weight concrete, and $f_y = 4200 \text{ kg/cm}^2$ and width of support is equal to 0.3 m.

**Solution:**

For section A-A:

\[ d = 70 - 4 - 0.8 - 2.2 - 2.5/2 = 61.75 \text{ cm} \]

For section B-B:
\[ d = 70 - 4 - 0.8 - 2.2/2 = 64.10 \text{ cm} \]

Moment capacity of section B-B, reinforced with 4 \( \phi 22 \text{ mm} \)

\[ \begin{align*}
M_{u} &= \frac{0.9 (15.2)(4200)}{10^5} \left[ 64.1 - \frac{15.2 (4200)}{1.7 (250)(30)} \right] = 33.95 \text{ t.m} 
\end{align*} \]
Theoretical cutoff points of $4 \phi 22 \text{ mm}$ are located at distance $x$ from the centerline of the left support, evaluated by equating the bending moment at distance $x$ to the moment capacity of the section, or

$$33.95 = 29 x - 1.5 x^2$$

or,

$$1.5 x^2 - 29 x + 33.95 = 0$$

Solving this quadratic equation in terms of $x$ gives

$$x = \frac{29 \pm \sqrt{(29)^2 - 4(1.5)(33.95)}}{2(1.5)}$$

$$x = \frac{29 \pm 25.24}{2(1.5)} \text{ and } x = 1.25 \text{ m}, \text{ or } x = 18.08 \text{ m (rejected)}$$

**ACI Code Requirements:**

1. Bars must be extended at least a distance equal to the larger of the effective depth $d = 61.75 \text{ cm}$, and $12 d_b = 12(2.2) = 26.4 \text{ cm}$, thus extension on both sides of the centerline of the beam is taken as $62 \text{ cm}$.

Length of cutoff bars = $2 (175 + 62) = 474 \text{ cm}$

2. Distance from point of maximum stress to end of cutoff bars on each side should be equal or larger than the development length of the bars in tension, $l_d$

For bars in tension

$$\psi_t = 1, \psi_e = 1, \psi_s = 1, \text{ and } \lambda = 1$$

$$c_b = 4.0 + 0.8 + 1.1 = 5.9 \text{ cm}$$

or

$$c_b = [30 - 2(4) - 2(0.8) - 2.2]/6 = 3.03 \text{ cm}$$

i.e., $c_b$ is taken as the smaller of the two values, is equal to $3.03 \text{ cm}$.

$$K_{tr} = \frac{40 A_{tr}}{sn} = \frac{40(2)(0.5)}{(20)(4)} = 0.5 \text{ cm}$$

$$\frac{c_b + K_{tr}}{d_b} = \frac{3.03 + 0.5}{2.2} = 1.6 < 2.5 \text{ O.K.}$$
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\[ l_d = \left( \frac{f_y \psi_t \psi_r \psi_s}{3.5 \lambda \left( \frac{c_b + K_{tr}}{d_b} \right) \sqrt{f'_c}} \right) \]

\[ d_b = \left( \frac{4200 (1.0)}{3.5 (1.6 \sqrt{250}) (2.2)} \right) = 104.36 \text{ cm} \]

Available development length = \( \frac{474}{2} = 237 \text{ cm} > 104.36 \text{ cm} \).

3. At least one-third of the positive moment reinforcement is extended 15 cm into the supports:

One-half of the total positive moment reinforcement is to be extended 15 cm into the supports. Thus, minimum length of these bars = 600 – 30 + 15 + 15 = 600 cm.

6.13 Standard Bend and Cutoff Points

Approximate bend and cutoff points can be used in continuous beams or one-way slabs where the following conditions are satisfied:

- Not more than half the reinforcement is bent or cutoff.
- Two or more spans.
- Spans are approximately equal with the larger of two adjacent spans not greater than the shorter by more than 20%.
- Loads are uniformly distributed.
- Unit live load does not exceed 3 times unit dead load.

The ACI Detailing Manual shows the following bend and cutoff points for continuous beams and one-way slabs reproduced in Figure 6.18.
Figure 6.18: (a) Beam (cut-off bars); (b) beam (bent-up bars); (c) one way slab (cut-off bars); (d) one way slab (bent-up bars)
Problems

P6.1 Design the reinforcement for the beam shown in Figure P6.1, using cutoff bars.

Use $f'_c = 350 \text{ kg} / \text{cm}^2$ and $f_y = 4200 \text{ kg} / \text{cm}^2$.

![Figure P6.1](image1)

P6.2 Design top reinforcement for the cantilever resisting the factored load shown in Figure P6.2, to satisfy bar anchorage requirements.

Use $f'_c = 280 \text{ kg} / \text{cm}^2$ and $f_y = 4200 \text{ kg} / \text{cm}^2$.

![Figure P6.2](image2)

P6.3 For the cantilever shown in Figure P6.3, develop the top reinforcement to satisfy code anchorage requirements.

Use $f'_c = 280 \text{ kg} / \text{cm}^2$ and $f_y = 4200 \text{ kg} / \text{cm}^2$.
Figure P6.3

**P6.4** For the beam shown in Figure P6.4, determine bar lengths $L_1$, $L_2$ and $L_3$, to satisfy code anchorage requirements.

Use $f'_c = 300 \text{ kg/cm}^2$ and $f_y = 4200 \text{ kg/cm}^2$.

Figure P6.4

**P6.5** For the beam shown in Figure P6.5, determine bar cutoff locations to satisfy code anchorage requirements.

Use $f'_c = 300 \text{ kg/cm}^2$ and $f_y = 4200 \text{ kg/cm}^2$. 
For the beam carrying the factored load shown in Figure P6.15.6, determine bar bend locations to satisfy code anchorage requirements, assuming that half the bottom reinforcement to be bent up.

Use $f'_{c} = 250 \text{ kg/cm}^2$ and $f_y = 4200 \text{ kg/cm}^2$.

For the beam carrying the factored loads shown in Figure P6.15.7, determine bar cutoff locations to satisfy code anchorage requirements.

Use $f'_{c} = 300 \text{ kg/cm}^2$ and $f_y = 4200 \text{ kg/cm}^2$. 

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**Figure P6.5**

**Figure P6.6**

**Figure P6.7**
Figure P6.7