Discussion of chapter 10) Compressibility of soil

1- Types of settlement:
1-1 Elastic (immediate) settlement (Se).
1-2 Primary consolidation settlement (Sc).
1-3 Secondary consolidation settlement (Sp).

• Calculation of elastic settlement:

\[ S_e = \Delta \sigma B \left( 1 - \mu_s^2 \right) I_p \]

\( \Delta \sigma \): Applied pressure = Load / area.
B: Width of the foundation

\( \mu_s \): Poisson’s ratio.
\( E_s \): Modulus of elasticity of soil.
\( I_p \): Influence factor given from table 10.1 P.262

Depend on the value \( m_1 = \text{Length of foundation} \) / \( \text{Width of foundation} \).
Problem 1) 10.1 Text book

Find the immediate settlement of rigid column footing 4.5 ft in diameter and carries 20 tons. \( E_s = 1500 \text{psi} \) and \( \mu_s = 0.25 \)

**Solution**

\[
S_e = \Delta \sigma B \frac{1 - \mu_s^2}{E_s} I_p
\]

\[
\Delta \sigma = \frac{P}{A} = \frac{20 \times 2000}{0.25 \pi \times 4.5^2} = 2515.04 \text{ psf}
\]

From table 10.1 P.262 \( I_p = 0.79 \)

\[
S_e = 2515.04 \times 4.5 \times \frac{1 - 0.25^2}{1500 / 12^2} \times 0.79 = 0.038 \text{ ft} = 0.465 \text{ inch}
\]

1 Metric ton = 2204.6 lb
1 ton or US ton = 2000 lb
Calculation of primary consolidation settlement:

In general,

\[ S_c = H \frac{\Delta e}{1 + e_o} \]

- For normal consolidated clay:

\[ S_c = C_c H \log \left( \frac{\sigma'_o + \Delta \sigma'}{\sigma'_o} \right) \]

\[ C_c : \text{Compressibility index.} \]

\[ \sigma'_o : \text{Present effective pressure at the middle of the layer under consideration.} \]

\[ \Delta \sigma' : \text{Increase in stress at the mid of the layer.} \]

- For over consolidated clay:

\[ \Delta \sigma = \frac{\Delta \sigma'_{\text{Top}} + 4 \Delta \sigma'_{\text{Mid}} + \Delta \sigma'_{\text{Bottom}}}{6} \]

\[ S_c = C_s H \frac{\sigma'_o + \Delta \sigma'}{1 + e_o} \log \left( \frac{\sigma'_o + \Delta \sigma'}{\sigma'_o} \right) \]

\[ C_s : \text{Swell index or called recompression index (Cr).} \]

\[ \sigma'_c : \text{Pre-consolidation pressure.} \]

\[ OCR = \frac{\sigma'_c}{\sigma'_o} \]

The most famous relation for \( C_c \) is:

\[ C_c = 0.009 (\text{LL} - 10) \]

\[ C_s = \left( \frac{1}{5} \rightarrow \frac{1}{10} \right) C_c \]
Problem 2) 10.8 text book

A soil profile as shown, the pre-consolidation of the clay is 3400 psf. Estimate the primary consolidation settlement that will take place as the result of surcharge equal 2200 psf.

Assume $C_s = \frac{1}{5} C_c$.

**Solution**

- Find the unit weights of the soil layers:
  - For dry sand:
    \[ \gamma_{\text{dry}} = \frac{G_i \gamma_w}{1 + e} = \frac{2.65 \times 62.4}{1 + 0.6} = 103.35 \text{pcf} \]
  - For saturated sand:
    \[ \gamma_{\text{sat}} = \frac{(G_i + e) \gamma_w}{1 + e} = \frac{(2.65 + 0.6) \times 62.4}{1 + 0.6} = 126.75 \text{pcf} \]
  - For saturated clay:
    \[ e = 2.7 \times 0.3 = 0.81 \]
    \[ \gamma_{\text{sat}} = \frac{(G_i + e) \gamma_w}{1 + e} = \frac{(2.65 + 0.81) \times 62.4}{1 + 0.81} = 121 \text{pcf} \]

- Find the present effective pressure:
  \[ \sigma'_o = 10 \times 103.35 + 10 \times (126.75 - 62.4) + \frac{10}{2} \times (121 - 62.4) = 1970 \text{ psf} \]
  \[ \sigma'_o + \Delta \sigma' = 1970 + 2200 = 4170 \text{ psf} \]
  \[ \sigma'_c = 3400 \text{ psf} \]
  \[ \sigma'_o (1970) \leq \sigma'_c (3400) \leq \sigma'_o + \Delta \sigma' (4170) \]

- Find $C_c$ and $C_s$:
  \[ C_c = 0.009 \times (35 - 10) = 0.225 \]
  \[ C_s = C_c / 5 = 0.225 / 5 = 0.045 \]
  \[ S_c = \frac{C_s H}{1 + e_o} \log \left( \frac{\sigma'_c}{\sigma'_o} \right) + \frac{C_c H}{1 + e_o} \log \left( \frac{\sigma'_o + \Delta \sigma'}{\sigma'_c} \right) \]
  \[ S_c = 0.045 \times 10 \log \left( \frac{3400}{1970} \right) + 0.225 \times 10 \log \left( \frac{4170}{3400} \right) = 0.169 \text{ ft} = 2.03 \text{ in} \]
Problem 2)
Calculate the final voids ratio of a thin clay layer when the vertical stress increases by 65 kPa if the initial void ratio is 1.11, the initial vertical effective stress is 50 kPa, OCR = 2, and the compression and recompression indexes are $C_c = 0.4$ and $C_r = 0.08$ respectively.

**Solution**

\[ S_c = H \frac{\Delta e}{1 + e_o} = H \frac{\Delta e}{1 + 1.11} = H \frac{\Delta e}{2.11} \]

\[ OCR = \frac{\sigma_c}{\sigma} \rightarrow 2 = \frac{\sigma_c}{50} \Rightarrow \sigma_c = 100 \text{kPa} \]

\[ \sigma_o + \Delta \sigma = 50 + 65 = 115 \text{kPa} \]

\[ \sigma_o = 50 \text{kPa} \]

\[ : \sigma_o (50) \leq \sigma_c (100) \leq \sigma_o + \Delta \sigma (115) \]

\[ S_c = \frac{C_s H}{1 + e_o} \log \left( \frac{\sigma_c}{\sigma_o} \right) + \frac{C_c H}{1 + e_o} \log \left( \frac{\sigma_o + \Delta \sigma}{\sigma_c} \right) \]

\[ S_c = \frac{0.08 \times H}{1 + 1.11} \log \left( \frac{100}{50} \right) + \frac{0.4 \times H}{1 + 1.11} \log \left( \frac{115}{100} \right) = 0.0229 H \]

\[ \Rightarrow 0.0229 H = H \frac{\Delta e}{2.11} \Rightarrow \Delta e = 2.11 \times 0.0229 = 0.04836 \rightarrow 1.11 - e_2 = 0.04836 \]

\[ \rightarrow e_2 = 1.11 - 0.04836 \Rightarrow e_2 = 1.0616 \]
Problem 3) 10.21 text book

For the shown rectangular footing, calculate the primary consolidation pressure given that:
B = 1.5 m
L = 2.5 m
Q = 120kN.

Solution

\[ q = \frac{120}{1.5 \times 2.5} = 32 \text{kpa}. \]

\[ \Delta \sigma = qI_4 \]

\[ \Delta \sigma = \frac{\Delta \sigma_{\text{Top}} + 4\Delta \sigma_{\text{mid}} + \Delta \sigma_{\text{bottom}}}{6} \]

<table>
<thead>
<tr>
<th>Layer</th>
<th>Z(m)</th>
<th>m1(L/B)</th>
<th>n1(2Z/B)</th>
<th>I</th>
<th>( \Delta \sigma ) (kpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>1.5</td>
<td>1.67</td>
<td>2</td>
<td>0.4058</td>
<td>13</td>
</tr>
<tr>
<td>Middle</td>
<td>2.75</td>
<td>1.67</td>
<td>3.67</td>
<td>0.1925</td>
<td>6.16</td>
</tr>
<tr>
<td>Bottom</td>
<td>4</td>
<td>1.67</td>
<td>5.33</td>
<td>0.09</td>
<td>2.88</td>
</tr>
</tbody>
</table>

\[ \Delta \sigma = \frac{13 + 4 \times 6.16 + 2.88}{6} = 18.38 \text{Kpa} \]
For clay layer, \( e = G_s w = 2.7 \times 0.35 = 0.945 \)

\[
\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1 + e} = \frac{(2.7 + 0.945) \times 9.81}{1 + 0.945} = 18.38 \text{ kN} / \text{m}^3.
\]

\(\sigma_o^* = 1.5 \times 15 + 1.5(18 - 9.81) + \frac{2.5}{2}(18.38 - 9.81) = 45.5 \text{ Kpa} \)

\( C_c = 0.009 \times (38 - 10) = 0.252 \)

It is normal consolidated clay:

\[
S_c = \frac{C_c H}{1 + e_o} \log \left( \frac{\sigma_o^* + \Delta\sigma^*}{\sigma_o^*} \right)
\]

\[
S_c = \frac{0.252 \times 2.5 \times \log \left( \frac{45.4 + 6.75}{45.5} \right)}{1 + 0.945} = 0.02m = 20mm
\]
• Time rate of consolidation:

\[ C_v = \frac{T_v H_{dr}^2}{t} \]

t: The required time to reach a specific degree of consolidation.
H_{dr}: Height of drainage.
C_v: Coefficient of consolidation.

To find C_v:

\[ C_v = \frac{k}{\gamma_v m_v} \]

K: Permeability coefficient.
m_v: Coefficient of volume compressibility.

\[ m_v = \frac{e_0 - e_1}{\sigma_1 - \sigma_0} \frac{1 + e_{avg}}{e_{avg}} \]

To find T_v: Look Table 10.5 P293

Or from equations:

\[ T_v = \left\{ \begin{array}{l}
\frac{\pi}{4} \left( \frac{U\%}{100} \right)^2 \rightarrow U = 0 \rightarrow 60\% \\
1.781 - 0.933 \log(100-U\%) \rightarrow U > 60\%
\end{array} \right. \]

<table>
<thead>
<tr>
<th>Table 10.6 Variation of T_v with U</th>
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</thead>
<tbody>
<tr>
<td>U (%)</td>
</tr>
<tr>
<td>-------</td>
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<td>24</td>
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<td>25</td>
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</tbody>
</table>
Problem 4) 10.10+10.11+10.12 Text book

The coordinates of two points in the virgin compression curve are as follow:

\[ e_1 = 1.82 \rightarrow \sigma_1 = 200kN/m^2 \]
\[ e_2 = 1.54 \rightarrow \sigma_1 = 400kN/m^2 \]

1- Determine the coefficient of volume compressibility for the pressure range stated above.

2- Given that \( C_v = 0.003 \text{ cm}^2/\text{sec} \), Determine (k) in cm/sec corresponding to average void ratio.

3- What would be the effective pressure \( \sigma' \) corresponding to \( e=1.7 \)

4- What would be the void ratio corresponding to effective pressure \( \sigma' = 500kN/m^2 \).

**Solution**

1- Find \( m_v \):

\[
\frac{e_1 - e_2}{\sigma_2 - \sigma_1} = \frac{1.82 - 1.54}{400 - 200} = \frac{1.82 + 1.54}{1 + e_{avg}} = 5.2238 \times 10^{-4} m^2 / kN
\]

2- Find K:

\[
C_v = \frac{k}{\gamma_m m_v}
\]

\[ 0.003 \times 10^{-4} = \frac{k}{9.81 \times 5.2238 \times 10^{-4}} \]

\[ \Rightarrow K = 1.537 \times 10^{-9} m/ \text{sec} = 1.537 \times 10^{-7} cm/ \text{sec} \]

3- Find \( \sigma' \) corresponding to \( e=1.7 \):

Slope of the virgin line:

\[ \frac{e_1 - e_2}{\log \sigma_2 - \log \sigma_1} = \frac{1.82 - 1.54}{\log 400 - \log 200} = 0.93 \]

At \( e = 1.7 \):

\[ \frac{1.82 - 1.7}{\log \sigma_3 - \log 200} \Rightarrow \sigma_3 = 269.19 kpa. \]

4- Find e corresponds to \( \sigma' = 500kN/m^2 \):

\[ \frac{1.82 - e_3}{\log 500 - \log 200} \Rightarrow e_3 = 1.45 \]
Problem 5) 10.17 Text book

For normally consolidated clay, the following results were obtained:
\[ \sigma'_o = 2tsf \rightarrow e = e'_o = 1.21 \]
\[ \sigma'_o + \Delta \sigma'_o = 4tsf \rightarrow e = 0.96 \]
\[ k = 1.8 \times 10^{-4} \text{ ft/day} \]

1- Hoe long will it take in days for 9 ft thick clay (Drained in one side) to reach 60% consolidation.
2- What is the settlement at that time.

Solution

1- Find \( t_{60} \):
\[ \frac{e_1 - e_2}{\sigma_2 - \sigma_1} = \frac{1.21 - 0.96}{4 - 2} = 0.06 \text{ ft}^2 / \text{ton} = 0.06 / 2000 = 3 \times 10^{-5} \text{ ft}^2 / \text{lb} \]
\[ m_v = \frac{\sigma_2 - \sigma_1}{1 + e_{avg}} = \frac{1.21 + 0.96}{2} = 0.635 \text{ ft}^2 / \text{in} \]
\[ c_v = \frac{k}{\gamma_w m_v} = \frac{1.8 \times 10^{-4}}{62.4 \times 3 \times 10^{-5}} = 0.096 \text{ ft}^2 / \text{day} \]
\[ U = 60\% \rightarrow T_{60} = 0.286 \]
\[ c_v = \frac{T_v H_{dr}^2}{t_{60}} \rightarrow 0.096 = \frac{0.286 \times 9^2}{t_{60}} \Rightarrow t_{60} = 240.8 \text{ days}. \]

2- \( S_c \) =? At 60% rate of consolidation:
\[ C_c = \frac{1.21 - 0.96}{\log 4 - \log 2} = 0.8304 \]
\[ S_c = \frac{C_c H}{1 + e'_o} \log \left( \frac{\sigma'_o + \Delta \sigma'_o}{\sigma'_o} \right) \]
\[ S_c = \frac{0.8304 \times 9}{1 + 1.21} \log \left( \frac{4}{2} \right) = 1.018 \text{ ft} \]
\[ U = 60\% \rightarrow S_c = 0.6 \times 1.018 = 0.61 \text{ ft} = 7.33 \text{ in} \]
Problem 6) 10.19 text book:

- For laboratory sample:
  
  H = 25mm (One way drainage).
  
  $t_{50} = 11$ min.

- For field:
  
  H = 4m (Two way drainage):
  
  In the field, find $t_{50}$ and $t_{90}$.

\[ C_v = \frac{T_s H^2}{dr} \]

**Solution**

At the lab:

\[ U = 50\% \rightarrow T_{50} = 0.197 \]

\[ C_v = \frac{0.197 \times 0.025^2}{11} = 1.119 \times 10^{-5} \text{ } \text{m}^2/\text{min} \]

At the field:

\[ 1.119 \times 10^{-5} = \frac{0.197 \times 2^2}{t_{50}} \Rightarrow t_{50} = 70420 \text{ min} = 49\text{days} \]

\[ 1.119 \times 10^{-5} = \frac{0.403 \times 2^2}{t_{70}} \Rightarrow t_{70} = 144057 \text{ min} = 100\text{days} \]
Problem 7)
A 3m thick clay layer is able to drain from both its upper and lower surface and after 3 years a settlement of 32 mm is observed. After many years a settlement of 40 mm is measured. If a specimen of this soil, 20 mm thick, was to be tested in an oedometer apparatus with two-way drainage what would be the time for 50% consolidation.

Solution

At 100 % consolidation → Sc = 40mm
At U %? Consolidation → Sc = 32 mm

\[ U = \frac{32 \times 100}{40} = 80\% \]

\[ C_v = \frac{T_v H_{dr}^2}{t} \]

\[ U = 80\% \rightarrow T_{v0} = 0.567 \]

\[ H_{dr} = 3/2 = 1.5m \]

\[ t = 3\ years = 1095\ days \]

\[ C_v = \frac{0.567 \times 1.5^2}{1095} = 1.165 \times 10^{-3} \text{ m}^2 / \text{days} \]

In the lab.: \[ C_v = \frac{T_v H_{dr}^2}{t} \]

\[ 1.165 \times 10^{-3} = \frac{0.197 \times 0.01^2}{t_{50}} = 0.0169\ days = 24.34\ min \]
Problem 8)
An oil tank, 20 m in diameter, is to be built on a site where the soil profile consists of 4 m of dense gravel overlying 3 m of clay overlying rigid impermeable rock, and the water table is 1 m below the soil surface. If the tank applies a uniform pressure of 100 kPa to the surface of the soil calculate the settlement in the clay layer after 1 year and after many years. For the gravel $\gamma_{\text{dry}} = 20 \text{ kN/m}^3$ and $\gamma_{\text{sat}} = 22 \text{ kN/m}^3$, and for the clay the properties are:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moisture content at centre of layer</td>
<td>50%</td>
</tr>
<tr>
<td>Specific Gravity, $G_s$</td>
<td>2.6</td>
</tr>
<tr>
<td>Pre-consolidation pressure, $\sigma_{pc}$</td>
<td>120 kPa</td>
</tr>
<tr>
<td>Compression Index, $C_c$</td>
<td>0.2</td>
</tr>
<tr>
<td>Re-compression Index, $C_r$</td>
<td>0.04</td>
</tr>
<tr>
<td>Coefficient of consolidation</td>
<td>$1 \text{ m}^2/\text{yr}$</td>
</tr>
</tbody>
</table>