**ANTENNA RADIATION**

Antennas radiate spherical waves that propagate in the radial direction for a coordinate system centered on the antenna. At large distances, spherical waves can be approximated by plane waves. Plane waves are useful because they simplify the problem.

They are not physical, however, because they require infinite power. The Poynting vector describes both the direction of propagation and the power density of the electromagnetic wave. It is found from the vector cross product of the electric and magnetic fields and is denoted \( S \):

\[
S = E \times H \quad \text{W/m}
\]

\[
|S| = S = \frac{|E|^2}{\eta} \quad \text{W/m}^2
\]

\[
\frac{|E|}{|H|} = \eta
\]

**Fundamental Antenna Parameters**

Describe the antenna performance with respect to space distribution of the radiated energy, power efficiency, matching to the feed circuitry, etc. Many of these parameters are interrelated.

- Radiation pattern.
- Pattern beamwidths.
- Radiation intensity.
- Directivity, Gain.
- Antenna efficiency and radiation efficiency.
- Frequency bandwidth.
- Input impedance and radiation resistance.
- Antenna effective area.
- Relationship between directivity and antenna effective area
Radiation pattern

- The radiation pattern of an antenna is a representation (pictorial or mathematical) of the distribution of the power radiated from the antenna as a function of direction angles from the antenna.
- Antenna radiation pattern (antenna pattern) is defined for large distances from the antenna, where the spatial (angular) distribution of the radiated power does not depend on the distance from the radiation source (in the far field).

Normalized pattern:

Often the field and power patterns are normalized with respect to their maximum value, yielding normalized field and power patterns. Also, the power pattern is usually plotted on a logarithmic scale or more commonly in decibels (dB). This scale is usually desirable because a logarithmic scale can accentuate in more details those parts of the pattern that have very low values, which later we will refer to as minor lobes. For an antenna, the

a) field pattern (in linear scale) typically represents a plot of the magnitude of the electric or magnetic field as a function of the angular space.
b) power pattern (in linear scale) typically represents a plot of the square of the magnitude of the electric or magnetic field as a function of the angular space.
c) power pattern (in dB) represents the magnitude of the electric or magnetic field, in decibels, as a function of the angular space.

*When the patterns are plotted on a linear scale, the field pattern and power pattern may look very different. However, when the patterns are plotted on a logarithmic scale (dB plot), both the normalized field and power patterns are the same since $10 \log(P/P_{\text{max}})$ is the same as $20 \log(E/E_{\text{max}})$. Thus, in practice, we often plot the patterns in dB scale, which also makes it easy to see details of the field or power over a large dynamic range, especially some minor side lobes.*

Radiation Pattern Lobes

Various parts of a radiation pattern are referred to as *lobes*, which may be subclassified

into major or main, minor, side, and back lobes
For an amplitude pattern of an antenna, there would be, in general, three electric-field components \((E_r, E_\theta, E_\phi)\) at each observation point on the surface of a sphere of constant radius \(r = r_c\). In the far field, the radial \(E_r\) component for all antennas is zero or vanishingly small compared to either one, or both, of the other two components. Some antennas, depending on their geometry and also observation distance, may have only one, two, or all three components. In general, the magnitude of the total electric field would be

\[
|E| = \sqrt{|E_r|^2 + |E_\theta|^2 + |E_\phi|^2}
\]

### 2.2.2 Isotropic, Directional, and Omnidirectional Patterns

- **An isotropic radiator** is defined as “a hypothetical lossless antenna having equal radiation in all directions.” Although it is ideal and not physically realizable, it is often taken as a reference for expressing the directive properties of actual antennas.

- **A directional antenna** is one “having the property of radiating or receiving electromagnetic waves more effectively in some directions than in others. This term is usually applied to an antenna whose maximum directivity is significantly greater than that of a half-wave dipole.”

- **omnidirectional**, and it is defined as one “having an essentially non-directional pattern in a given plane and a directional pattern in any orthogonal plane.” An omnidirectional pattern is then a special type of a directional pattern.
Figure 2.5 Principal E- and H-plane patterns for a pyramidal horn antenna.

Figure 2.6 Omnidirectional antenna pattern.
2.2.4 Field Regions

![Diagram of field regions](image)

Figure 2.7 Field regions of an antenna.

2.2.5 Radian and Steradian

The measure of a plane angle is a radian. One radian is defined as the plane angle with its vertex at the center of a circle of radius \( r \) that is subtended by an arc whose length is \( r \). A graphical illustration is shown in Figure 2.10(a). Since the circumference of a circle of radius \( r \) is \( C = 2\pi r \), there are \( 2\pi \) rad \( (2\pi r/r) \) in a full circle.

The measure of a solid angle is a steradian. One steradian is defined as the solid angle with its vertex at the center of a sphere of radius \( r \) that is subtended by a spherical surface area equal to that of a square with each side of length \( r \). A graphical illustration is shown in Figure 2.10(b). Since the area of a sphere of radius \( r \) is \( A = 4\pi r^2 \), there are \( 4\pi \) sr \( (4\pi r^2/r^2) \) in a closed sphere.

\[
R_1 = 0.62 \sqrt{D/\lambda},
\]
\[
R_2 = 2D^2/\lambda.
\]
2.3 RADIATION POWER DENSITY

The quantity used to describe the power associated with an electromagnetic wave is the instantaneous Poynting vector defined as
\[ W = \mathbf{E} \times \mathbf{H} \]

\[ W = \text{instantaneous Poynting vector} \quad (W/m^2) \]
\[ \mathbf{E} = \text{instantaneous electric-field intensity} \quad (V/m) \]
\[ \mathbf{H} = \text{instantaneous magnetic-field intensity} \quad (A/m) \]

Since the Poynting vector is a power density, the total power crossing a closed surface can be obtained by integrating the normal component of the Poynting vector over the entire surface. In equation form

\[
\mathcal{P} = \oint_S \mathbf{W} \cdot d\mathbf{s} = \oint_S \mathbf{W} \cdot \mathbf{n} \, da
\]

\[ \mathcal{P} = \text{instantaneous total power} \quad (W) \]
\[ \mathbf{n} = \text{unit vector normal to the surface} \]
\[ da = \text{infinitesimal area of the closed surface} \quad (m^2) \]

For applications of time-varying fields, it is often more desirable to find the average power density which is obtained by integrating the instantaneous Poynting vector over one period and dividing by the period.

\[
W_{av}(x, y, z) = \left[ \mathcal{W}(x, y, z; t) \right]_{av} = \frac{1}{2} \text{Re}[\mathbf{E} \times \mathbf{H}^*] \quad (W/m^2)
\]

If the real part of \((\mathbf{E} \times \mathbf{H}^*)/2\) represents the average (real) power density, what does the imaginary part of the same quantity represent? At this point it will be very natural to assume that the imaginary part must represent the reactive (stored) power density associated with the electromagnetic fields. In later chapters, it will be shown that the power density associated with the electromagnetic fields of an antenna in its far-field region is predominately real and will be referred to as radiation density.

the average power radiated by an antenna (radiated power) can be written as
2.4 RADIATION INTENSITY

Radiation intensity in a given direction is defined as “the power radiated from an antenna per unit solid angle.” The radiation intensity is a far-field parameter, and it can be obtained by simply multiplying the radiation density by the square of the distance. Since in a radiated wave is proportional to $1/R^2$. It is convenient to define radiation intensity to remove the $1/R^2$ dependence: In mathematical form it is expressed as

$$U = r^2 W_{\text{rad}}$$

where

$U$=radiation intensity (W/unit solid angle)

$W_{\text{rad}}$ =radiation density (W/m²)

*Radiation intensity depends only on the direction of radiation and remains the same at all distances.* A probe antenna measures the relative radiation intensity (pattern) by moving in a circle (constant $R$) around the antenna.

$$P_{\text{rad}} = \iiint_{\Omega} U \, d\Omega = \int_0^{2\pi} \int_0^{\pi} U \, \sin \theta \, d\theta \, d\phi$$

where $d\Omega = \text{element of solid angle} = \sin \theta \, d\theta \, d\phi$.

For anisotropic source $U$ will be independent of the angles $\theta$ and $\varphi$, as was the case for $W_{\text{rad}}$. 
The radiation intensity is also related to the far-zone electric field of an antenna,

\[ W_o = \frac{P}{4\pi r^2} \left( \frac{W}{m^2} \right) \]

\[ U_o = \frac{P}{4\pi} \left( \frac{W}{Sr} \right) \]

The radiation intensity is also related to the far-zone electric field of an antenna,

\[ U(\theta, \phi) \approx \frac{1}{2\eta} \left[ |E_\theta(\theta, \phi)|^2 + |E_\phi(\theta, \phi)|^2 \right] \]

\( E_\theta, E_\phi \) = far-zone electric-field components of the antenna

\( \eta \) = intrinsic impedance of the medium

The radial electric-field component \( (E_r) \) is assumed, if present, to be small in the far zone.

The total power is obtained by integrating the radiation intensity, as given by over the entire solid angle of \( 4\pi \). Thus

\[ P_{rad} = \int_U U \, d\Omega = \int_0^{2\pi} \int_0^\pi U \sin \theta \, d\theta \, d\phi \]

where \( d\Omega = \text{element of solid angle} = \sin \theta \, d\theta \, d\phi \).

**Beamwidth, BW**

**Half-power beamwidth (HPBW)** also called the 3dB beam width or just the beam width (to identify how sharp the beam is) is the angle between two vectors from the pattern’s origin to the points of the major lobe where the radiation intensity is half its maximum
**First-null beamwidth (FNBW)** is the angle between two vectors, originating at the pattern’s origin and tangent to the main beam at its base.

Often \( \text{FNBW} \approx 2\times \text{HPBW} \)

---

**Figure 4.8**  A radiation pattern illustrated in a conventional 2D plot
2.6 DIRECTIVITY

Every real antenna radiates more energy in some directions than in others (i.e. has directional properties. Therefore directivity of an antenna defined as “the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions. The average radiation intensity is equal to the total power radiated by the antenna divided by 4π.

\[
D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}}
\]

If the direction is not specified, it implies the direction of maximum radiation intensity (maximum directivity) expressed as

\[
D_{\text{max}} = D_0 = \frac{U_{\text{max}}}{U_0} = \frac{U_{\text{max}}}{U_0} = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}}
\]

D=directivity (dimensionless)
D0 =maximum directivity (dimensionless)
U=radiation intensity (W/unit solid angle)
Umax=maximum radiation intensity (W/unit solid angle)
U0 =radiation intensity of isotropic source (W/unit solid angle)
Prad =total radiated power (W)

The directivity of an isotropic source is unity since its power is radiated equally well in all directions. For all other sources, the maximum directivity will always be greater than unity, and it is a relative “figure of merit” which gives an indication of the directional properties of the antenna as compared with those of an isotropic source. The directivity can be smaller than unity; in fact it can be equal to zero. The values of directivity will be equal to or greater than zero and equal to or less than the maximum directivity (0≤D≤D0)
Let the radiation intensity of an antenna be of the form

\[ U = B_0 F(\theta, \phi) \simeq \frac{1}{2\eta} \left( |E^0_\theta(\theta, \phi)|^2 + |E^0_\phi(\theta, \phi)|^2 \right) \quad (2-19) \]

where \( B_0 \) is a constant, and \( E^0_\theta \) and \( E^0_\phi \) are the antenna’s far-zone electric-field components. The maximum value of (2-19) is given by

\[ U_{\text{max}} = B_0 F(\theta, \phi)|_{\text{max}} = B_0 F_{\text{max}}(\theta, \phi) \quad (2-19a) \]

The total radiated power is found using

\[ P_{\text{rad}} = \iiint_{\Omega} U(\theta, \phi) d\Omega = B_0 \int_0^{2\pi} \int_0^\pi F(\theta, \phi) \sin \theta \, d\theta \, d\phi \quad (2-20) \]

We now write the general expression for the directivity and maximum directivity using (2-16) and (2-16a), respectively, as

\[ D(\theta, \phi) = 4\pi \frac{F(\theta, \phi)}{\int_0^{2\pi} \int_0^\pi F(\theta, \phi) \sin \theta \, d\theta \, d\phi} \quad (2-21) \]

\[ D_0 = 4\pi \frac{F(\theta, \phi)|_{\text{max}}}{\int_0^{2\pi} \int_0^\pi F(\theta, \phi) \sin \theta \, d\theta \, d\phi} \quad (2-22) \]

Equation (2-22) can also be written as

\[ D_0 = \frac{4\pi}{\left[ \int_0^{2\pi} \int_0^\pi F(\theta, \phi) \sin \theta \, d\theta \, d\phi \right] / \left( F(\theta, \phi)|_{\text{max}} \right)} = \frac{4\pi}{\Omega_A} \quad (2-23) \]

where \( \Omega_A \) is the beam solid angle, and it is given by

\[ \Omega_A = \frac{1}{F(\theta, \phi)|_{\text{max}}} \int_0^{2\pi} \int_0^\pi F(\theta, \phi) \sin \theta \, d\theta \, d\phi = \int_0^{2\pi} \int_0^\pi F_n(\theta, \phi) \sin \theta \, d\theta \, d\phi \quad (2-24) \]

\[ F_n(\theta, \phi) = \frac{F(\theta, \phi)}{F(\theta, \phi)|_{\text{max}}} \quad (2-25) \]

Dividing by \( F(\theta, \phi)|_{\text{max}} \) merely normalizes the radiation intensity \( F(\theta, \phi) \), and it makes its maximum value unity.

The beam solid angle \( \Omega_A \) is defined as the solid angle through which all the power of the antenna would flow if its radiation intensity is constant (and equal to the maximum value of \( U \)) for all angles within \( \Omega_A \).
The beam area (or beam solid angle) $\Omega_A$ for an antenna is the integral of the normalized power pattern over a sphere ($4\pi sr$). That is,

$$
\Omega_A = \int_0^{2\pi} \int_0^\pi P_A(\theta, \phi) \sin \theta \, d\theta \, d\phi \quad \text{(sr)} \quad \ldots \text{(2.3.2)}
$$

where $d\Omega = \sin \theta \, d\theta \, d\phi$

The beam area $\Omega_A$ of an actual pattern is equivalent to the same solid angle subtended by the spherical cap of the cone-shaped (triangular cross-section) pattern. This is diagrammatically shown in Fig. 2.3.3.

Diagrammatically:

---

**Equivalent solid angle $\Omega_A$**

**Actual pattern of beam area $\Omega_A$**

**Half-power beam width $\theta_{HP}$**

---

Polar Plot of $P(\theta)$
### 2.6.1 Directional Patterns

Instead of using the exact expression of (2-23) to compute the directivity, it is often convenient to derive simpler expressions, even if they are approximate, to compute the directivity. These can also be used for design purposes. For antennas with one narrow major lobe and very negligible minor lobes, the beam solid angle is approximately equal to the product of the half-power beamwidths in two perpendicular planes.

![Diagram of directional patterns](image)

**Figure 2.14** Beam solid angles for nonsymmetrical and symmetrical radiation patterns.

With this approximation, can be approximated by **Kraus’ formula**

$$D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}}$$  \hspace{1cm} (2-26)

The beam solid angle $\Omega_A$ has been approximated by

$$\Omega_A \simeq \Theta_{1r}\Theta_{2r}$$  \hspace{1cm} (2-26a)

where

- $\Theta_{1r}$ = half-power beamwidth in one plane  \hspace{0.5cm} (rad)
- $\Theta_{2r}$ = half-power beamwidth in a plane at a right angle to the other  \hspace{0.5cm} (rad)
The validity of the previous equation is based on a pattern that has only one major lobe and any minor lobes, if present, should be of very low intensity.

Or by Tai & Pereira formula

\[
D_0 \simeq \frac{4\pi (180/\pi)^2}{\Theta_{1d} \Theta_{2d}} = \frac{41,253}{\Theta_{1d} \Theta_{2d}}
\]

\[
D_0 \simeq \frac{32 \ln 2}{\Theta_{1r}^2 + \Theta_{2r}^2} = \frac{22.181}{\Theta_{1r}^2 + \Theta_{2r}^2}
\]

\[
D_0 \simeq \frac{22.181 (180/\pi)^2}{\Theta_{1d}^2 + \Theta_{2d}^2} = \frac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2}
\]

### 2.8 ANTENNA EFFICIENCY

Associated with an antenna are a number of efficiencies and can be defined using Figure 2.22. The total antenna efficiency \(e_0\) is used to take into account losses at the input terminals and within the structure of the antenna. Such losses may be due, referring to Figure 2.22(b), to

1. reflections because of the mismatch between the transmission line and the antenna
2. \(I^2R\) losses (conduction and dielectric)
In general, the overall efficiency can be written as

$$e_0 = e_re_ce_d$$  \hspace{1cm} (2-44)

where

- $e_0$ = total efficiency \hspace{1cm} \text{(dimensionless)}
- $e_r$ = reflection (mismatch) efficiency \hspace{1cm} (1 - |\Gamma|^2) \hspace{1cm} \text{(dimensionless)}
- $e_c$ = conduction efficiency \hspace{1cm} \text{(dimensionless)}
- $e_d$ = dielectric efficiency \hspace{1cm} \text{(dimensionless)}
- $\Gamma$ = voltage reflection coefficient at the input terminals of the antenna
  \hspace{1cm} [\Gamma = (Z_{in} - Z_0)/(Z_{in} + Z_0) \text{ where } Z_{in} = \text{antenna input impedance,}]
  \hspace{1cm} Z_0 = \text{characteristic impedance of the transmission line}]
- VSWR = voltage standing wave ratio \hspace{1cm} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$
Usually $e_c$ and $e_d$ are very difficult to compute, but they can be determined experimentally. Even by measurements they cannot be separated, and it is usually

$$e_0 = e_r e_{cd} = e_{cd} (1 - |\Gamma|^2)$$

more convenient to write as

where $e_{cd} = e_c e_d$ = antenna radiation efficiency, which is used to relate the gain and directivity.

**GAIN**

Another useful measure describing the performance of an antenna is the gain. Although the gain of the antenna is closely related to the directivity, it is a measure that takes into account the efficiency of the antenna as well as its directional capabilities. Remember that directivity is a measure that describes only the directional properties of the antenna and it is therefore controlled only by the pattern.

Gain of an antenna (in a given direction) is defined as “the ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically. The radiation intensity corresponding to the isotropically radiated power is equal to the power accepted (input) by the antenna divided by $4\pi$.”

$$\text{Gain} = 4\pi \frac{\text{radiation intensity}}{\text{total input (accepted) power}} = 4\pi \frac{U(\theta, \phi)}{P_{in}} \quad \text{(dimensionless)}$$

“gain does not include losses arising from impedance mismatches (reflection losses) and polarization mismatches (losses).”
Thus, we can introduce an absolute gain $G_{abs}$ that takes into account the reflection/mismatch losses (due to the connection of the antenna element to the transmission line), and it can be written as

$$G_{abs} (\theta, \phi) = e_r G (\theta, \phi) = (1 - |\Gamma|^2) G (\theta, \phi)$$

$$= e_r e_{cd} D (\theta, \phi) = e_o D (\theta, \phi)$$

Thus, we can introduce an absolute gain $G_{abs}$ that takes into account the reflection/mismatch losses (due to the connection of the antenna element to the transmission line), and it can be written as

- If lossless antenna, $G=D$
- If the antenna is matched to the transmission line, that is, the antenna input impedance $Z_{in}$ is equal to the characteristic impedance $Z_c$ of the line($|\Gamma|=0$), then the two gains are equal($G_{abs}=G$).
22

- Usually the gain is given in terms of decibels instead of the dimensionless quantity. The conversion formula is given by

\[ G_0(\text{dB}) = 10 \log_{10}[e_{cd}D_0 \text{ (dimensionless)}] \]

For many practical antennas an approximate formula for the gain,

\[ G_0 \approx \frac{30,000}{\Theta_1d \Theta_2d} \]

In practice, whenever the term “gain” is used, it usually refers to the maximum gain.

2.11 BANDWIDTH

The bandwidth of an antenna is defined as “the range of frequencies within which the performance of the antenna, with respect to some characteristic, conforms to a specified standard.” The bandwidth can be considered to be the range of frequencies, on either side of a center frequency (usually the resonance frequency for a dipole), where the antenna characteristics (such as input impedance, pattern, beamwidth, polarization, side lobe level, gain, beam direction, radiation efficiency) are within an acceptable value of those at the center frequency. Because the characteristics (input impedance, pattern, gain, polarization, etc.) of an antenna do not necessarily vary in the same manner or are even critically affected by the frequency, there is no unique characterization of the bandwidth. The specifications are set in each case to meet the needs of the particular application. Usually there is a distinction made between pattern and input impedance variations. Accordingly, pattern bandwidth and impedance bandwidth are used to emphasize this distinction. Associated with pattern bandwidth are gain, side lobe level, beamwidth, polarization, and beam direction while input impedance and radiation efficiency are related to impedance bandwidth.
2.12 POLARIZATION

Polarization of an antenna in a given direction is defined as “the polarization of the wave transmitted (radiated) by the antenna.

At any point in the far field of an antenna the radiated wave can be represented by a plane wave whose electric-field strength is the same as that of the wave and whose direction of propagation is in the radial direction from the antenna. As the radial distance approaches infinity, the radius of curvature of the radiated wave’s phase front also approaches infinity and thus in any specified direction the wave appears locally as a plane wave.” Polarization may be classified as linear, circular, or elliptical. If the vector that describes the electric field at a point in space as a function of time is always directed along a line, the field is said to be linearly polarized.
In general, however, the figure that the electric field traces is an ellipse, and the field is said to be elliptically polarized. Linear and circular polarizations are special cases of elliptical, and they can be obtained when the ellipse becomes a straight line or a circle, respectively. The figure of the electric field is traced in a clockwise (CW) or counterclockwise (CCW) sense. Clockwise rotation of the electric-field vector is also designated as right-hand polarization and counterclockwise as left-hand polarization.

2.12.1 Linear, Circular, and Elliptical Polarizations

The instantaneous field of a plane wave, traveling in the negative z direction, can be written as

$$\mathbf{E}(z; t) = \mathbf{a}_x \mathbf{E}_x(z; t) + \mathbf{a}_y \mathbf{E}_y(z; t)$$

$$\mathbf{E}_x(z; t) = \text{Re}[E_x e^{j(\omega t + k z)}] = \text{Re}[E_{xo} e^{j(\omega t + k z + \phi_x)}]$$
$$= E_{xo} \cos(\omega t + k z + \phi_x)$$

$$\mathbf{E}_y(z; t) = \text{Re}[E_y e^{j(\omega t + k z)}] = \text{Re}[E_{yo} e^{j(\omega t + k z + \phi_y)}]$$
$$= E_{yo} \cos(\omega t + k z + \phi_y)$$

A. Linear Polarization

For the wave to have linear polarization, the time-phase difference between the two components must be

$$\Delta \phi = \phi_y - \phi_x = n \pi, \quad n = 0, 1, 2, 3, \ldots$$

the field vector (electric or magnetic) possesses:

a. Only one component, or

b. Two orthogonal linear components that are in time phase or 180 (or multiples of 180°) out-of-phase.
\[ \delta_L = 2k\pi \Rightarrow \tau > 0 \]

\[ \delta_L = (2k + 1)\pi \Rightarrow \tau < 0 \]

\[ \tau = \pm \arctan \left( \frac{E_y}{E_x} \right) \]

(a) \hspace{2cm} (b)

The sum of the E field vectors determines the sense of polarization

\[ e(t) = \hat{x} \cdot E_x \cos(\omega t) + \hat{y} \cdot E_y \cos(\omega t \pm n\pi) \]

\[ \Rightarrow E = \hat{x} \cdot E_x \pm \hat{y} \cdot E_y \]
B. Circular Polarization

Circular polarization can be achieved only when the magnitudes of the two components are the same and the time-phase difference between them is odd multiples of $\pi/2$. That is,

$$|\mathcal{E}_x| = |\mathcal{E}_y| \Rightarrow E_{xo} = E_{yo}$$

$$\Delta \phi = \phi_y - \phi_x = \begin{cases} + \left(\frac{1}{2} + 2n\right)\pi, n = 0, 1, 2, \ldots & \text{for CW} \\ - \left(\frac{1}{2} + 2n\right)\pi, n = 0, 1, 2, \ldots & \text{for CCW} \end{cases}$$

If the direction of wave propagation is reversed (i.e., $+z$ direction), the phases for CW and CCW rotation must be interchanged.

The necessary and sufficient conditions to accomplish this are if the field vector (electric or magnetic) possesses all of the following:

a. The field must have two orthogonal linear components, and

b. The two components must have the same magnitude, and

c. The two components must have a time-phase difference of odd multiples of 90

$$e(t) = \hat{x}E_x \cos(\omega t) + \hat{y}E_y \cos[\omega t \pm (\pi / 2 + n\pi)]$$

$$\Rightarrow \mathbf{E} = E_m (\hat{x} \pm j\hat{y})$$
If \(+\hat{z}\) is the direction of propagation: **counterclockwise (CCW)** or **left-hand** polarization

If \(+\hat{z}\) is the direction of propagation: **clockwise (CW)** or **right-hand** polarization

Note that the sense of rotation changes if the direction of propagation changes. In the example above, if the wave propagates along \(-\hat{z}\), the plot to the left, where \(\mathbf{E} = E_m(\hat{x} + j\hat{y})\), corresponds to a right-hand wave, while the plot to the right, where \(\mathbf{E} = E_m(\hat{x} - j\hat{y})\), corresponds to a left-hand wave.
C. Elliptical Polarization

A wave is elliptically polarized if it is not linearly or circularly polarized.

Elliptical polarization can be attained only when the time-phase difference between the two components is odd multiples of $\pi/2$ and their magnitudes are not the same or when the time-phase difference between the two components is not equal to multiples of $\pi/2$ (irrespective of their magnitudes). That is,

$$|\mathbf{E}_x| \neq |\mathbf{E}_y| \Rightarrow E_{xo} \neq E_{yo}$$

when $\Delta \phi = \phi_y - \phi_x = \left\{ \begin{array}{ll} + \left( \frac{1}{2} + 2n \right) \pi & \text{for CW} \\ - \left( \frac{1}{2} + 2n \right) \pi & \text{for CCW} \end{array} \right.$$

or

$$\Delta \phi = \phi_y - \phi_x \neq \pm \frac{n}{2} \pi = \left\{ \begin{array}{ll} > 0 & \text{for CW} \\ < 0 & \text{for CCW} \end{array} \right.$$

The ratio of the major axis to the minor axis is referred to as the axial ratio (AR), and

$$\mathbf{e}(t) = \hat{\mathbf{x}}E_x \cos \omega t + \hat{\mathbf{y}}E_y \cos(\omega t + \delta_L)$$

$$\Rightarrow \mathbf{E} = \hat{\mathbf{x}}E_x + \hat{\mathbf{y}}E_y e^{j\delta_L}$$

it is equal to

$$\frac{\text{major axis}}{\text{minor axis}} = \frac{OA}{OB}, \quad 1 \leq \text{AR} \leq \infty \quad (2-65)$$

where

$$OA = \left[ \frac{1}{2} \left( E_{xo}^2 + E_{yo}^2 + \left| E_{xo}^4 + E_{yo}^4 + 2E_{xo}^2E_{yo}^2 \cos(2\Delta \phi) \right|^{1/2} \right) \right]^{1/2} \quad (2-66)$$

$$OB = \left[ \frac{1}{2} \left( E_{xo}^2 + E_{yo}^2 - \left| E_{xo}^4 + E_{yo}^4 + 2E_{xo}^2E_{yo}^2 \cos(2\Delta \phi) \right|^{1/2} \right) \right]^{1/2} \quad (2-67)$$

The tilt of the ellipse, relative to the y axis, is represented by the angle $\tau$ given by

$$\tau = \frac{\pi}{2} - \frac{1}{2} \tan^{-1} \left[ \frac{2E_{xo}E_{yo}}{E_{xo}^2 - E_{yo}^2} \cos(\Delta \phi) \right] \quad (2-68)$$
The necessary and sufficient conditions to accomplish this are if the field vector (electric or magnetic) possesses all of the following:

a. The field must have two orthogonal linear components, and

b. The two components can be of the same or different magnitude.

c. (1) If the two components are not of the same magnitude, the time-phase difference between the two components must not be 0 or multiples of 180 (because it will then be linear). (2) If the two components are of the same magnitude, the time-phase difference between the two components must not be odd multiples of 90 (because it will then be circular).

The sense of rotation is always determined by rotating the phase-leading component toward the phase-lagging component and observing the field rotation as the wave is viewed as it travels away from the observer. If the rotation is clockwise, the wave is right-hand (or clockwise) circularly polarized; if the rotation is counterclockwise, the wave is left-hand (or counterclockwise) circularly polarized. The rotation of the phase-leading component toward the phase-lagging component should be done along the angular separation between the two components that is less than 180.

Phases equal to or greater than 0 and less than 180 should be considered leading whereas those equal to or greater than 180 and less than 360 should be considered lagging.
2.12.2 Polarization Loss Factor and Efficiency

In general, the polarization of the receiving antenna will not be the same as the polarization of the incoming (incident) wave. This is commonly stated as “polarization mismatch.” The amount of power extracted by the antenna from the incoming signal will not be maximum because of the polarization loss. Assuming that the electric field of the incoming wave can be written as

\[ \mathbf{E}_i = \hat{\rho}_w E_i \]

where \( \hat{\rho}_w \) is the unit vector of the wave, and the polarization of the electric field of the receiving antenna can be expressed as

\[ \mathbf{E}_a = \hat{\rho}_a E_a \]

where \( \hat{\rho}_a \) is its unit vector (polarization vector),

the polarization loss can be taken into account by introducing a polarization loss factor (PLF). It is defined, based on the polarization of the antenna in its transmitting mode, as

\[ \text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |\cos \psi_p|^2 \text{ (dimensionless)} \]

![Figure 2.24](image)

**Figure 2.24** Polarization unit vectors of incident wave (\( \hat{\rho}_w \)) and antenna (\( \hat{\rho}_a \)), and polarization loss factor (PLF).

polarization efficiency
Another figure of merit that is used to describe the polarization characteristics of a wave and that of an antenna is the **polarization efficiency** (polarization mismatch or loss factor) which is defined as “the ratio of the power received by an antenna from a given plane wave of arbitrary polarization to the power that would be received by the same antenna from a plane wave of the same power flux density and direction of propagation, whose state of polarization has been adjusted for a maximum received power.” This is similar to the PLF and it is expressed as

\[
p_e = \frac{\| \ell_e \cdot \mathbf{E}^{\text{inc}} \|^2}{\| \ell_e \|^2 \| \mathbf{E}^{\text{inc}} \|^2}
\]

where

\[\ell_e = \text{vector effective length of the antenna}\]
\[\mathbf{E}^{\text{inc}} = \text{incident electric field}\]

![Diagram showing PLF for transmitting and receiving linear wire antennas](image)

**Figure 2.25** Polarization loss factors (PLF) for aperture and linear wire antennas.

**Example** The electric field of a linearly polarized EM wave is

\[\mathbf{E}^l = \hat{x} \cdot E_m(x, y)e^{-j\beta z}\]

It is incident upon a linearly polarized antenna whose polarization is

\[\mathbf{E}_a = (\hat{x} + \hat{y}) \cdot E(r, \theta, \varphi)\]

Find the PLF.
\[
\text{PLF} = \left| \hat{x} \cdot \frac{1}{\sqrt{2}} (\hat{x} + \hat{y}) \right|^2 = \frac{1}{2}
\]
\[
\text{PLF}_{[\text{dB}]} = 10 \log_{10} 0.5 = -3 \text{ dB}
\]

**Example**  
A transmitting antenna produces a far-zone field, which is RH circularly polarized. This field impinges upon a receiving antenna, whose polarization (in transmitting mode) is also RH circular. Determine the PLF.

Both antennas (the transmitting one and the receiving one) are RH circularly polarized in transmitting mode. Assume that a transmitting antenna is located at the center of a spherical coordinate system. The far-zone field it would produce is described as
\[
\mathbf{E}_{\text{far}} = E_m \left[ \hat{\theta} \cdot \cos \omega t + \hat{\phi} \cdot \cos(\omega t - \pi / 2) \right]
\]
It is a RH circularly polarized field with respect to the outward radial direction. Its polarization vector is
\[
\hat{\rho} = \frac{\hat{\theta} - j\hat{\phi}}{\sqrt{2}}.
\]
This is exactly the polarization vector of the transmitting antenna.
This same field $E^{far}$ is incident upon a receiving antenna, which has the polarization vector $\hat{\rho}_a = (\hat{\theta}_a - j\hat{\phi}_a)/\sqrt{2}$ in its own coordinate system $(r_a, \theta_a, \phi_a)$. However, $E^{far}$ propagates along $-\hat{r}_a$ in the $(r_a, \theta_a, \phi_a)$ coordinate system, and, therefore, its polarization vector becomes
\[
\hat{\rho}_i = \frac{\hat{\theta}_a + j\hat{\phi}_a}{\sqrt{2}}.
\]

The PLF is calculated as
\[
\text{PLF} = |\hat{\rho}_i \cdot \hat{\rho}_a|^2 = \left|\frac{(\hat{\theta}_a + j\hat{\phi}_a)(\hat{\theta}_a - j\hat{\phi}_a)}{4}\right|^2 = 1,
\]
\[
\text{PLF}_{[dB]} = 10\log_{10} 1 = 0.
\]

There are no polarization losses.

**Example 2.12**

A right-hand (clockwise) circularly polarized wave radiated by an antenna, placed at some distance away from the origin of a spherical coordinate system, is traveling in the inward radial direction at an angle $(\theta, \phi)$ and it is impinging upon a right-hand circularly polarized receiving antenna placed at the origin (see Figures 2.1 and 17.23 for the geometry of the coordinate system). The polarization of the receiving antenna is defined in the transmitting
mode, as desired by the definition of the IEEE. Assuming the polarization of the incident wave is represented by
\[ E_\omega = (\hat{a}_0 + j\hat{a}_y) \tilde{E}(\tau, \theta, \phi) \]

Determine the polarization loss factor (PLF).

**Solution:** The polarization of the incident right-hand circularly polarized wave traveling along the °-r direction is described by the unit vector
\[ \hat{\rho}_\omega = \left( \frac{\hat{a}_0 + j\hat{a}_y}{\sqrt{2}} \right) \]

while that of the receiving antenna, in the transmitting mode, is represented by the unit vector
\[ \hat{\rho}_d = \left( \frac{\hat{a}_0 - j\hat{a}_y}{\sqrt{2}} \right) \]

Therefore the polarization loss factor is
\[ PLF = |\hat{\rho}_\omega \cdot \hat{\rho}_d|^2 = \frac{1}{4} |1 + 1|^2 = 1 \text{ dB} \]

Since the polarization of the incoming wave matches (including the sense of rotation) the polarization of the receiving antenna there should not be any losses. Obviously the answer matches the expectation.

**Example:**

A uniform plane wave is propagating in direction of the positive z-axis. Find the polarizatic (linear, circular, or elliptical), sense of rotation (CW or CCW), axial ratio (AR), and tilt ang \( \tau \) (in degrees) for

- a) \( E_x = E_y \) and \( \Delta \phi = 0 \),
- b) \( E_x \neq E_y \) and \( \Delta \phi = 0 \),
- c) \( E_x = E_y \) and \( \Delta \phi = \pi / 2 \),
- d) \( E_x = E_y \) and \( \Delta \phi = -\pi / 2 \),
- e) \( E_x = E_y \) and \( \Delta \phi = \pi / 4 \),
- f) \( E_x = E_y \) and \( \Delta \phi = -\pi / 4 \),
- g) \( E_x = 0.5E_y \) and \( \Delta \phi = \pi / 2 \),
- h) \( E_x = 0.5E_y \) and \( \Delta \phi = -\pi / 2 \).

In all cases justify the answer.
a) Linear because $\Delta \phi = 0$.

b) Linear because $\Delta \phi = 0$.

c) Circular because $E_x = E_y$ and $\Delta \phi = \pi/2$.
   LHCP / CCW, AR = 1.

d) Circular because $E_x = E_y$ and $\Delta \phi = -\pi/2$.
   RHCP / CW, AR = 1.

e) Elliptical because $\Delta \phi$ is not multiples of $\pi/2$.
   CCW,
   \[
   OA = E_0 \cdot \left[0.5\left(1 + 1 + \sqrt{2}\right)\right]^{1/2} = 1.30656 \cdot E_0
   \]
   \[
   AR : \ E_x = E_y = E_0 \cdot \left[0.5\left(1 + 1 - \sqrt{2}\right)\right]^{1/2} = 0.541196 \cdot E_0
   \Rightarrow AR = \frac{1.30656}{0.541196} = 2.414
   \]
   \[
   \tau = 90^\circ - \frac{1}{2} \tan^{-1}\left(\frac{2 \cdot 1 \cdot \cos(45^\circ)}{1 - 1}\right) = 90^\circ - \frac{1}{2} \cdot (90^\circ) = 45^\circ
   \]

f) Elliptical because $\Delta \phi$ is not multiples of $\pi/2$.
   CW,
   \[
   AR = OA/OB : \ OA = 1.30656 \cdot E_0 \]
   \[
   OB = 0.541196 \cdot E_0 \Rightarrow AR = \frac{1.30656}{0.541196} = 2.414
   \]
   \[
   \tau = 90^\circ - \frac{1}{2} \cdot (90^\circ) = 45^\circ
   \]

g) Elliptical because: $E_x = E_y$ AND $\Delta \phi$ is not zero nor multiples of $\pi$.
   CCW,
   \[
   AR = OA/OB.
   \]
   \[
   OA = E_y \cdot \left[0.5\left(0.25 + 1 + 0.75\right)\right]^{1/2} = E_y
   \]
   \[
   OB = E_y \cdot \left[0.5\left(0.25 + 1 - 0.75\right)\right]^{1/2} = 0.5 \cdot E_y \Rightarrow AR = \frac{1}{0.5} = 2
   \]
   \[
   \tau = 90^\circ - \frac{1}{2} \tan^{-1}\left(\frac{0}{-0.75}\right) = 90^\circ - \frac{1}{2} \cdot (0^\circ) = 90^\circ
   \]

h) Elliptical because: $E_x = E_y$ AND $\Delta \phi$ is not zero nor multiples of $\pi$.
   CW,
   \[
   AR = OA/OB.
   \]
   \[
   OA = E_y
   \]
   \[
   OB = 0.5 \cdot E_y \Rightarrow AR = \frac{1}{0.5} = 2
   \]
   \[
   \tau = 90^\circ - \frac{1}{2} \cdot (0^\circ) = 90^\circ
   \]
A linearly polarized wave traveling in the negative z-direction has a tilt angle $\tau$ of $45^\circ$. It is incident upon an antenna whose polarization characteristics are given by

$$\bar{\rho}_a = \frac{4\bar{a}_x + j\bar{a}_y}{\sqrt{17}}$$

Find the polarization loss factor (PLF) dimensionless and in dB.

Polarization vector of the linearly polarized wave:

$$\bar{\rho}_w = \frac{\bar{a}_x + \bar{a}_y}{\sqrt{2}}$$

Polarization vector of the elliptically polarized wave:

$$\bar{\rho}_a = \frac{4\bar{a}_x + j\bar{a}_y}{\sqrt{17}}$$

$$PLF = |\bar{\rho}_w \cdot \bar{\rho}_a|^2 = \left| \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & j \end{pmatrix} \right|^2 \left| \frac{1}{\sqrt{17}} \begin{pmatrix} 4 \\ j \end{pmatrix} \right|^2 = \frac{17}{34} = 0.5 = -3\text{dB}$$
2.13 INPUT IMPEDANCE

Input impedance is defined as “the impedance presented by an antenna at its terminals or the ratio of the voltage to current at a pair of terminals or the ratio of the appropriate components of the electric to magnetic fields at a point.” In Figure 2.27(a) these terminals are designated as a–b. The ratio of the voltage to current at these terminals, with no load attached, defines the impedance of the antenna as

\[ Z_A = R_A + jX_A \]
where

\[ Z_A = \text{antenna impedance at terminals } a-b \text{ (ohms)} \]
\[ R_A = \text{antenna resistance at terminals } a-b \text{ (ohms)} \]
\[ X_A = \text{antenna reactance at terminals } a-b \text{ (ohms)} \]

In general the resistive part of (2-72) consists of two components; that is

\[ R_A = R_r + R_L \]

where

\[ R_r = \text{radiation resistance of the antenna} \]
\[ R_L = \text{loss resistance of the antenna} \]
If we assume that the antenna is attached to a generator with internal impedance

\[ Z_g = R_g + jX_g \]  

(2-74)

where

\[ R_g = \text{resistance of generator impedance (ohms)} \]
\[ X_g = \text{reactance of generator impedance (ohms)} \]

\[ I_g = \frac{V_g}{Z_A + Z_g} = \frac{V_g}{(R_r + R_L + R_g) + j(X_A + X_g)} \]  

(A)  

(2-75)

and its magnitude by

\[ |I_g| = \frac{|V_g|}{\sqrt{(R_r + R_L + R_g)^2 + (X_A + X_g)^2}} \]  

(2-75a)

where \( V_g \) is the peak generator voltage. The power delivered to the antenna for radiation is given by

\[ P_r = \frac{1}{2} |I_g|^2 R_r = \frac{|V_g|^2}{2} \left[ \frac{R_r}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2} \right] \]  

(W)  

(2-76)

and that dissipated as heat by

\[ P_L = \frac{1}{2} |I_g|^2 R_L = \frac{|V_g|^2}{2} \left[ \frac{R_L}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2} \right] \]  

(W)  

(2-77)

The remaining power is dissipated as heat on the internal resistance \( R_g \) of the generator, and it is given by

\[ P_g = \frac{|V_g|^2}{2} \left[ \frac{R_g}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2} \right] \]  

(W)  

(2-78)

The maximum power delivered to the antenna occurs when we have conjugate matching; that is when

\[ R_r + R_L = R_g \]  

(2-79)
\[ X_A = -X_g \]  

(2-80)
Of the power that is provided by the generator, half is dissipated as heat in the internal resistance (Rg) of the generator and the other half is delivered to the antenna. This only happens when we have conjugate matching. Of the power that is delivered to the antenna, part is radiated through the mechanism provided by the radiation resistance and the other is dissipated as heat which influences part of the overall efficiency of the antenna. If the antenna is lossless and matched to the transmission line (eo=1), then half of the total power supplied by the generator is radiated by the antenna during conjugate matching, and the other half is dissipated as heat in the generator. Thus, to radiate half of the available power through Rr you must dissipate the other half as heat in the generator through Rg. These two powers are, respectively, analogous to the power transferred to the load and the power scattered by the antenna in the receiving mode.

The use of the antenna in the receiving mode is shown in Figure 2.28(a). The incident wave impinges upon the antenna, and it induces a voltage VT which is analogous to Vg of the transmitting mode. The Thevenin equivalent circuit of the antenna and its...
load is shown in Figure 2.28(b) in the receiving mode under conjugate matching (Rr + RL = RT and XA = −XT) the

powers delivered to RT, Rr, and RL are given, respectively, by

\[
P_T = \frac{|V_T|^2}{8} \left( \frac{R_T}{(R_r + R_L)^2} \right) = \frac{|V_T|^2}{8} \left( \frac{1}{R_r + R_L} \right) = \frac{|V_T|^2}{8RT} \tag{2-86}
\]

\[
P_r = \frac{|V_T|^2}{2} \left( \frac{R_r}{4(R_r + R_L)^2} \right) = \frac{|V_T|^2}{8} \left( \frac{R_r}{(R_r + R_L)^2} \right) \tag{2-87}
\]

\[
P_L = \frac{|V_T|^2}{8} \left( \frac{R_L}{(R_r + R_L)^2} \right) \tag{2-88}
\]

while the induced (collected or captured) is

\[
P_c = \frac{1}{2} V_T I_T^* = \frac{1}{2} V_T \left( \frac{V_T^*}{2(R_r + R_L)} \right) = \frac{|V_T|^2}{4} \left( \frac{1}{R_r + R_L} \right) \tag{2-89}
\]

These are analogous, respectively, to (2-81) – (2-83) and (2-85). The power Pr of (2-87) delivered to Rr is referred to as scattered (or reradiated) power. It is clear through (2-86) – (2-89) that under conjugate matching of the total power collected or captured [Pc of (2-89)] half is delivered to the load RT [PT of (2-86)] and the other half is scattered or reradiated through Rr [Pr of (2-87)] and dissipated as heat through RL [PL of (2-88)]. If the losses are zero (RL = 0), then half of the captured power is delivered to the load and the other half is scattered.
2.14 ANTENNA RADIATION EFFICIENCY

The conduction and dielectric losses of an antenna are very difficult to compute and in most cases they are measured. Even with measurements, they are difficult to separate and they are usually lumped together to form the *ecd* efficiency. The resistance *RL* is used to represent the conduction-dielectric losses. The *conduction-dielectric efficiency ecd* is defined as the ratio of the power delivered to the radiation resistance *Rr* to the power delivered to *Rr* and *RL*.

\[
ecd = \left[ \frac{R_r}{R_L + R_r} \right]
\]

(dimensionless)

\[
R_{dc} = \frac{1}{\sigma A} \quad \text{(ohms)}
\]

Therefore the high-frequency resistance can be written, based on a *uniform current distribution*, as

\[
R_{hf} = \frac{l}{P} R_s = \frac{l}{P} \sqrt{\frac{\omega \mu_0}{2\sigma}} \quad \text{(ohms)}
\]

where *P* is the perimeter of the cross section of the rod (*P = C = 2\pi b* for a circular wire of radius *b*). *R_s* is the conductor surface resistance, \(\omega\) is the angular frequency, \(\mu_0\) is the permeability of free-space, and *\sigma* is the conductivity of the metal.

For a \(\lambda/2\) dipole with a sinusoidal current distribution \(RL=0.5\ Rhf\)
2.15.1 Vector Effective Length

The effective length of an antenna, whether it be a linear or an aperture antenna, is a quantity that is used to determine the voltage induced on the open-circuit terminals of the antenna when a wave impinges upon it. It should be noted that it is also referred to as the effective height.

This relation can be expressed as

\[ V_{oc} = E^i \cdot \ell_e \]

where

- \( V_{oc} \) = open-circuit voltage at antenna terminals
- \( E^i \) = incident electric field
- \( \ell_e \) = vector effective length
$V_{oc}$ can be thought of as the voltage induced in a linear antenna of length $G_e$ when $\mathbf{E}$ and $\mathbf{E}^i$ are linearly polarized [19], [20]. From the relation of (2-93) the effective length of a linearly polarized antenna receiving a plane wave in a given direction is defined as “the ratio of the magnitude of the open-circuit voltage developed at the terminals of the antenna to the magnitude of the electric-field strength in the direction of the antenna polarization.

2.15.2 Antenna Equivalent Areas

With each antenna, we can associate a number of equivalent areas. These are used to describe the power capturing characteristics of the antenna when a wave impinges on it. One of these equivalent areas is the effective area (aperture), which in a given direction is defined as “the ratio of the available power at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction, the wave being polarization-matched to the antenna.

\[ A_e = \frac{P_T}{W_i} = \frac{|I_T|^2 R_T/2}{W_i} \]  

(2-94)

where

- $A_e =$ effective area (effective aperture) (m²)
- $P_T =$ power delivered to the load (W)
- $W_i =$ power density of incident wave (W/m²)

The effective aperture is the area which when multiplied by the incident power density gives the power delivered to the load

\[ A_e = \frac{|V_T|^2}{2W_i} \left[ \frac{R_T}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2} \right] \]

Under conditions of maximum power transfer (conjugate matching), $R_r + R_L = R_T$ and $X_A = -X_T$, the effective area of (2-95) reduces to the maximum effective aperture given by

\[ A_{em} = \frac{|V_T|^2}{8W_i} \left[ \frac{R_T}{(R_L + R_r)^2} \right] = \frac{|V_T|^2}{8W_i} \left[ \frac{1}{R_r + R_L} \right] \]

The scattering area is defined as the equivalent area when multiplied by the incident power density is equal to the scattered or reradiated power. Under conjugate matching this is written, similar to (2-96), as
The loss area is defined as the equivalent area, which when multiplied by the incident power density leads to the power dissipated as heat through \( R_L \). Under conjugate matching this is written, similar to (2-96), as

\[
A_L = \frac{|V_T|^2}{8W} \left[ \frac{R_L}{(R_L + R_r)^2} \right]
\]

Finally the capture area is defined as the equivalent area, which when multiplied by the incident power density leads to the total power captured, collected, or intercepted by the antenna. Under conjugate matching this is written, similar to (2-96), as

\[
A_c = \frac{|V_T|^2}{8W} \left[ \frac{R_T + R_r + R_L}{(R_L + R_r)^2} \right]
\]

\textit{Capture Area} = \textit{Effective Area} + \textit{Scattering Area} + \textit{Loss Area}

Now that the equivalent areas have been defined, let us introduce the aperture efficiency \( \epsilon_{ap} \) of an antenna, which is defined as the ratio of the maximum effective area \( A_{em} \) of the antenna to its physical area \( A_p \), or

\[
\epsilon_{ap} = \frac{A_{em}}{A_p} = \frac{\text{maximum effective area}}{\text{physical area}}
\]

For aperture type antennas, such as waveguides, horns, and reflectors, the maximum effective area cannot exceed the physical area but it can equal it \( (A_{em} \leq A_p \text{ or } 0 \leq \epsilon_{ap} \leq 1) \). Therefore the maximum value of the aperture efficiency cannot exceed unity (100%). For a lossless antenna \( (R_L = 0) \) the maximum value of the scattering area is also equal to the physical area. Therefore even though the aperture efficiency is greater than 50%, for a lossless antenna under conjugate matching only half of the captured power is delivered to the load and the other half is scattered.
2.16 MAXIMUM DIRECTIVITY AND MAXIMUM EFFECTIVE AREA

In general then, the maximum effective aperture ($A_{em}$) of any antenna is related to its maximum directivity ($D_0$) by

$$A_{em} = \frac{\lambda^2}{4\pi} D_0$$

Thus, when (2-110) is multiplied by the power density of the incident wave it leads to the maximum power that can be delivered to the load. This assumes that there are no conduction-dielectric losses (radiation efficiency $e_{cd}$ is unity), the antenna is matched to the load (reflection efficiency $e_r$ is unity), and the polarization of the impinging wave matches that of the antenna (polarization loss factor PLF and polarization efficiency $p_e$ are unity).

If there are losses associated with an antenna and reflection and polarization losses are also included, then the maximum effective area of $\hat{A}_{em}$ is represented by

$$A_{em} = e_0 \left( \frac{\lambda^2}{4\pi} \right) D_0 |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$
$$= e_{cd} (1 - |\Gamma|^2) \left( \frac{\lambda^2}{4\pi} \right) D_0 |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$