Equilibrium of Floating Bodies:
To be the floating body in equilibrium, two conditions must be satisfied:

- The buoyant Force \( F_b \) must equal the weight of the floating body \( W \).
- \( F_b \) and \( W \) must act in the same straight line.

So, for equilibrium:
\[ F_b = W_{\text{object}} \]

The equilibrium of a body may be:
- Stable.
- Unstable.
- Neutral (could be considered stable)

Stability of a Submerge Bodies

**Stable equilibrium:** if when displaced, it returns to its original equilibrium position.

**Unstable equilibrium:** if when displaced, it returns to a new equilibrium position

**Notes:**
- In this case (body is fully immersed in water) when the body is tilted, the shape of the displaced fluid doesn’t change, so the center of buoyancy remains unchanged relative to the body.
- The weight of the body is located at the center of gravity of the body \( G \) and the buoyant force located at the center of buoyancy \( B \).

**Stable Equilibrium:**
A small angular displacement \( \varphi \) from the equilibrium position will generate a moment equals: \( (W \times BG \times \varphi) \).
The immersed body is considered **Stable** if \( G \) is below \( B \), this will generate **righting moment** and the body will tend to return to its **original equilibrium position**.
Unstable Equilibrium:
The immersed body is considered **Unstable** if G is above B, this will generate an **overturning moment** and the body will tend to be in a **new equilibrium position**.

Stability of Floating Bodies
Here, the volume of the liquid remains unchanged since $F_b=W$, but the **shape of this volume changes** and thereby its center of buoyancy will differ.

When the body is displaced through an angle $\theta$ the center of buoyancy moves from B to $B_1$ and a turning moment is produced.

**Metacenter (M):**
The point at which the line of action of the buoyant force ($F_b$) **cuts** the original vertical line through G.
So, **Moment Generated is** $(W \times GM \times \theta)$.
$GM$ is known as a **metacentric height**.

**Stability:**
**Stable**
If M lies above G, a righting moment is produced, equilibrium is stable and GM is regarded as positive. (GM=+VE)
Unstable
If M lies below G, an overturning moment is produced, equilibrium is unstable and GM is regarded as negative. (GM= —VE).

Neutral:
If M coincides with G, the body is in neutral equilibrium.

Determination of the Position of Metacenter Relative to Centre of Buoyancy:

\[ BM = \frac{I}{V_{\text{displaced}}} \]

I = the smallest moment of inertia of the waterline plane

Procedures for Evaluating the Stability of Floating Bodies
1. Determine the position of the floating body (Draft) using the principles of buoyancy (Total Weight = Buoyant Force).
2. Locate the center of buoyance B and compute the distance from some datum to point B ($y_B$). The bottom of the object is usually taken as a datum.
3. Locate the center of gravity G and compute ($y_G$) measured from the same datum.
4. Determine the shape of the area at the fluid surface (plane view) and compute I for that shape.
5. Compute the displace volume ($V_d$)
6. Compute BM distance ($BM = I / V_d$).
7. Compute ($y_M = y_B + BM$)
8. If ($y_M > y_G$) >> the body is stable. ($GM = +VE$)
9. If ($y_M < y_G$) >> the body is unstable. ($GM = -VE$)

**Important Note:**
If $y_M = y_G$ (GM = 0), this case is called neutral and the object could be considered stable.
Problems

1.
For the shown figure below, a cube of wood of side length (L) is float in water. If the specific gravity of the wood is 0.88. Determine if this cube is stable or not.

Solution
2. A barge is 4.5m wide and 12m long and floats with a draft of 1.2m. It is piled so high with gravel so that its center of gravity became 1m above the waterline. Is it stable?

Solution

3. The figure below shows a cross section of boat. Show if the boat is stable or not. If the boat is stable compute the righting moment if the angle of heel is 10°. The boat is float in water and its 6m long.
Solution
4.

A wooden cone floats in water in the position shown in figure below. The specific gravity of the wood is 0.6. Would the cone be stable?

Solution
Fluid Mechanics

Static Forces on Surfaces-Buoyancy

\[ X = 0.001272 \rightarrow X = 0.152 \text{m} \]

\[ D = 1.388 \times 0.152 = 0.21 \text{m} \]

\[ y_0 = 3 \times 0.1575 = 0.4725 \text{m} \] (from tip of cone)

\[ y_0 = 3 \times 0.1875 = 0.5625 \text{m} \] (from tip of cone)

Calculation of BM:

\[ BM = I \cdot V \]

\[ I = \pi \cdot \frac{64}{64} \times 0.152 \times 0.152 = 2.62 \times 10^{-3} \text{m}^4 \]

\[ V = 0.001272 \text{m}^3 \rightarrow BM = 2.62 \times 10^{-3} \times 0.001272 = 0.0031 \text{m} \]

\[ y_\text{GM} = y_0 - y_0 = 0.1875 - 0.1875 = 0.1875 \text{m} \]

\[ GM = y_\text{GM} - y_0 = 0.1875 - 0.1875 = 0.1875 \text{m} \] (VE)

\[ \rightarrow GM = 0.1875 (\text{VE}) \]