Chapter (9)
Sheet Pile Walls
Introduction

Sheet piles are a temporary structures used to retain a soil or water for a specific period of time, to build a structure in the other side of this wall. For example; if we want to build a structure with three basement floors (underground) and this structure surrounded by other structures, when the excavation process starts, if the soil under the surrounding structures doesn’t retained by a sheet pile, this soil will fail and will moves to the excavation site, and the structure above this soil may collapse suddenly, so before establishment of excavation process, sheet pile must be constructed to retain this soil and prevent it from fails and after completion of constructed the structure, we can remove this sheet pile because it’s function was end. Another example; if we wanna build a structure in the sea (waterfront structures) we can use sheet piles to retain sea water from flowing to the required area, and then withdraw the water confined between sheet piles and thereby build the required structures, finally remove sheet piles because there functions were end.

The following figures are some explanation of the applications of sheet piles and the shape of sheet pile itself:

Sheet Pile in basement
Sections of steel Sheet Piles
Notes:
1. Sheet piles may be made from steel, concrete or wood.
2. As seen in the above pictures, sheet piles must penetrate a specified distance in earth (from both sides) to be stable against applied lateral loads, this depth called depth of penetration, and the following figure explain the main parts of sheet piles:

The line at which the sheet pile starts penetrating in soil from both sides is known by dredge line, and the depth of penetration of sheet pile under this line is D: depth of penetration.
Designing of sheet piles mainly is to calculate the depth of penetration $D$ and determining the section of sheet pile as will be discussed later.

**Types of Sheet Piles**

There are two main types of sheet piles:
1. Cantilever Sheet Piles.
2. Anchored Sheet piles.

Now, we will learn how to analyze and design each type.

**Cantilever Sheet Piles**

Cantilever sheet pile walls are usually recommended to use for walls of moderate height ($\leq 6\,\text{m}$) measured above the dredge line. In such walls, the sheet piles are act as a wide *cantilever beam* above the dredge line. The main step in analyzing cantilever sheet pile is to knowing the deflection of cantilever sheet pile with depth, and knowing (from deflection shape) the type of LEP (active or passive).

The following figure clarifies the deflection of the cantilever sheet pile due to lateral earth pressure:
As you see, due to the lateral earth pressure the wall will pushed out the soil above the dredge line so the type of LEP above the dredge line is active pressure and no passive pressure because there is no soil exist in the other side (Zone A) in the above figure.

Below the dredge line there exists a soil in both sides of the wall the wall and the wall still moves out (left side) till reaching point O (point of rotation) after point O the wall will moves to right side as shown.

So, soil below dredge line can be divided into two zones; zone B between dredge line and point O, in this zone the wall moves to the left, so the soil on the right exerts active pressure and the soil on the left exerts passive pressure. Zone C between point O to the end of sheet pile, in this zone the wall moves to the right, so the soil on the right exerts passive pressure and the soil on the left exerts active pressure as seen in the figure above.

There are three cases for cantilever sheet piles:
✓ Cantilever Sheet Pile penetrating in Sandy Soil.
✓ Cantilever Sheet Pile penetrating in Clayey Soil.
✓ Cantilever Sheet Pile penetrating in $C - \phi$ Soil.

Before discussing each type, the following notes are very important:
➢ The first step in designing the sheet pile is to draw the net LEP distribution with depth along the sheet pile, the net LEP is the difference between passive LEP and active LEP at every change in soil with depth i.e. net pressure $= \Delta \sigma = \sigma_{\text{passive}} - \sigma_{\text{active}}$

➢ Designing a sheet pile consists of the following two steps:
1. Calculation the depth of penetration (D).
2. Determining the section Modulus (S) where: $S = \frac{M_{\text{max}}}{\sigma_{\text{all}}}$

$M_{\text{max}}$ = maximum moment along sheet pile
$\sigma_{\text{all}}$ = maximum allwable flexural stress (for sheet pile material).

➢ Always in this chapter we will use Rankine LEP theory.

The most important one, is drawing the LEP distribution along sheet pile (especially below the dredge line) correctly (If you did, completion of designing process will be easy), so now we want to learn how to draw the LEP distribution for all cases of cantilever sheet pile.
Cantilever Sheet Piles Penetrating Sandy Soil

Consider the following example:

The first step always is calculating $K_a$ and $K_p$ for each layer, but here all layers have the same friction angle, so:

$$K_a = \tan^2 \left( 45 - \frac{\phi}{2} \right) = \tan^2 \left( 45 - \frac{30}{2} \right) = 0.333$$

$$K_p = \tan^2 \left( 45 + \frac{\phi}{2} \right) = \tan^2 \left( 45 + \frac{30}{2} \right) = 3$$

Now we calculate the LEP at each depth:

$$\sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a}$$

$$\sigma_{h,p} = (q + \gamma H)K_p + 2c\sqrt{K_p}$$

@ $z = 0.0$ (above dredge line → active pressure only)

$$\sigma_{h,a} = (0 + 16 \times 0) \times 0.333 - 0 = 0.0$$

$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0.0$$

@ $z = 2m$ (above dredge line → active pressure only)

just before = just after because $K_a$ is the same

$$\sigma_{h,a} = (0 + 16 \times 2) \times 0.333 - 0 = 10.65 kN/m^2$$

$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 10.65 = -10.65 kN/m^2$$

The negative sign means we draw this value at right side (side of active pressure because is the largest pressure at this depth.)
@ \( z = 5 \text{m} \) (just before \( \rightarrow \) active pressure only)

\[
\sigma_{h,a} = (0 + 16 \times 2 + (19 - 10) \times 3) \times 0.333 - 0 = 19.65 \text{kN/m}^2
\]

\[
\Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 19.65 = -19.65 \text{kN/m}^2
\]

(Do\’t forget, effective pressure always)

@ \( z = 5 \text{m} \) (just after \( \rightarrow \) active pressure at right, passive pressure at left)
Since \( K_a \) is the same before and after, the pressure will be the same

\[
\sigma_{h,a} = (0 + 16 \times 2 + (19 - 10) \times 3) \times 0.333 - 0 = 19.65 \text{kN/m}^2
\]

\[
\sigma_{h,p} = (0 + (19 - 10) \times 0)3 + 0 = 0.0
\]

\[
\Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 19.65 = -19.65 \text{kN/m}^2
\]

@ \( z = 5 + D \) (passive pressure at right, active pressure at left)
Note that at this depth the types of pressure changes as explained above

Active pressure at left side:

\[
\sigma_{h,a} = (0 + (19 - 10) \times D) \times 0.333 - 0 = 3D
\]

Passive pressure at right side:

\[
\sigma_{h,p} = (0 + 16 \times 2 + (19 - 10) \times 3 + (19 - 10) \times D)3 + 0 = 177 + 27D
\]

\[
\Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = (177 + 27D) - 3D = 177 + 24D
\]

Note that the value of \((177 + 24D)\) is positive which means we draw this value as a line in the passive zone at this depth (right side).

**Now, we draw the LEP distribution along the wall:**
Now, under dredge line the net lateral pressure $\Delta \sigma_h$ always will increase by the value of $\gamma z'(K_p - K_a)$ where $z'$: depth below dredge line at any point. And since $K_p$ is always larger than $K_a$ (when $\phi > 0$) the increase in pressure will always in the direction of passive zones. So, at the specific point (point O) the pressure will changes from active to passive (right side) and thereby the LEP distribution will tend to moves in the direction of passive zone as shown.

**Cantilever Sheet Piles Penetrating Clay**

Consider the following example:

The first step always is calculating $K_a$ and $K_p$ for each layer:

- $K_a = \tan^2 \left( 45 - \frac{\phi}{2} \right)$
- $K_{a1} = \tan^2 \left( 45 - \frac{30}{2} \right) = 0.333$
- $K_{a2} = \tan^2 \left( 45 - \frac{35}{2} \right) = 0.27$
- $K_{a3} = \tan^2(45 - 0) = 1$

Dredge Line

<table>
<thead>
<tr>
<th>Layer</th>
<th>$\gamma$ (kN/m$^3$)</th>
<th>$\phi$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
<td>0</td>
</tr>
</tbody>
</table>

$C = 40$ kN/m$^2$

$\phi = 0.0$
\[ K_p = \tan^2 \left( 45 + \frac{\phi}{2} \right) \]
\[ K_{p1} = \tan^2 \left( 45 + \frac{30}{2} \right) = 3 \]
\[ K_{p2} = \tan^2 \left( 45 + \frac{35}{2} \right) = 3.69 \]
\[ K_{p3} = \tan^2 (45 + 0) = 1 \]

Now we calculate the LEP at each depth:

\[ \sigma_{h,a} = (q + \gamma H) K_a - 2c \sqrt{K_a} \]
\[ \sigma_{h,p} = (q + \gamma H) K_p + 2c \sqrt{K_p} \]

@ \( z = 0.0 \) (above dredge line → active pressure only)
\[ \sigma_{h,a} = (0 + 16 \times 0) \times 0.333 - 0 = 0.0 \]
\[ \Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 \]

@ \( z = 2m \) (above dredge line → active pressure only)
just before
\[ \sigma_{h,a} = (0 + 16 \times 2) \times 0.333 - 0 = 10.65 \text{ kN/m}^2 \]
\[ \Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 10.65 = -10.65 \text{ kN/m}^2 \]
just after
\[ \sigma_{h,a} = (0 + 16 \times 2) \times 0.27 - 0 = 8.64 \text{ kN/m}^2 \]
\[ \Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 8.64 = -8.64 \text{ kN/m}^2 \]

@ \( z = 5m \) (just before → active pressure only)
\[ \sigma_{h,a} = (0 + 16 \times 2 + 18 \times 3) \times 0.27 - 0 = 23.2 \text{ kN/m}^2 \]
\[ \Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 23.2 = -23.2 \text{ kN/m}^2 \]

@ \( z = 5m \) (just after → active pressure at right, passive pressure at left)
And there is a value for cohesion( \( C = 40 \)) below the drege line
\[ \sigma_{h,a} = (0 + 16 \times 2 + 18 \times 3) \times 1 - 2 \times 40 \times \sqrt{1} = 6 \text{ kN/m}^2 \]
\[ \sigma_{h,p} = (0 + 0) \times 1 + 2 \times 40 \times \sqrt{1} = 80 \]
\[ \Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 80 - 6 = 74 \text{ kN/m}^2 \]
The positive sign means we draw this value at left side (side of passive pressure because is the largest pressure at this depth.)
@ $z = 5 + D$ (passive pressure at right, active pressure at left)

Active pressure at left side:

$\sigma_{h,a} = (0 + 17 \times D) \times 1 - 2 \times 40 \times \sqrt{1} = 17D - 80$

Passive pressure at right side:

$\sigma_{h,p} = (0 + 16 \times 2 + 18 \times 3 + 17 \times D) \times 1 + 2 \times 40 \times \sqrt{1} = 166 + 17D$

$\Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = (166 + 17D) - (17D - 80) = 246$

Note that the value of (246) is positive which means we draw this value as a line in the passive zone at this depth (right side).

Now, we draw the LEP distribution along the wall:

Now, under dredge line the net lateral pressure $\Delta \sigma_h$ always will increase by the value of $\gamma z' (K_p - K_a)$. But here since ($\phi = 0$) $K_p = K_a = 1$. So, the increase in pressure will be zero till reaching point O (i.e. the pressure will be constant at this depth). At the specific point (point O) the pressure will changes from active to passive (right side) and thereby the LEP distribution will tend to moves in the direction of passive zone as shown.
Cantilever Sheet Piles Penetrating C − ϕ Soil

Consider the following example:

\[
\begin{align*}
\gamma &= 16 \text{ kN/m}^3 \\
\phi &= 30^\circ \\
\gamma &= 18 \text{ kN/m}^3 \\
\phi &= 35^\circ \\
\gamma &= 17 \text{ kN/m}^3 \\
C &= 40 \text{ kN/m}^2 \\
\phi &= 20^\circ \\
\end{align*}
\]

The first step always is calculating \( K_a \) and \( K_p \) for each layer:

\[
\begin{align*}
K_a &= \tan^2 \left( 45 - \frac{\phi}{2} \right) \\
K_{a1} &= \tan^2 \left( 45 - \frac{30}{2} \right) = 0.333 \\
K_{a2} &= \tan^2 \left( 45 - \frac{35}{2} \right) = 0.27 \\
K_{a3} &= \tan^2 \left( 45 - \frac{20}{2} \right) = 0.49 \\
K_p &= \tan^2 \left( 45 + \frac{\phi}{2} \right) \\
K_{p1} &= \tan^2 \left( 45 + \frac{30}{2} \right) = 3 \\
K_{p2} &= \tan^2 \left( 45 + \frac{35}{2} \right) = 3.69 \\
K_{p3} &= \tan^2 \left( 45 + \frac{20}{2} \right) = 2.04
\end{align*}
\]
Now we calculate the LEP at each depth:

\[ \sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a} \]
\[ \sigma_{h,p} = (q + \gamma H)K_p + 2c\sqrt{K_p} \]

@ \( z = 0.0 \) (above dredge line → active pressure only)
\[ \sigma_{h,a} = (0 + 16 \times 0) \times 0.333 - 0 = 0.0 \]
\[ \Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 \]

@ \( z = 2m \) (above dredge line → active pressure only)
just before
\[ \sigma_{h,a} = (0 + 16 \times 2) \times 0.333 - 0 = 10.65 \text{ kN/m}^2 \]
\[ \Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 10.65 = -10.65 \text{ kN/m}^2 \]
just after
\[ \sigma_{h,a} = (0 + 16 \times 2) \times 0.27 - 0 = 8.64 \text{ kN/m}^2 \]
\[ \Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 8.64 = -8.64 \text{ kN/m}^2 \]

@ \( z = 5m \) (just before → active pressure only)
\[ \sigma_{h,a} = (0 + 16 \times 2 + 18 \times 3) \times 0.27 - 0 = 23.2 \text{ kN/m}^2 \]
\[ \Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 23.2 = -23.2 \text{ kN/m}^2 \]

@ \( z = 5m \) (just after → active pressure at right, passive pressure at left)
And there is a value for cohesion (\( C = 40 \)) below the drege line
\[ \sigma_{h,a} = (0 + 16 \times 2 + 18 \times 3) \times 0.49 - 2 \times 40 \times \sqrt{0.49} = -13.86 \text{ kN/m}^2 \]
\[ \sigma_{h,p} = (0 + 0) \times 2.04 + 2 \times 40 \times \sqrt{2.04} = 114.3 \text{ kN/m}^2 \]
\[ \Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 114.3 - (-13.86) = 128.12 \text{ kN/m}^2 \]

@ \( z = 5 + D \) (passive pressure at right, active pressure at left)
Active pressure at left side:
\[ \sigma_{h,a} = (0 + 17 \times D) \times 0.49 - 2 \times 40 \times \sqrt{0.49} = 8.33D - 56 \]
Passive pressure at right side:
\[ \sigma_{h,p} = (0 + 16 \times 2 + 18 \times 3 + 17 \times D) \times 2.04 + 2 \times 40 \times \sqrt{2.04} \]
\[ = 200.26 + 34.68D \]
\[ \Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = (200.26 + 34.68D) - (8.33D - 56) \]
\[ = 256.26 + 26.35D \]
Note that the value of \((256.26 + 26.35D)\) is positive which means we draw this value as a line in the passive zone at this depth (right side).

Now, we draw the LEP distribution along the wall:

Now, under dredge line the net lateral pressure \(\Delta \sigma_h\) always will increase by the value of \(\gamma z' (K_p - K_a)\). And since \(\phi = 20 \rightarrow K_p > K_a \rightarrow \) the increase in pressure will be in the direction of passive zones. So, after the value of 128.12 the stress will be increased gradually in the direction of passive zone till reaching (point O) the pressure will changes from active to passive (right side) and thereby the LEP distribution will tend to moves in the direction of passive zone as shown.

Now If we draw the net LEP distribution correctly, we can calculate the depth of penetration \(D\) by applying equilibrium equations:

\[
\sum F_x = 0.0 \text{ (Along the sheet pile)}
\]
\[
\sum M = 0.0 \text{ (At the bottom of sheet pile)}
\]

**Note:**
If \(M_{\text{max}}\) is required always take a section with distance \((x)\) above point O Because in cantilever sheet piles the maximum moment always above O.

See examples 9.1 and 9.3
**Anchored Sheet Piles**

When the height of the backfill soil behind a sheet pile exceeds 6m, the deflection on the sheet pile will be great and thereby the depth of penetration and the section of sheet pile will be large to meet this large deflection. To reduce this deflection, sheet pile should be supported from its upper edge (usually at distance 1m-2m from the top), this support is called anchor and the sheet pile with anchor called **Anchored Sheet Pile**.

There are two ways for analysis of anchored sheet piles:

- **Free Earth Support Method.**
- **Fixed Earth Support Method.**

In our discussion we will mainly discuss free earth support method.

The following figure shows anchored sheet pile:

![Anchored Sheet pile (Free Earth Support Method)](image)

**Anchored Sheet pile (Free Earth Support Method)**

In this method, the soil is assumed as a simply support (pin support) at the end of sheet pile, and also the wall is simply supported from its upper edge by anchor. So the deflection of sheet pile will be similar to the deflection of simply supported beam as shown in the following figure:
Important notes on the above figure:

1. As you see, the deflection of the sheet pile is similar to the deflection of simply supported beam, so if we need $M_{\text{max}}$ we take a section above the dredge line (at point of maximum deflection “zero shear”).

2. Note that the soil in right side at all depths pushes the wall to the left side, so the soil in the right side will exerts active LEP at all depths of sheet pile and no inflection point (as in cantilever sheet pile), also, under the dredge line, the soil on the wall will pushed into the left side soil, thus the LEP of the left soil is passive pressure to the end without any inflection (انقلاب).

3. Depending on note(2), when we drawing the net pressure distribution under the dredge line, the increase in pressure will be always in the left side (passive side) to the end without any inflection.

Now, we will sketch the pressure distribution for anchored sheet pile using free earth support method when the sheet pile penetrating in different types of soil.
Anchored Sheet Piles Penetrating Sandy Soil
Consider the following example:

The net pressure distribution along the sheet pile will be as following:
Below the dredge line:
The net lateral pressure $\Delta \sigma_h$ always will increased by the value of $\gamma z'(K_p - K_a)$. And since the soil is pure sand ($\phi > 0$) $\rightarrow K_p > K_a$ $\rightarrow$ the increase in pressure will be in the direction of passive zones. So, below dredge line, the stress will be increased gradually in the direction of passive zone till reaching the end of sheet pile because there is no inflection point (no change in LEP types) below the dredge line.

**Anchored Sheet Piles Penetrating Clay**

Consider the following example:

The net pressure distribution along the sheet pile will be as following:
Below the dredge line:
The net lateral pressure $\Delta \sigma_h$ always will increased by the value of $\gamma z' (K_p - K_a)$. But since the soil is pure clay ($\phi = 0$) $\rightarrow K_p = K_a$ the increase in pressure will zero till reaching the end of sheet pile because there is no changes in the type of LEP.

**Anchored Sheet Piles Penetrating C - $\phi$ Soil**
Consider the following example:
The net pressure distribution along the sheet pile will be as following:

Below the dredge line:
The net lateral pressure $\Delta \sigma_h$ always will increase by the value of $\gamma z'(K_p - K_a)$. And since the soil is $C - \phi$ soil ($\phi > 0$) $\rightarrow K_p > K_a \rightarrow$ the increase in pressure will be in the direction of passive zones. So, below dredge line, the stress will be increased gradually in the direction of passive zone till reaching the end of sheet pile because there is no inflection point (no change in LEP types) below the dredge line.
Problems
1.
For the anchored sheet pile shown below, do the following:
1. Draw the lateral earth pressure distribution with depth.
2. Calculate the depth of penetration (D).
3. Calculate the anchor force per unit length of the sheet pile.
4. Calculate section modulus if $\sigma_{ull}=175$ MPa.

Solution
The first is calculating $K_a$ and $K_p$ for each layer:

$K_a = \tan^2 \left( 45 - \frac{\phi}{2} \right)$

$K_{a1} = \tan^2 \left( 45 - \frac{33}{2} \right) = 0.29$

$K_{a2} = \tan^2 \left( 45 - \frac{28}{2} \right) = 0.36$
\[ K_{a3} = \tan^2 \left( 45 - \frac{22}{2} \right) = 0.45 \]

\[ K_p = \tan^2 \left( 45 + \frac{\Phi}{2} \right) \]

The required value of \( K_p \) is the value of the third layer (layer below the dredge line)

\[ K_{p3} = \tan^2 \left( 45 + \frac{22}{2} \right) = 2.2 \]

**Now we calculate the net LEP at each depth:**

\[ \sigma_{h,a} = (q + \gamma H) K_a - 2c\sqrt{K_a} \]

\[ \sigma_{h,p} = (q + \gamma H) K_p + 2c\sqrt{K_p} \]

@ \( z = 0 \) (above dredge line → active pressure only)

\[ \sigma_{h,a} = (70 + 18 \times 0) \times 0.29 - 2 \times 17 \times \sqrt{0.29} = 2 \text{ kN/m}^2 \]

\[ \Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 2 = -2 \] (2 in active direction)

@ \( z = 3 \text{m} \) (above dredge line → active pressure only)

just before \((K_a = 0.29 \ , \ C = 17)\)

\[ \sigma_{h,a} = (70 + 18 \times 3) \times 0.29 - 2 \times 17 \times \sqrt{0.29} = 17.65 \text{ kN/m}^2 \]

\[ \Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 17.65 = -17.65 \text{ kN/m}^2 \]

just after \((K_a = 0.36 \ , \ C = 27)\)

\[ \sigma_{h,a} = (70 + 18 \times 3) \times 0.36 - 2 \times 27 \times \sqrt{0.36} = 12.24 \text{ kN/m}^2 \]

\[ \Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 12.24 = -12.24 \text{ kN/m}^2 \]

@ \( z = 9 \text{m} \) (just before → active pressure only)\((K_a = 0.36 \ , \ C = 27)\)

\[ \sigma_{h,a} = (70 + 18 \times 3 + (19 - 10) \times 6) \times 0.36 - 2 \times 27 \times \sqrt{0.36} \]

\[ = 31.68 \text{ kN/m}^2 \]

\[ \Delta \sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 31.68 = -31.68 \text{ kN/m}^2 \]

@ \( z = 9 \text{m} \) (just after → active pressure at right, passive pressure at left)

And\((K_a = 0.45 \ , \ K_p = 2.2 \ , \ C = 50)\) below the drege line

Active pressure (at right)

\[ \sigma_{h,a} = (70 + 18 \times 3 + (19 - 10) \times 6) \times 0.45 - 2 \times 50 \times \sqrt{0.45} \]

\[ = 13 \text{ kN/m}^2 \]
Passive pressure (at left)
\[
\sigma_{h,p} = (0 + (19 - 10) \times 0) \times 2.2 + 2 \times 50 \times \sqrt{2.2} \\
= 148.3 \text{ kN/m}^2
\]
\[
\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 148.3 - 13 = 135.3 \text{ kN/m}^2
\]

@ \( z = 9 + D \) (Active pressure at right, Passive pressure at left "no inflection")

Active pressure at (at right):
\[
\sigma_{h,a} = (70 + 18 \times 3 + (19 - 10) \times 6 + (19 - 10) \times D) \times 0.45 \\
-2 \times 50 \times \sqrt{0.45} = 13 + 4.05 D
\]

Passive pressure (at left):
\[
\sigma_{h,p} = (0 + (19 - 10) \times D) \times 2.2 + 2 \times 50 \times \sqrt{2.2} \\
= 148.3 + 19.8 D
\]
\[
\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = (148.3 + 19.8 D) - (13 + 4.05 D) \\
= 135.3 + 15.75 D
\]

Note that the value of \((135.3 + 15.75 D)\) is positive which means we draw this value as a line in the passive zone at this depth (left side).

Now we draw the net stress distribution along the sheet pile:
As you see, there are two unknowns (D and F).
The most suitable method to find D and F is:
To find D → take \( \sum M_{@F} = 0.0 \)
To find F → take \( \sum F_{x} = 0.0 \)
Assume forces 1, 2, 3 and 4 are positive and forces F, 5 and 6 are negative.
We prepare the following table (to simplified the solution):

<table>
<thead>
<tr>
<th>Force</th>
<th>Magnitude</th>
<th>Arm from F</th>
<th>Moment about F</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>2 \times 3 = 6 (+)</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>0.5 \times (17.65 - 2) \times 3 = 23.48 (+)</td>
<td>1.5 - \frac{3}{3} = 0.5</td>
<td>11.74 (+)</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>12.24 \times 6 = 73.44 (+)</td>
<td>3 + 1.5 = 4.5</td>
<td>330.48 (+)</td>
</tr>
<tr>
<td>( P_4 )</td>
<td>0.5 \times (31.68 - 12.24) \times 6 = 58.32 (+)</td>
<td>6 - \frac{6}{3} + 1.5 = 5.5</td>
<td>320.76 (+)</td>
</tr>
<tr>
<td>( P_5 )</td>
<td>135.3 D (−)</td>
<td>7.5 + 0.5 D</td>
<td>1014.75D + 67.65D^2 (−)</td>
</tr>
<tr>
<td>( P_6 )</td>
<td>0.5 \times (15.75D) \times D = 7.88D^2 (−)</td>
<td>7.5 + \frac{2}{3}D</td>
<td>59.1D^2 + 5.25D^3 (−)</td>
</tr>
<tr>
<td>F</td>
<td>F (−)</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

\[ \sum M_{@F} = 0.0 \rightarrow 11.74 + 330.48 + 320.76 = 1014.75D + 67.65D^2 + 59.1D^2 + 5.25D^3 \]
\[ \rightarrow 5.25D^3 + 126.75D^2 + 1014.75D - 663 = 0.0 \]

By trial and error or by calculating → D = 0.606 m ✓.

\[ \sum M_{Fx} = 0.0 \rightarrow 6 + 23.48 + 73.44 + 58.32 = 135.3 \times 0.606 + 7.88 \times 0.606^2 + F \rightarrow F = 76.35 kN ✓. \]

Now to find section modulus:
\[ S = \frac{M_{\text{max}}}{\sigma_{\text{all}}} \]

So we need to calculate \( M_{\text{max}} \), and we mentioned previously, the maximum moment in anchored sheet pile will be above dredge line (at point of zero shear), so we can make a section above the dredge line and calculate \( M_{\text{max}} \) as following:
Now, to calculate the areas 3 and 4 in terms of X we must calculate the pressure at the distance X \( (\sigma_X) \) by interpolation.

\[
@X = 0.0 \rightarrow \sigma = 12.24, \quad @X = 6 \rightarrow \sigma = 31.68 \quad @X = X \rightarrow \sigma = \sigma_X
\]

\[
\frac{31.68 - 12.24}{6 - 0} = \frac{\sigma_X - 12.24}{X - 0} \rightarrow \sigma_X = 19.44X + 12.24
\]

Now we can calculate the force 3 and 4 in terms of X:

\[
P_3 = 12.24X
\]

\[
P_4 = 0.5 \times (\sigma_X - 12.24)X = 0.5 \times (19.44X + 12.24 - 12.24)X = 9.72X^2
\]

Now at distance X, summation forces must be zero (point of zero shear) to get maximum moment:

\[
F = P_1 + P_2 + P_3 + P_4 \quad (F, P_1 \ and \ P_2 \ are \ taken \ from \ table \ above)
\]

\[
76.35 = 6 + 23.48 + 12.24X + 9.72X^2
\]

\[
\rightarrow 9.72X^2 + 12.24X - 46.87 = 0
\]

\[
\rightarrow X = 1.65 \text{ m}
\]

\[
\rightarrow P_3 = 12.24 \times 1.65 = 20.2 \quad \text{and} \quad P_4 = 9.72 \times 1.65^2 = 26.46
\]
Now to get maximum moment, take summation moment at point E (point of zero shear):

\[ M_{\text{max}} = \sum M_{@E} \]

\[ M_{\text{max}} = 76.35 \times (1.5 + 1.65) - 6 \times (1.5 + 1.65) - 23.48 \times \left(\frac{3}{3} + 1.65\right) \]
\[ -20.2 \times \left(\frac{1.65}{2}\right) - 26.46 \times \left(\frac{1.65}{3}\right) \]

\[ \rightarrow M_{\text{max}} = 128.2 \text{ kN.m} \]

\[ \sigma_{\text{all}} = 175 \text{ MPa.} = 175,000 \text{ kPa. (kN/m}^2\text{)} \]

\[ S = \frac{M_{\text{max}}}{\sigma_{\text{all}}} = \frac{128.2}{175000} = 0.73 \times 10^{-3} \text{ m}^3/\text{m of wall} \checkmark. \]

**Important Note:**

In the above problem, the water table is at both sides of sheet pile, so pore water pressure will canceled from both sides, however if the water table is at one side of the sheet pile (right side) as shown in the following figure:

When calculating the pressure at right side (active pressure), pore water pressure must be added as following:

\[ \sigma_{h,a} = (q + \gamma H) K_a - 2c \sqrt{K_a} + \gamma_w \times h_w \]  (at each depth below the GWT)
2.
For the anchored sheet pile shown below, do the following:
1. Draw the lateral earth pressure distribution with depth.
2. Calculate the depth of penetration (D).
3. Calculate the anchor force per unit length of the sheet pile.

![Sheet pile diagram](image)

\[ q = 30 \text{kN/m}^2 \]

\[ \gamma = 17 \text{kN/m}^3 \]

\[ \phi = 36^\circ \]

\[ \gamma = 17 \text{kN/m}^3 \]

\[ \phi = 32^\circ \]

\[ \gamma = 18 \text{kN/m}^3 \]

\[ \phi = 20^\circ \]

\[ C = 30 \text{kN/m}^2 \]

**Solution**

Note that there is no water table in this problem.
Solve the problem by yourself with the same procedures in the problem above.
Final Answers:
D = 2.06 m  
F = 149 kN  
\( M_{\text{max}} = 351 \text{kN.m} \)

**See Examples 9.5 and 9.10** in your textbook