Chapter 5: 
Methods and Philosophy of Statistical Process Control

Learning Outcomes

After careful study of this chapter You should be able to:
• Understand chance and assignable causes of variation,
• Explain statistical basis of Shewhart control chart,
• Explain the rational subgroup concept,
• Understand the basic tools of SPC,
• Explain Phase I and Phase II use of control charts,
• Explain patterns on control charts,
Introduction

- **Statistical process control** is a collection of tools that when used together can result in process stability and variability reduction.

The **seven major tools** are:

1) Histogram or Stem and Leaf plot
2) Check Sheet
3) Pareto Chart
4) Cause and Effect Diagram
5) Defect Concentration Diagram
6) Scatter Diagram
7) Control Chart

Statistical Process Control (SPC)

- Uses statistics & control charts to tell when to adjust process.
- Developed by Shewhart in 1920’s.
- Involves:
  - Creating standards (upper & lower limits)
  - Measuring sample output (e.g. mean wgt.)
  - Taking corrective action (if necessary)
- Done while product is being produced.
A process that is operating with only chance causes of variation present is said to be in statistical control.

A process that is operating in the presence of assignable causes is said to be out of control.

The eventual goal of SPC is reduction or elimination of variability in the process by identification of assignable causes.

Tampering can make the process variation 1.41 times worse than not doing anything!!!
Basic Principles

A typical control chart has control limits set at values such that if the process is in control, nearly all points will lie between the upper control limit (UCL) and the lower control limit (LCL).
Out-of-Control Situations

• If at least one point plots beyond the control limits, the process is out of control
• If the points behave in a systematic or nonrandom manner, then the process could be out of control.

Relationship between hypothesis testing and control charts

• We have a process that we assume the true process mean is $\mu = 74$ and the process standard deviation is $\sigma = 0.01$. Samples of size 5 are taken giving a standard deviation of the sample average, $\bar{X}$, as

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.01}{\sqrt{5}} = 0.0045$$
**Statistical Basis of the Control Chart**

**Relationship between hypothesis testing and control charts**

- Control limits can be set at 3 standard deviations from the mean.
- This results in “3-Sigma Control Limits”
  \[ UCL = 74 + 3(0.0045) = 74.0135 \]
  \[ CL = 74 \]
  \[ LCL = 74 - 3(0.0045) = 73.9865 \]

- Choosing the control limits is equivalent to setting up the critical region for testing hypothesis
  \[ H_0: \mu = 75 \]
  \[ H_1: \mu \neq 75 \]
Important uses of the control chart

- Most processes do not operate in a state of statistical control.
- Consequently, the routine and attentive use of control charts will identify assignable causes. If these causes can be eliminated from the process, variability will be reduced and the process will be improved.
- The control chart only detects assignable causes. Management, operator, and engineering action will be necessary to eliminate the assignable causes.
- Out-of-control action plans (OCAPs) are an important aspect of successful control chart usage.
- Refer to the process improvement cycle.
Statistical Basis of the Control Chart

Types the control chart

- **Variables Control Charts**
  - These charts are applied to data that follow a continuous distribution (measurement data).

- **Attributes Control Charts**
  - These charts are applied to data that follow a discrete distribution.

Type of Process Variability

- Stationary behavior, uncorrelated data
- Stationary behavior, auto-correlated data
- Non-stationary behavior
Statistical Basis of the Control Chart

Type of Variability

- Shewhart control charts are most effective when the in-control process data is stationary and uncorrelated.

Statistical Basis of the Control Chart

Popularity of control charts

1) Control charts are a proven technique for improving productivity.
2) Control charts are effective in defect prevention.
3) Control charts prevent unnecessary process adjustment.
4) Control charts provide diagnostic information.
5) Control charts provide information about process capability.
Choice of Control Limits

*General model of a control chart*

\[ UCL = \mu_w + L\sigma_w \]
\[ \text{Center Line} = \mu_w \]
\[ LCL = \mu_w - L\sigma_w \]

where \( L \) = distance of the control limit from the center line
\( \mu_w \) = mean of the sample statistic, \( w \).
\( \sigma_w \) = standard deviation of the statistic, \( w \).

“99.7% of the Data”

- If approximately 99.7% of the data lies within \( 3\sigma \) of the mean (i.e., 99.7% of the data should lie within the control limits), then \( 1 - 0.997 = 0.003 \) or 0.3% of the data can fall outside \( 3\sigma \) (or 0.3% of the data lies outside the control limits). (Actually, we should use the more exact value 0.0027)

- 0.0027 is the probability of a Type I error or a false alarm in this situation.
Choice of Control Limits

Three-Sigma Limits

- The use of 3-sigma limits generally gives good results in practice.
- If the distribution of the quality characteristic is reasonably well approximated by the normal distribution, then the use of 3-sigma limits is applicable.
- These limits are often referred to as action limits.

Warning Limits on Control Charts

- Warning limits (if used) are typically set at 2 standard deviations from the mean.
- If one or more points fall between the warning limits and the control limits, or close to the warning limits the process may not be operating properly.
- Good thing: warning limits often increase the sensitivity of the control chart.
- Bad thing: warning limits could result in an increased risk of false alarms.
Sample Size and Sampling Frequency

- In designing a control chart, both the sample size to be selected and the frequency of selection must be specified.
- Larger samples make it easier to detect small shifts in the process.
- Current practice tends to favor smaller, more frequent samples.

Average Run Length

- The average run length (ARL) is a very important way of determining the appropriate sample size and sampling frequency.
- Let \( p \) = probability that any point exceeds the control limits. Then,

\[
ARL = \frac{1}{p}
\]
Consider a problem with control limits set at 3 standard deviations from the mean. The probability that a point plots beyond the control limits is again, 0.0027 (i.e., \( p = 0.0027 \)). Then the average run length is

\[
ARL = \frac{1}{0.0027} = 370
\]
Sample Size and Sampling Frequency

**Average Time to Signal**

- Sometimes it is more appropriate to express the performance of the control chart in terms of the average time to signal (ATS). Say that samples are taken at fixed intervals, h hours apart.

\[
\text{ATS} = \text{ARL} (h)
\]

Rational Subgroups

- Subgroups or samples should be selected so that if assignable causes are present, the chance for differences between subgroups will be maximized, while the chance for differences due to these assignable causes within a subgroup will be minimized.
Rational Subgroups

Selection of Rational Subgroups

- Select consecutive units of production.
  - Provides a “snapshot” of the process.
  - Effective at detecting process shifts.

- Select a random sample over the entire sampling interval.
  - Can be effective at detecting if the mean has wandered out-of-control and then back in-control.

Analysis of Patterns on Control Charts

Nonrandom patterns can indicate out-of-control conditions

- Patterns such as cycles, trends, are often of considerable diagnostic value (more about this in Chapter 6)
- Look for “runs” - this is a sequence of observations of the same type (all above the center line, or all below the center line)
- Runs of say 8 observations or more could indicate an out-of-control situation.
  - Run up: a series of observations are increasing
  - Run down: a series of observations are decreasing
Analysis of Patterns on Control Charts

Western Electric Handbook Rules (Should be used to increased the chance of detecting Mean shifts)

A process is considered out of control if any of the following occur:
1) One point plots outside the 3-sigma control limits.
2) Two out of three consecutive points plot beyond the 2-sigma warning limits.
3) Four out of five consecutive points plot at a distance of 1-sigma or beyond from the center line.
4) Eight consecutive points plot on one side of the center line.

The control chart

- The control chart is most effective when integrated into a comprehensive SPC program.
- The seven major SPC problem-solving tools should be used routinely to identify improvement opportunities.
- The seven major SPC problem-solving tools should be used to assist in reducing variability and eliminating waste.
The **seven major tools**

The **seven major tools** are

1) Histogram or Stem and Leaf plot
2) Check Sheet
3) Pareto Chart
4) Cause and Effect Diagram
5) Defect Concentration Diagram
6) Scatter Diagram
7) Control Chart

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**The “Magnificent Seven”**
Check Sheets
• Useful for collecting historical or current operating data about the process under investigation.
• Can provide a useful time-oriented summary of data

Pareto Chart
• The Pareto chart is a frequency distribution (or histogram) of attribute data arranged by category.
• Plot the frequency of occurrence of each defect type against the various defect types.
• Once a defect, error, or problem has been identified and isolated for further study, potential causes of this undesirable effect must be analyzed.
• Cause and effect diagrams are sometimes called fishbone diagrams because of their appearance.

How to Construct a Cause-and-Effect Diagram?
1. Define the problem or effect to be analyzed.
2. Form the team to perform the analysis. Often the team will uncover potential causes through brainstorming.
3. Draw the effect box and the center line.
4. Specify the major potential cause categories and join them as boxes connected to the center line.
5. Identify the possible causes and classify them into the categories in step 4. Create new categories, if necessary.
6. Rank order the causes to identify those that seem most likely to impact the problem.
7. Take corrective action.
Defect Concentration Diagram

- A **defect concentration diagram** is a picture of the unit, showing all relevant views.
- Various types of defects that can occur are drawn on the picture.
- The diagram is then analyzed to determine if the location of the defects on the unit provides any useful information about the potential causes of the defects.
The Rest of the “Magnificent Seven”

Scatter Diagram

- The scatter diagram is a plot of two variables that can be used to identify any potential relationship between the variables.
- The shape of the scatter diagram often indicates what type of relationship may exist.

Patterns to Look for in Control Charts

- Upper control limit
  - Target
  - Normal behavior
  - One plot out above (or below), investigate for cause.
  - Trends in either direction, 5 plots, investigate for cause of progressive change.

- Lower control limit
  - Two plots near lower (or upper) control. Investigate for cause.
  - Run of 5 above (or below) central line. Investigate for cause.
  - Erratic behavior, investigate.
End of Chapter 5