Chapter 7 – Control Charts Attributes

Learning Outcomes

After careful study of this chapter you should be able to do the following:

- Understand the statistical basis of the attributes control chart,
- Know how to design attributes control chart,
- Know how to set up and use the p-chart for fraction non-conforming,
- Know how to set up and use the np-control chart for the number of nonconforming items,
- Know how to set up and use the c-control chart for defects,
Learning Outcomes (cont.)

- Know how to set up and use the u-control chart for defects per unit,
- Use attributes control charts with variable sample size,
- Understand the advantages and disadvantages of attributes versus variables control charts,
- Understand the rational subgroup concept for attributes control charts,
- Determine the average run length for attributes control charts.

Introduction

- Data that can be classified into one of several categories or classifications is known as attribute data.
- Classifications such as conforming and nonconforming are commonly used in quality control.
- Another example of attributes data is the count of defects.
A nonconformity is a departure of a quality characteristic from its intended level or state that occurs with a severity sufficient to cause an associated product or service not to meet a specification requirement.

Defect is concerned with satisfying intended normal, or reasonably foreseeable, usage requirement.

Defect is appropriate for use when evaluation is in terms of usage.

Defective is analogous to defect and is appropriate for use when unit of product or service is evaluated in terms of usage rather than conformance to specifications.

Limitations of variable control charts: These charts cannot be used for quality characteristics which are attributes.

Nonconformity is appropriate for conformance to specifications.

The term Nonconforming Unit is used to describe a unit of product or service containing at least one nonconformity.
Control Charts for Fraction Nonconforming

- **Fraction nonconforming** is the ratio of the number of nonconforming items in a population to the total number of items in that population.
- Control charts for fraction nonconforming are based on the binomial distribution.

Recall: A quality characteristic follows a binomial distribution if:

1. All trials are independent.
2. Each outcome is either a “success” or “failure”.
3. The probability of success on any trial is given as p. The probability of a failure is 1-p.
4. The probability of a success is constant.
Control Charts for Fraction Nonconforming

- The binomial distribution with parameters $n \neq 0$ and $0 < p < 1$, is given by
  $$p(x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

- The mean and variance of the binomial distribution are
  $$\mu = np \quad \sigma^2 = np (1 - p)$$

Development of the Fraction Nonconforming Control Chart

Assume
- $n =$ number of units of product selected at random.
- $D =$ number of nonconforming units from the sample
- $p =$ probability of selecting a nonconforming unit from the sample.
- Then:
  $$P(D = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$
Control Charts for Fraction Nonconforming

Development of the Fraction Nonconforming Control Chart

- The sample fraction nonconforming is given as

\[ \hat{p} = \frac{D}{n} \]

where \( D \) is a random variable with mean and variance \( \hat{p} \)

\[ \mu = p \quad \sigma^2 = \frac{p(1-p)}{n} \]

The P Chart

- The P Chart is used for data that consist of the proportion of the number of occurrences of an event to the total number of occurrences.
- It is used in quality to report the fraction or percent nonconforming in a product, quality characteristic, or group of quality characteristics.
- The fraction nonconforming, \( p \), is usually small, say, 0.10 or less.
- Because the fraction nonconforming is very small, the subgroup sizes must be quite large to produce a meaningful chart.
The P Chart (cont.)

- It can be used to control one quality characteristic, as is done with X bar and R chart,
- Or to control a group of quality characteristics of the same type or of the same part,
- Or to control the entire product.
- It can be established to measure the quality produced by a work center, by a department, by a shift, or by an entire plant.
- The subgroup size of the P chart can be either variable or constant.

The P Chart (cont.) and Objectives of the P Chart

- It is frequently used to report the performance of an operator, group of operators, or management as a means of evaluating their quality performance.

Objectives of the P Chart:

1. Determine the average quality level: This information provides the process capability in terms of attributes.
2. Bring to the attention of management any changes in the average.
3. Improve the product quality: Ideas for quality improvement.
Objectives of the P Chart (cont.)

Objectives of the $P$ Chart cont’d:
4. Evaluate the quality performance of operating and management personnel.
5. Suggest places to use X bar and R chart: They are more sensitive to variation.
6. Determine acceptance criteria of a product before shipment to the customer.

The P Chart for Constant Subgroup Size

1. Select the quality characteristic(s):
   a) Single quality characteristic.
   b) Group of quality characteristics.
   c) A part.
   d) An entire product.
   e) A number of products.
   f) It can be established for performance control of an operator, work center, department, shift, plant, or corporation.
2. Determine the subgroup size and method:
   - The size of the subgroup is a function of the proportion nonconforming.
   - A minimum size of 50 is suggested as a starting point.

3. Collect the data:
   - At least 25 subgroups.
   - Different sources (Check sheet).
   - For each subgroup the proportion nonconforming is calculated by the formula $P = np/n$

4. Calculate the trial central line and the control limits:

5. Establish the revised central line and control limits.
The P Chart

- The P Chart is most effective if it is posted where operating and quality personnel can view it.
- The control limits are usually three standard deviations from the central value. Therefore, approximately 99% of the plotted points, $P$, will fall between the upper and lower control limits.
- A P Chart will also indicate long-range trends in quality, which will help to evaluate changes in personnel, methods, equipment, tooling, materials, and inspection techniques.
- $P$-chart is based on the binomial distribution.

The P Chart Variable Subgroup Size

1. Collect the data.
2. Determine the trial central line and control limits: Since the subgroup size changes each day, limits must be calculated for each day.
3. As the subgroup size gets larger, the control limits are closer together.
4. Establish revised central line and control limits:
If $P_0$ is known, the process of data collection and trial control limits is not necessary.

$P$ is the proportion (fraction) nonconforming in a single subgroup.

$P-\bar{Bar}$ is the average proportion (fraction) nonconforming of many subgroup.

$P_0$ is the standard or reference value of the proportion (fraction) nonconforming based on the best estimate of $P-\bar{Bar}$.

The $P$ Chart Variable Subgroup Size (cont.)

There are three different approaches to construct and operate a control chart with a variable sample size:

1. Variable- Width Control Limits,
2. Control Limits based on an average sample size,
3. The Standardized Control Chart
Control Charts for Fraction Nonconforming

**Standard Given**
- If a standard value of \( p \) is given, then the control limits for the fraction nonconforming are

\[
\begin{align*}
\text{UCL} &= p + 3\sqrt{\frac{p(1-p)}{n}} \\
\text{CL} &= p \\
\text{LCL} &= p - 3\sqrt{\frac{p(1-p)}{n}}
\end{align*}
\]

**No Standard Given**
- If no standard value of \( p \) is given, then the control limits for the fraction nonconforming are

\[
\begin{align*}
\text{UCL} &= \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \\
\text{CL} &= \bar{p} \\
\text{LCL} &= \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}
\end{align*}
\]

where

\[
\begin{align*}
\bar{p} &= \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m}
\end{align*}
\]
Control Charts for Fraction Nonconforming

Trial Control Limits

• Control limits that are based on a preliminary set of data can often be referred to as trial control limits.

The quality characteristic is plotted against the trial limits, if any points plot out of control, assignable causes should be investigated and points removed.

• With removal of the points, the limits are then recalculated.

\[
\text{Center Line (} \bar{p} \text{) } = (\bar{p}) = \frac{\text{Total No. of Defective in All Subgroups Under Investigation}}{\text{Total No. of Units Examined in All Subgroups Under Investigation}}
\]

\[
UCL (\bar{p}) = \bar{p} + 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}
\]

\[
UCL (p) = p - 3 \sqrt{\frac{p(1-p)}{n}}
\]
Example

- A process that produces bearing housings is investigated. Ten samples of size 100 are selected.

<table>
<thead>
<tr>
<th>Sample #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td># Nonconf.</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>8</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

- Is this process operating in statistical control?

How to Interpret Attribute Charts

- Points beyond limits- primary test.
  - Below lower limits means process has improved.
- Zone rules do not apply.
- Rules for trends, shifts do apply.

Only get One Chart !!
Control Charts for Fraction Nonconforming

Interpretation of Points on the Control Chart for Fraction Nonconforming

- Care must be exercised in interpreting points that plot below the lower control limit.
  - They often do not indicate a real improvement in process quality.
  - They are frequently caused by errors in the inspection process or improperly calibrated test and inspection equipment.

The P Chart Variable Subgroup Size (Example)

\[
\text{UCL} = \bar{p} + 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}} \\
\text{CL} = \bar{p} \\
\text{LCL} = \bar{p} - 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}
\]
The *np* control chart

Number Nonconforming Chart (*np*):

- The *np* chart is easier for operating personnel to understand than the *p* chart.
- The limitation that this chart has, is that the subgroup size needs to be constant.

The *np* control chart (cont.)

- The actual number of nonconforming can also be charted. Let *n* = sample size, *p* = proportion of nonconforming. The control limits are:

\[
\begin{align*}
\text{UCL} &= np + 3\sqrt{np(1-p)} \\
\text{CL} &= np \\
\text{LCL} &= np - 3\sqrt{np(1-p)}
\end{align*}
\]

(if a standard, *p*, is not given, use \(\overline{p}\))
Traditionally np charts are used only when subgroup size are constant. Data collected for an np chart will be a series of integer, each representing the number of nonconforming (or conforming) item in its subgroup.

**Center Line**

\[
\text{Center Line (np)} = \frac{\text{np}}{\text{np}} = \text{Subgroup Size} \div \text{Subgroups Under Investigation}
\]

**UCL (np)**

\[
\text{UCL (np)} = \text{np} + 3 \sqrt{\frac{\text{np}(1-p)}{\text{Total No. of Units Examined in All Subgroups Under Investigation}}}
\]

**LCL (np)**

\[
\text{LCL (np)} = \text{np} - 3 \sqrt{\frac{\text{np}(1-p)}{\text{Total No. of Units Examined in All Subgroups Under Investigation}}}
\]

The **np control chart (cont.)**

\[
\begin{align*}
\text{UCL} &= np + 3\sqrt{np(1-p)} \\
\text{CL} &= np \\
\text{LCL} &= np - 3\sqrt{np(1-p)}
\end{align*}
\]
The np control chart (Example)

\[
\begin{align*}
UCL &= np + 3 \sqrt{np(1-p)} \\
CL &= np \\
LCL &= np - 3 \sqrt{np(1-p)} 
\end{align*}
\]

Defect vs. Defective

- ‘Defect’ – a single nonconforming quality characteristic.
- ‘Defective’ – items having one or more defects.
Comparison of Variables v. Attributes

- **Variables**
  - Fit certain cases.
  - Both mean and variation information.
  - More expensive?
  - Identify mean shifts sooner before large number nonconforming.

- **Attributes**
  - Fit certain cases – taste, color, etc.
  - Larger sample sizes.
  - Provides summary level performance.
  - Must define nonconformity.

Advantages of attribute control charts

- Allowing for quick summaries, that is, the engineer may simply classify products as acceptable or unacceptable, based on various quality criteria.
- Thus, attribute charts sometimes bypass the need for expensive, precise devices and time-consuming measurement procedures.
- More easily understood by managers unfamiliar with quality control procedures.
Advantages of attribute control charts (cont.)

- More sensitive than variable control charts.
- Therefore, variable control charts may alert us to quality problems before any actual "unacceptable" (as detected by the attribute chart) will occur.
- Montgomery (1985) calls the variable control charts leading indicators of trouble that will sound an alarm before the number of rejects (scrap) increases in the production process.

Example

\[ n = 100, \ m = 10 \]

<table>
<thead>
<tr>
<th>Sample #</th>
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<td>4</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Fraction Nonconf.</td>
<td>0.05</td>
<td>0.02</td>
<td>0.03</td>
<td>0.08</td>
<td>0.04</td>
<td>0.01</td>
<td>0.02</td>
<td>0.06</td>
<td>0.03</td>
<td>0.04</td>
</tr>
</tbody>
</table>

\[
\bar{p} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m} = 0.038
\]
Control Charts for Fraction Nonconforming

**Example**

Control Limits are:

\[
\begin{align*}
UCL &= 0.038 + 3 \sqrt{\frac{0.038(1-0.038)}{100}} = 0.095 \\
CL &= 0.038 \\
LCL &= 0.038 - 3 \sqrt{\frac{0.038(1-0.038)}{100}} = -0.02 \rightarrow 0
\end{align*}
\]
End of Chapters (7 -1)