Chapter 14: Inventory Management

Learning Outcomes

After successful studying this chapter You should be able to:

• Discuss the role of information technology in managing inventories.
• Describe the functions and costs of an inventory system.
• Determine the order quantity.
• Determine the reorder point and safety stock for inventory systems.
• Design a continuous or periodic review inventory-control system.
• Conduct an ABC analysis of inventory items.
• Determine the order quantity for the single-period inventory case.
• Describe the rationale behind the retail discounting model.
Overview

- Opposing Views of Inventories
- Nature of Inventories
- Fixed Order Quantity Systems
- Fixed Order Period Systems
- Other Inventory Models
- Some Realities of Inventory Planning
- Wrap-Up: What World-Class Companies Do

Opposing Views of Inventory

- Why We Want to Hold Inventories
- Why We Not Want to Hold Inventories
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**Why We Want to Hold Inventories?**

- Improve customer service
- Reduce certain costs such as
  - ordering costs
  - stockout costs
  - acquisition costs
  - start-up quality costs
- Contribute to the efficient and effective operation of the production system

**Why We Want to Hold Inventories?**

- Finished Goods
  - Essential in produce-to-stock positioning strategies
  - Necessary in level aggregate capacity plans
  - Products can be displayed to customers
- Work-in-Process
  - Necessary in process-focused production
  - May reduce material-handling & production costs
- Raw Material
  - Suppliers may produce/ship materials in batches
  - Quantity discounts and freight/handling $$ savings
Why We do not Want to Hold Inventories?

- Certain costs increase such as
  - carrying costs
  - cost of customer responsiveness
  - cost of coordinating production
  - cost of diluted return on investment
  - reduced-capacity costs
  - large-lot quality cost
  - cost of production problems

Nature of Inventory

- Two Fundamental Inventory Decisions
- Terminology of Inventories
- Independent Demand Inventory Systems
- Dependent Demand Inventory Systems
- Inventory Costs
Two Fundamental Inventory Decisions

- How much to order of each material when orders are placed with either outside suppliers or production departments within organizations
- When to place the orders

Independent Demand Inventory Systems

- Demand for an item carried in inventory is independent of the demand for any other item in inventory
- Finished goods inventory is an example
- Demands are estimated from forecasts (Chapter 3) and/or customer orders
**Dependent Demand Inventory Systems**

- Items whose demand depends on the demands for other items
- For example, the demand for raw materials and components can be calculated from the demand for finished goods
- The systems used to manage these inventories (Chapter 15) are different from those used to manage independent demand items

**Inventory Costs**

- Costs associated with ordering too much (represented by carrying costs)
- Costs associated with ordering too little (represented by ordering costs)
- These costs are opposing costs, i.e., as one increases the other decreases
- . . . more
The sum of the two costs is the total stocking cost (TSC). When plotted against order quantity, the TSC decreases to a minimum cost and then increases. This cost behavior is the basis for answering the first fundamental question: how much to order. It is known as the economic order quantity (EOQ).
Fixed Order Quantity Systems

- Behavior of Economic Order Quantity (EOQ) Systems
- Determining Order Quantities
- Determining Order Points

Behavior of EOQ Systems

- As demand for the inventoried item occurs, the inventory level drops
- When the inventory level drops to a critical point, the order point, the ordering process is triggered
- The amount ordered each time an order is placed is fixed or constant
- When the ordered quantity is received, the inventory level increases
- . . . more
Behavior of EOQ Systems

- An application of this type system is the two-bin system
- A perpetual inventory accounting system is usually associated with this type of system

Determining Order Quantities

- Basic EOQ
- EOQ for Production Lots
- EOQ with Quantity Discounts
Model I: Basic EOQ

• Typical assumptions made
  – annual demand (D), carrying cost (C) and ordering cost (S) can be estimated
  – average inventory level is the fixed order quantity (Q) divided by 2 which implies
    • no safety stock
    • orders are received all at once
    • demand occurs at a uniform rate
    • no inventory when an order arrives
  – . . . more

• Assumptions (continued)
  – Stockout, customer responsiveness, and other costs are inconsequential
  – acquisition cost is fixed, i.e., no quantity discounts

• Annual carrying cost = (average inventory level) x (carrying cost) = (Q/2)C
• Annual ordering cost = (average number of orders per year) x (ordering cost) = (D/Q)S
• . . . more
Model I: Basic EOQ

- Total annual stocking cost (TSC) = annual carrying cost + annual ordering cost = \( \frac{Q}{2}C + \frac{D}{Q}S \)
- The order quantity where the TSC is at a minimum (EOQ) can be found using calculus (take the first derivative, set it equal to zero and solve for Q)

\[
EOQ = \sqrt{\frac{2DS}{C}}
\]

Example: Basic EOQ

Zartex Co. produces fertilizer to sell to wholesalers. One raw material – calcium nitrate – is purchased from a nearby supplier at $22.50 per ton. Zartex estimates it will need 5,750,000 tons of calcium nitrate next year.

The annual carrying cost for this material is 40% of the acquisition cost, and the ordering cost is $595.

a) What is the most economical order quantity?
b) How many orders will be placed per year?
c) How much time will elapse between orders?
Example: Basic EOQ

- Economical Order Quantity (EOQ)
  
  \[ D = 5,750,000 \text{ tons/year} \]
  
  \[ C = 0.40(22.50) = $9.00/\text{ton/year} \]
  
  \[ S = $595/\text{order} \]

\[
\text{EOQ} = \sqrt{\frac{2DS}{C}}
\]

\[
= \sqrt{\frac{2(5,750,000)(595)}{9.00}}
\]

\[
= 27,573.135 \text{ tons per order}
\]

Example: Basic EOQ

- Total Annual Stocking Cost (TSC)

\[
\text{TSC} = \frac{Q}{2}C + \frac{D}{Q}S
\]

\[
= \left( \frac{27,573.135}{2} \right)(9.00)
\]

\[
+ \left( \frac{5,750,000}{27,573.135} \right)(595)
\]

\[
= 124,079.11 + 124,079.11
\]

\[
= $248,158.22
\]

Note: Total Carrying Cost equals Total Ordering Cost
Example: Basic EOQ

- Number of Orders Per Year
  \[ \text{Number of Orders Per Year} = \frac{D}{Q} \]
  \[ = \frac{5,750,000}{27,573.135} \]
  \[ = 208.5 \text{ orders/year} \]

- Time Between Orders
  \[ \text{Time Between Orders} = \frac{Q}{D} \]
  \[ = \frac{1}{208.5} \]
  \[ = 0.004796 \text{ years/order} \]
  \[ = 0.004796 \times 365 \text{ days/year} = 1.75 \text{ days/order} \]

Note: This is the inverse of the formula above.

Model II: EOQ for Production Lots

- Used to determine the order size, production lot, if an item is produced at one stage of production, stored in inventory, and then sent to the next stage or the customer
- Differs from Model I because orders are assumed to be supplied or produced at a uniform rate (p) rate rather than the order being received all at once
- ... more
Model II: EOQ for Production Lots

- It is also assumed that the supply rate, p, is greater than the demand rate, d
- The change in maximum inventory level requires modification of the TSC equation
- \( TSC = \frac{Q}{2} \left[ \frac{(p-d)}{p} \right] C + \frac{D}{Q} S \)
- The optimization results in

\[
EOQ = \sqrt{\frac{2DS}{C} \left[ \frac{p}{p-d} \right]}
\]

Example: EOQ for Production Lots

Highland Electric Co. buys coal from Cedar Creek Coal Co. to generate electricity. CCCC can supply coal at the rate of 3,500 tons per day for $10.50 per ton. HEC uses the coal at a rate of 800 tons per day and operates 365 days per year.

HEC’s annual carrying cost for coal is 20% of the acquisition cost, and the ordering cost is $5,000.

a) What is the economical production lot size?
b) What is HEC’s maximum inventory level for coal?
Example: EOQ for Production Lots

- Economical Production Lot Size
  
  \[ d = 800 \text{ tons/day}; \quad D = 365(800) = 292,000 \text{ tons/year} \]
  
  \[ p = 3,500 \text{ tons/day} \]
  
  \[ S = $5,000/\text{order} \quad C = .20(10.50) = $2.10/\text{ton/year} \]

  \[
  EOQ = \sqrt{\frac{2DS}{C} \cdot \frac{p}{p-d}} \\
  EOQ = \sqrt{\frac{2(292,000)(5,000)}{2.10} \cdot \frac{3,500}{3,500-800}} \\
  = 42,455.5 \text{ tons per order}
  \]

- Total Annual Stocking Cost (TSC)

  \[
  TSC = \frac{Q}{2} \cdot \frac{(p-d)}{p} \cdot C + \frac{D}{Q} \cdot S \\
  = \frac{(42,455.5/2)(3,500-800)/3,500)(2.10)}{292,000/42,455.5)(5,000) \\
  = 34,388.95 + 34,388.95 \\
  = 68,777.90
  \]

  Note: Total Carrying Cost equals Total Ordering Cost
Example: EOQ for Production Lots

• Maximum Inventory Level
  \[ \frac{Q(p-d)}{p} = \frac{42,455.5(3,500 - 800)}{3,500} = 42,455.5(0.771429) = 32,751.4 \text{ tons} \]

Note: HEC will use 23% of the production lot by the time it receives the full lot.

Model III: EOQ with Quantity Discounts

• Under quantity discounts, a supplier offers a lower unit price if larger quantities are ordered at one time
• This is presented as a price or discount schedule, i.e., a certain unit price over a certain order quantity range
• This means this model differs from Model I because the acquisition cost (ac) may vary with the quantity ordered, i.e., it is not necessarily constant
• . . . more
Model III: EOQ with Quantity Discounts

- Under this condition, acquisition cost becomes an incremental cost and must be considered in the determination of the EOQ.
- The total annual material costs (TMC) = Total annual stocking costs (TSC) + annual acquisition cost

\[ TSC = \left(\frac{Q}{2}\right)C + \left(\frac{D}{Q}\right)S + (D)ac \]

- ...more

To find the EOQ, the following procedure is used:

1. Compute the EOQ using the lowest acquisition cost.
   - If the resulting EOQ is feasible (the quantity can be purchased at the acquisition cost used), this quantity is optimal and you are finished.
   - If the resulting EOQ is not feasible, go to Step 2
2. Identify the next higher acquisition cost.
Model III: EOQ with Quantity Discounts

3. Compute the EOQ using the acquisition cost from Step 2.
   – If the resulting EOQ is feasible, go to Step 4.
   – Otherwise, go to Step 2.
4. Compute the TMC for the feasible EOQ (just found in Step 3) and its corresponding acquisition cost.
5. Compute the TMC for each of the lower acquisition costs using the minimum allowed order quantity for each cost.
6. The quantity with the lowest TMC is optimal.

Example: EOQ with Quantity Discounts

A-1 Auto Parts has a regional tire warehouse in Atlanta. One popular tire, the XRX75, has estimated demand of 25,000 next year. It costs A-1 $100 to place an order for the tires, and the annual carrying cost is 30% of the acquisition cost. The supplier quotes these prices for the tire:

<table>
<thead>
<tr>
<th>Q</th>
<th>ac</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 499</td>
<td>$21.60</td>
</tr>
<tr>
<td>500 – 999</td>
<td>20.95</td>
</tr>
<tr>
<td>1,000 +</td>
<td>20.90</td>
</tr>
</tbody>
</table>
Example: EOQ with Quantity Discounts

- Economical Order Quantity
  \[ EOQ_i = \sqrt{\frac{2DS}{C_i}} \]

  \[ EOQ_3 = \sqrt{2(25,000)100/(.3(20.90))} = 893.00 \]

  This quantity is not feasible, so try \( ac = $20.95 \)

  \[ EOQ_2 = \sqrt{2(25,000)100/(.3(20.95))} = 891.93 \]

  This quantity is feasible, so there is no reason to try \( ac = $21.60 \)

Example: EOQ with Quantity Discounts

- Compare Total Annual Material Costs (TMCs)
  \[ TMC = \left(\frac{Q}{2}\right)C + \frac{(D/Q)S}{(D)}ac \]

  Compute TMC for \( Q = 891.93 \) and \( ac = $20.95 \)

  \[ TMC_2 = \left(\frac{891.93}{2}\right)(.3)(20.95) + (25,000/891.93)100 \]
  \[ + (25,000)20.95 \]
  \[ = 2,802.89 + 2,802.91 + 523,750 \]
  \[ = $529,355.80 \]

... more
Example: EOQ with Quantity Discounts

Compute TMC for $Q = 1,000$ and $ac = $20.90

$$TMC_3 = \left(\frac{1,000}{2}\right)(.3)(20.90) + \left(\frac{25,000}{1,000}\right)100 + (25,000)20.90$$

$$= 3,135.00 + 2,500.00 + 522,500$$

$$= $528,135.00 \text{ (lower than } TMC_2)$$

The EOQ is 1,000 tires at an acquisition cost of $20.90.

Determining Order Points

• Basis for Setting the Order Point
• DDLT Distributions
• Setting Order Points
Basis for Setting the Order Point

• In the fixed order quantity system, the ordering process is triggered when the inventory level drops to a critical point, the order point.
• This starts the lead time for the item.
• Lead time is the time to complete all activities associated with placing, filling and receiving the order.

• During the lead time, customers continue to draw down the inventory.
• It is during this period that the inventory is vulnerable to stock out (run out of inventory).
• Customer service level is the probability that a stock out will not occur during the lead time.

• ... more
**Basis for Setting the Order Point**

- The order point is set based on
  - the demand during lead time (DDLT) and
  - the desired customer service level
- Order point (OP) = Expected demand during lead time (EDDLT) + Safety stock (SS)
- The amount of safety stock needed is based on the degree of uncertainty in the DDLT and the customer service level desired

**DDLT Distributions**

- If there is variability in the DDLT, the DDLT is expressed as a distribution
  - discrete
  - continuous
- In a discrete DDLT distribution, values (demands) can only be integers
- A continuous DDLT distribution is appropriate when the demand is very high
Setting Order Point for a Discrete DDLT Distribution

- Assume a probability distribution of actual DDLTs is given or can be developed from a frequency distribution
- Starting with the lowest DDLT, accumulate the probabilities. These are the service levels for DDLTs
- Select the DDLT that will provide the desired customer level as the order point

Example: OP for Discrete DDLT Distribution

One of Sharp Retailer’s inventory items is now being analyzed to determine an appropriate level of safety stock. The manager wants an 80% service level during lead time. The item’s historical DDLT is:

<table>
<thead>
<tr>
<th>DDLT (cases)</th>
<th>Occurrences</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>
OP for Discrete DDLT Distribution

• Construct a Cumulative DDLT Distribution

<table>
<thead>
<tr>
<th>DDLT (cases)</th>
<th>Probability of DDLT</th>
<th>Probability of DDLT or Less</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>.4</td>
<td>.4</td>
</tr>
<tr>
<td>4</td>
<td>.3</td>
<td>.7</td>
</tr>
<tr>
<td>5</td>
<td>.2</td>
<td>.9</td>
</tr>
<tr>
<td>6</td>
<td>.1</td>
<td>1.0</td>
</tr>
</tbody>
</table>

To provide 80% service level, OP = 5 cases

OP for Discrete DDLT Distribution

• Safety Stock (SS)

\[
\begin{align*}
\text{OP} &= \text{EDDLT} + \text{SS} \\
\text{SS} &= \text{OP} - \text{EDDLT} \\
\text{EDDLT} &= .4(3) + .3(4) + .2(5) + .1(6) = 4.0 \\
\text{SS} &= 5 - 4 = 1
\end{align*}
\]
Setting Order Point for a Continuous DDLT Distribution

- Assume that the lead time (LT) is constant
- Assume that the demand per day is normally distributed with the mean ($d$) and the standard deviation ($\sigma_d$)
- The DDLT distribution is developed by “adding” together the daily demand distributions across the lead time
- . . . more

\[
\sigma_{\text{DDLT}} = \sqrt{\text{LT}(\sigma_d)^2}
\]

EDDLT = LT(d)
Setting Order Point for a Continuous DDLT Distribution

- The customer service level is converted into a Z value using the normal distribution table.
- The safety stock is computed by multiplying the Z value by $\sigma_{DDL}$.
- The order point is set using $OP = EDDL + SS$, or by substitution.

$$OP = LT(d) + z \sqrt{LT(\sigma_d)^2}$$

Example: OP - Continuous DDLT Distribution

Auto Zone sells auto parts and supplies including a popular multi-grade motor oil. When the stock of this oil drops to 20 gallons, a replenishment order is placed. The store manager is concerned that sales are being lost due to stock outs while waiting for an order. It has been determined that lead time demand is normally distributed with a mean of 15 gallons and a standard deviation of 6 gallons.

The manager would like to know the probability of a stock out during lead time.
Example: OP - Continuous DDLT Distribution

- EDDLТ = 15 gallons
- $\sigma_{DDLТ} = 6$ gallons

$$OP = EDDLТ + Z(\sigma_{DDLТ})$$

20 = 15 + Z(6)
5 = Z(6)
Z = 5/6
Z = .833

Example: OP - Continuous DDLТ Distribution

- Standard Normal Distribution

Area = .2967
Area = .2033
Area = .5
0 833
Example: OP - Continuous DDLT Distribution

- The Standard Normal table shows an area of .2967 for the region between the $z = 0$ line and the $z = .833$ line. The shaded tail area is $.5 - .2967 = .2033$.
- The probability of a stockout during lead time is .2033

Rules of Thumb in Setting OP

- Set safety stock level at a percentage of EDDLT
  
  $\text{OP} = \text{EDDLT} + j(\text{EDDLT})$

  where $j$ is a factor between 0 and 3.

- Set safety stock level at square root of EDDLT
  
  $\text{OP} = \text{EDDLT} + \sqrt{\text{EDDLT}}$
Fixed Order Period Systems

- Behavior of Economic Order Period (EOP) Systems
- Economic Order Period Model

Behavior of Economic Order Period Systems

- As demand for the inventoried item occurs, the inventory level drops
- When a prescribed period of time (EOP) has elapsed, the ordering process is triggered, i.e., the time between orders is fixed or constant
- At that time the order quantity is determined using order quantity = upper inventory target - inventory level + EDDLT
- . . . more
Behavior of Economic Order Period Systems

- After the lead time elapses, the ordered quantity is received, and the inventory level increases.
- The upper inventory level may be determined by the amount of space allocated to an item.
- This system is used where it is desirable to physically count inventory each time an order is placed.

Determining the EOP

- Using an approach similar to that used to derive EOQ, the optimal value of the fixed time between orders is derived to be

\[ EOP = \frac{\sqrt{2S}}{DC} \]
Other Inventory Models

- Hybrid Inventory Models
- Single-Period Inventory Models

Hybrid Inventory Models

- Optional replenishment model
  - Similar to the fixed order period model
  - Unless inventory has dropped below a prescribed level when the order period has elapsed, no order is placed
  - Protects against placing very small orders
  - Attractive when review and ordering costs are large

... more
Hybrid Inventory Models

- **Base stock model**
  - Start with a certain inventory level
  - Whenever a withdrawal is made, an order of equal size is placed
  - Ensures that inventory maintained at an approximately constant level
  - Appropriate for very expensive items with small ordering costs

Single Period Inventory Models

- Order quantity decision covers only one period
- Appropriate for perishable items, e.g., fashion goods, certain foods, magazines
- Payoff tables may be used to analyze the decision under uncertainty
- ... more
Single Period Inventory Models

- One of the following rules can be used in the analysis
  - greatest profit
  - least total expected long and short costs
  - least total expected costs

Some Realities of Inventory Planning

- ABC Classification
- EOQ and Uncertainty
- Dynamics of Inventory Planning
ABC Classification

• Start with the inventoried items ranked by dollar value in inventory in descending order
• Plot the cumulative dollar value in inventory versus the cumulative items in inventory
• ... more

ABC Classification

• Typical observations
  – A small percentage of the items (Class A) make up a large percentage of the inventory value
  – A large percentage of the items (Class C) make up a small percentage of the inventory value
• These classifications determine how much attention should be given to controlling the inventory of different items
EOQ and Uncertainty

- The TSC and TMC curves are relatively flat, therefore moving left or right of the optimal order quantity on the order quantity axis has little effect on the costs
- Estimation errors of the values of parameter used to compute an EOQ usually do not have a significant impact on total costs
- . . . more

EOQ and Uncertainty

- Many costs are not directly incorporated in the EOQ and EOP formulas, but could be important factors
- Emergency procedures to replenish inventories quickly should be established
Dynamics of Inventory Planning

- Continually review ordering practices and decisions
- Modify to fit the firm's demand and supply patterns
- Constraints, such as storage capacity and available funds, can impact inventory planning
- Computers and information technology are used extensively in inventory planning

Wrap-Up: World-Class Practice

- Inventory cycle is the central focus of independent demand inventory systems
- Production planning and control systems are changing to support lean inventory strategies
- Information systems electronically link supply chain
End of Chapter 14