1

TRIANGULATION AND TRILATERATION

1.1 GENERAL

The horizontal positions of points is a network developed to provide accurate control for topographic mapping, charting lakes, rivers and ocean coast lines, and for the surveys required for the design and construction of public and private works of large extent. The horizontal positions of the points can be obtained in a number of different ways in addition to traversing. These methods are triangulation, trilateration, intersection, resection, and satellite positioning.

The method of surveying called *triangulation* is based on the trigonometric proposition that if one side and two angles of a triangle are known, the remaining sides can be computed. Furthermore, if the direction of one side is known, the directions of the remaining sides can be determined. A triangulation system consists of a series of joined or overlapping triangles in which an occasional side is measured and remaining sides are calculated from angles measured at the vertices of the triangles. The vertices of the triangles are known as *triangulation stations*. The side of the triangle whose length is predetermined, is called the *base line*. The lines of triangulation system form a network that ties together all the triangulation stations (Fig. 1.1).

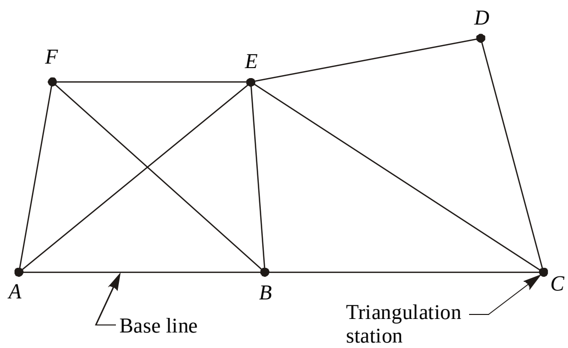

Fig. 1.1 Triangulation network

A *trilateration system* also consists of a series of joined or overlapping triangles. However, for trilateration the lengths of all the sides of the triangle are measured and few directions or angles are measured to establish azimuth. Trilateration has become feasible with the development of electronic distance measuring (EDM) equipment which has made possible the measurement of all lengths with high order of accuracy under almost all field conditions.

A combined triangulation and trilateration system consists of a network of triangles in which all the angles and all the lengths are measured. Such a combined system represents the strongest network for creating horizontal control.

Since a triangulation or trilateration system covers very large area, the curvature of the earth has to be taken into account. These surveys are, therefore, invariably geodetic. Triangulation surveys were first carried out by Snell, a Dutchman, in 1615.

Field procedures for the establishment of trilateration station are similar to the procedures used for triangulation, and therefore, henceforth in this chapter the term triangulation will only be used.

1.2 PRINCIPLE OF TRIANGULATION

Fig. 1.2 shows two interconnected triangles *ABC* and *BCD*. All the angles in both the triangles and the length *L* of the side *AB*, have been measured.

Also the azimuth θ of *AB* has been measured at the triangulation station *A*, whose coordinates (X_A, Y_A), are known.

The objective is to determine the coordinates of the triangulation stations *B*, *C*, and *D* by the method of triangulation. Let us first calculate the lengths of all the lines.

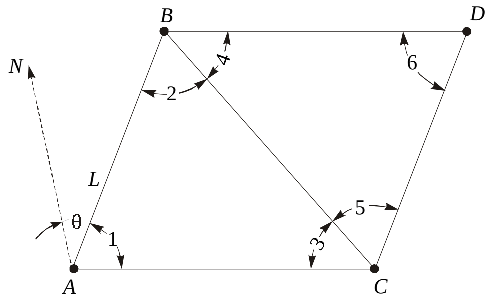

Fig. 1.2 Principle of triangulation

By sine rule in ΔABC, we have

$$\frac{AB}{\sin 3} = \frac{BC}{\sin 1} = \frac{CA}{\sin 2}$$

We have $\quad AB = L = l_{AB}$

or $\quad BC = \dfrac{L \sin 1}{\sin 3} = l_{BC}$

and $\quad CA = \dfrac{L \sin 2}{\sin 3} = l_{CA}$

Now the side *BC* being known in ΔBCD, by sine rule, we have

$$\frac{BC}{\sin 6} = \frac{CD}{\sin 4} = \frac{BD}{\sin 5}$$

We have $\quad BC = \dfrac{L \sin 1}{\sin 3} = l_{BC}$

or $\quad CD = \left(\dfrac{L \sin 1}{\sin 3}\right)\dfrac{\sin 4}{\sin 6} = l_{CD}$

and $\quad BC = \left(\dfrac{L \sin 1}{\sin 3}\right)\dfrac{\sin 5}{\sin 6} = l_{BD}$

Let us now calculate the azimuths of all the lines.

Azimuth of $AB = \theta = \theta_{AB}$

Azimuth of $AC = \theta + \angle 1 = \theta_{AC}$

Azimuth of $BC = \theta + 180° - \angle 2 = \theta_{BC}$

Azimuth of $BD = \theta + 180° - (\angle 2 + \angle 4) = \theta_{BD}$

Azimuth of $CD = \theta - \angle 2 + \angle 5 = \theta_{CD}$

From the known lengths of the sides and the azimuths, the consecutive coordinates can be computed as below.

Latitude of $AB = l_{AB} \cos\theta_{AB} = L_{AB}$

Departure of $AB = l_{AB} \sin\theta_{AB} = D_{AB}$

Latitude of $AC = l_{AC} \cos\theta_{AC} = L_{AC}$

Departure of $AC = l_{AC} \sin\theta_{AC} = D_{AC}$

Latitude of $BD = l_{BD} \cos\theta_{BD} = L_{BD}$

Departure of $BD = l_{BD} \sin\theta_{BD} = L_{BD}$

Latitude of $CD = l_{CD} \cos\theta_{CD} = L_{CD}$
Departure of $CD = l_{CD} \sin\theta_{CD} = D_{CD}$

The desired coordinates of the triangulation stations *B*, *C*, and *D* are as follows :

X-coordinate of *B*, $X_B = X_A + D_{AB}$

Y-coordinate of *B*, $Y_B = Y_B + L_{AB}$

X-coordinate of *C*, $X_C = X_A + D_{AC}$

Y-coordinate of *C*, $Y_C = Y_A + L_{AC}$

X-coordinate of *D*, $X_D = X_B + D_{BD}$

Y-coordinate of *D*, $Y_D = Y_B + L_{BD}$

It would be found that the length of side can be computed more than once following different routes, and therefore, to achieve a better accuracy, the mean of the computed lengths of a side is to be considered.

1.3 OBJECTIVE OF TRIANGULATION SURVEYS

The main objective of triangulation or trilateration surveys is to provide a number of stations whose relative and absolute positions, horizontal as well as vertical, are accurately established. More detailed location or engineering survey are then carried out from these stations.

The triangulation surveys are carried out

(*i*) to establish accurate control for plane and geodetic surveys of large areas, by terrestrial methods,

(*ii*) to establish accurate control for photogrammetric surveys of large areas,

(*iii*) to assist in the determination of the size and shape of the earth by making observations for latitude, longitude and gravity, and

(*iv*) to determine accurate locations of points in engineering works such as :

- (*a*) Fixing centre line and abutments of long bridges over large rivers.
- (*b*) Fixing centre line, terminal points, and shafts for long tunnels.
- (*c*) Transferring the control points across wide sea channels, large water bodies, etc.
- (*d*) Detection of crustal movements, etc.
- (*e*) Finding the direction of the movement of clouds.

1.4 CLASSIFICATION OF TRIANGULATION SYSTEM

Based on the extent and purpose of the survey, and consequently on the degree of accuracy desired, triangulation surveys are classified as *first-order* or *primary*, *second-order* or *secondary*, and *third-order* or *tertiary*. First-order triangulation is used to determine the shape and size of the earth or to cover a vast area like a whole country with control points to which a second-order triangulation system can be connected. A second-order triangulation system consists of a network within a first-order triangulation. It is used to cover areas of the order of a region, small country, or province. A third-order triangulation is a framework fixed within and connected to a second-order triangulation system. It serves the purpose of furnishing the immediate control for detailed engineering and location surveys.

Table 1.1 Triangulation system

S.No.	Characteristics	First-order triangulation	Second-order triangulation	Third-order triangulation
1.	Length of base lines	8 to 12 km	2 to 5 km	100 to 500 m
2.	Lengths of sides	16 to 150 km	10 to 25 km	2 to 10 km
3.	Average triangular error (after correction for spherical excess)	less than 1″	3″	12″
4.	Maximum station closure	not more than 3″	8″	15″
5.	Actual error of base	1 in 50,000	1 in 25,000	1 in 10,000
6.	Probable error of base	1 in 10,00,000	1 in 500,000	1 in 250,000
7.	Discrepancy between two measures (k is distance in kilometre)	$5\sqrt{k}$ mm	$10\sqrt{k}$ mm	$25\sqrt{k}$ mm
8.	Probable error of the computed distances	1 in 50,000 to 1 in 250,000	1 in 20,000 to 1 in 50,000	1 in 5,000 to 1 in 20,000
9.	Probable error in astronomical azimuth	0.5″	5″	10″

Table 1.1 presents the general specifications for the three types of triangulation systems.

1.5 TRIANGULATION FIGURES AND LAYOUTS

The basic figures used in triangulation networks are the triangle, braced or geodetic quadilateral, and the polygon with a central station (Fig. 1.3).

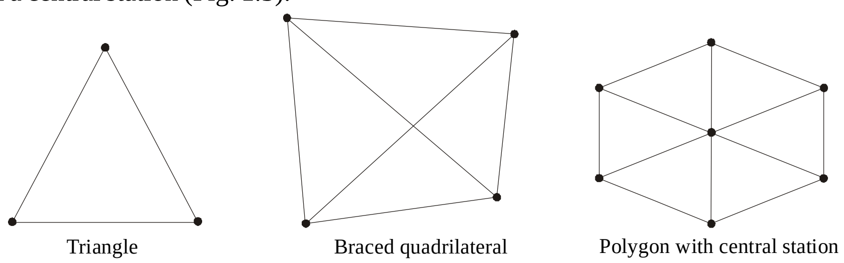

Fig. 1.3 Basic triangulation figures

The triangles in a triangulation system can be arranged in a number of ways. Some of the commonly used arrangements, also called *layouts*, are as follows :

1. Single chain of triangles
2. Double chain of triangles
3. Braced quadrilaterals
4. Centered triangles and polygons
5. A combination of above systems.

1.5.1 Single chain of triangles

When the control points are required to be established in a narrow strip of terrain such as a valley between ridges, a layout consisting of single chain of triangles is generally used as shown in Fig. 1.4. This system is rapid and economical due to its simplicity of sighting only four other stations, and does not involve observations of long diagonals. On the other hand, simple triangles of a triangulation system provide only one route through which distances can be computed, and hence, this system does not provide any check on the accuracy of observations. Check base lines and astronomical observations for azimuths have to be provided at frequent intervals to avoid excessive accumulation of errors in this layout.

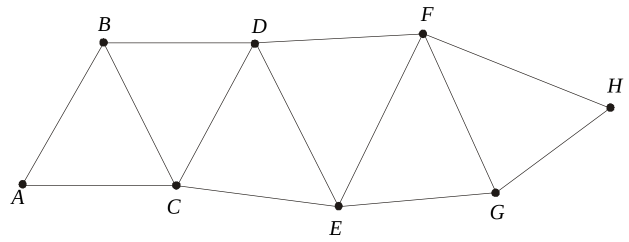

Fig. 1.4 Single of triangles

1.5.2 Double chain of triangles

A layout of double chain of triangles is shown in Fig. 1.5. This arrangement is used for covering the larger width of a belt. This system also has disadvantages of single chain of triangles system.

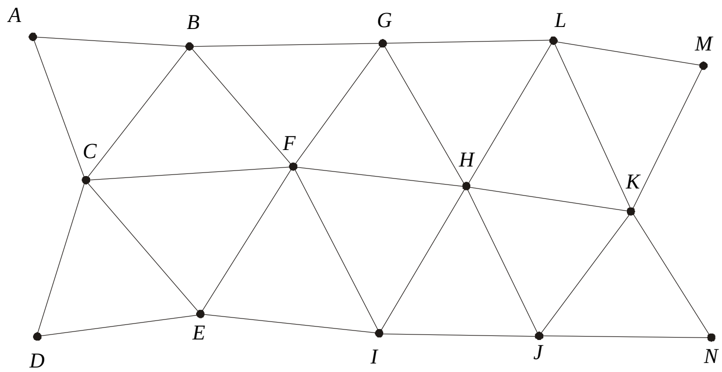

Fig. 1.5 Double chain of triangles

1.5.3 Braced quadrilaterals

A triangulation system consisting of figures containing four corner stations and observed diagonals shown in Fig. 1.6, is known as a layout of braced quadrilaterals. In fact, braced quadrilateral consists of overlapping triangles. This system is treated to be the strongest and the best arrangement of triangles, and it provides a means of computing the lengths of the sides using different combinations of sides and angles. Most of the triangulation systems use this arrangement.

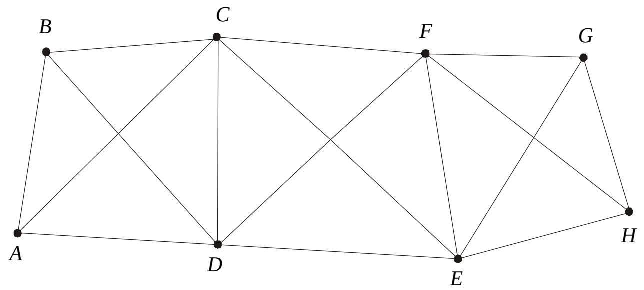

Fig. 1.6 Braced quadrilaterals

1.5.4 Centered triangles and polygons

A triangulation system which consists of figures containing interior stations in triangle and polygon as shown in Fig. 1.7, is known as centered triangles and polygons.

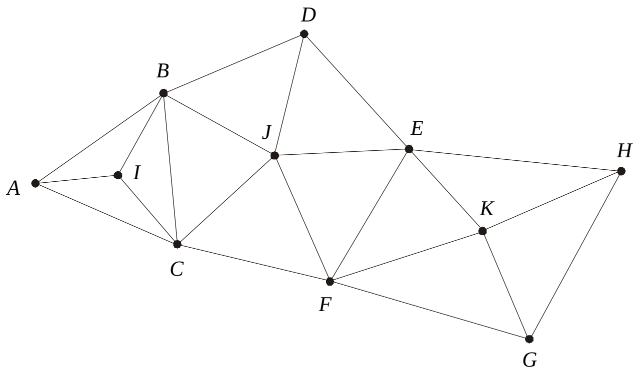

Fig. 1.7 Centered triangles and polygons

This layout in a triangulation system is generally used when vast area in all directions is required to be covered. The centered figures generally are quadrilaterals, pentagons, or hexagons with central stations. Though this system provides checks on the accuracy of the work, generally it is not as strong as the braced quadrilateral arrangement. Moreover, the progress of work is quite slow due to the fact that more settings of the instrument are required.

1.5.5 A combination of all above systems

Sometimes a combination of above systems may be used which may be according to the shape of the area and the accuracy requirements.

1.6 LAYOUT OF PRIMARY TRIANGULATION FOR LARGE COUNTRIES

The following two types of frameworks of primary triangulation are provided for a large country to cover the entire area.

1. Grid iron system
2. Central system.

1.6.1 Grid iron system

In this system, the primary triangulation is laid in series of chains of triangles, which usually runs roughly along meridians (north-south) and along perpendiculars to the meridians (east-west), throughout the country (Fig. 1.8). The distance between two such chains may vary from 150 to 250 km. The area between the parallel and perpendicular series of primary triangulation, are filled by the secondary and tertiary triangulation systems. Grid iron system has been adopted in India and other countries like Austria, Spain, France, etc.

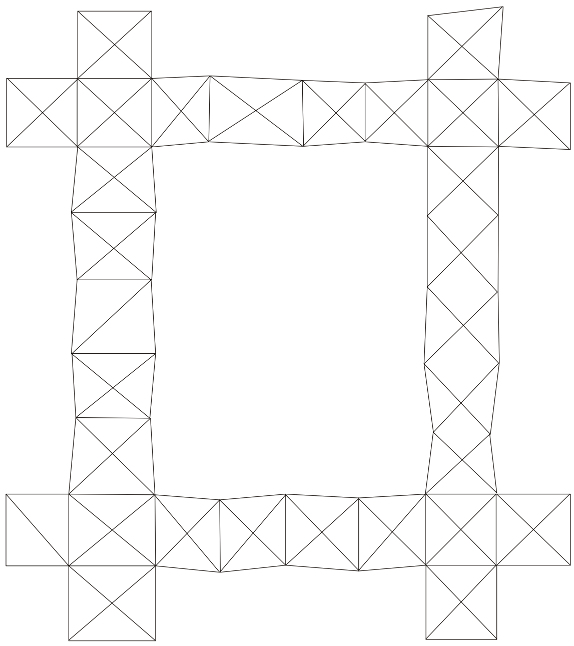

Fig. 1.8 Grid iron system of triangulation

1.6.2 Central system

In this system, the whole area is covered by a network of primary triangulation extending in all directions from the initial triangulation figure *ABC*, which is generally laid at the centre of the country (Fig. 1.9).

This system is generally used for the survey of an area of moderate extent. It has been adopted in United Kingdom and various other countries.

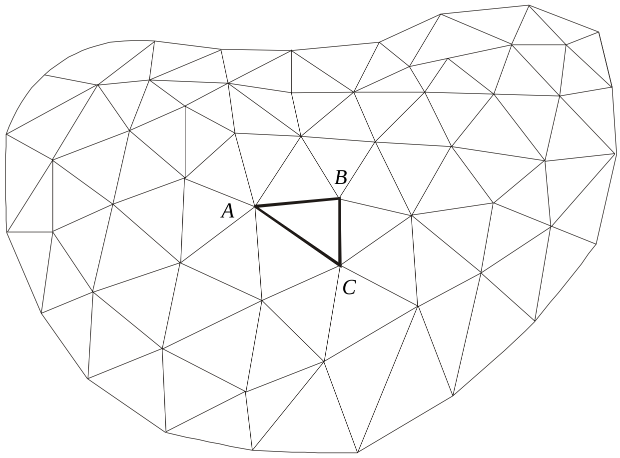

Fig. 1.9 Central system of triangulation

1.7 CRITERIA FOR SELECTION OF THE LAYOUT OF TRIANGLES

The under mentioned points should be considered while deciding and selecting a suitable layout of triangles.

1. Simple triangles should be preferably equilateral.
2. Braced quadrilaterals should be preferably approximate squares.
3. Centered polygons should be regular.
4. The arrangement should be such that the computations can be done through two or more independent routes.
5. The arrangement should be such that at least one route and preferably two routes form well-conditioned triangles.
6. No angle of the figure, opposite a known side should be small, whichever end of the series is used for computation.
7. Angles of simple triangles should not be less than 45°, and in the case of quadrilaterals, no angle should be less than 30°. In the case of centered polygons, no angle should be less than 40°.
8. The sides of the figures should be of comparable lengths. Very long lines and very short lines should be avoided.
9. The layout should be such that it requires least work to achieve maximum progress.
10. As far as possible, complex figures should not involve more than 12 conditions.

It may be noted that if a very small angle of a triangle does not fall opposite the known side it does not affect the accuracy of triangulation.

1.8 WELL-CONDITIONED TRIANGLES

The accuracy of a triangulation system is greatly affected by the arrangement of triangles in the layout and the magnitude of the angles in individual triangles. The triangles of such a shape, in which any error in angular measurement has a minimum effect upon the computed lengths, is known as *well-conditioned triangle*.

In any triangle of a triangulation system, the length of one side is generally obtained from computation of the adjacent triangle. The error in the other two sides if any, will affect the sides of the triangles whose computation is based upon their values. Due to accumulated errors, entire triangulation system is thus affected thereafter. To ensure that two sides of any triangle are equally affected, these should, therefore, be equal in length. This condition suggests that all the triangles must, therefore, be isoceles.

Let us consider an isosceles triangle *ABC* whose one side *AB* is of known length (Fig. 1.10). Let *A*, *B*, and *C* be the three angles of the triangle and *a*, *b*, and *c* are the three sides opposite to the angles, respectively.

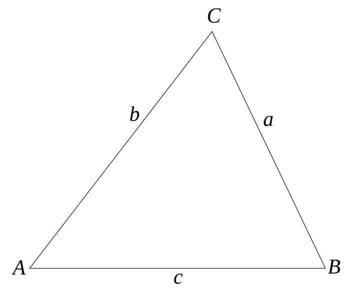

Fig. 1.10 Triangle in a triangulation system

As the triangle is isosceles, let the sides *a* and *b* be equal.

Applying sine rule to ΔABC, we have

$$\frac{a}{\sin A} = \frac{c}{\sin C} \qquad \text{... (1.1)}$$

or

$$a = c\,\frac{\sin A}{\sin C} \qquad \text{... (1.2)}$$

If an error of δA in the angle *A*, and δC in angle *C* introduce the errors δa_1 and δa_2, respectively, in the side *a*, then differentiating Eq. (1.2) partially, we get

$$\delta a_1 = c\,\frac{\cos A\,\delta A}{\sin C} \qquad \text{... (1.3)}$$

and

$$\delta a_2 = -c\,\frac{\sin A \cos C\,\delta C}{\sin^2 C} \qquad \text{... (1.4)}$$

Dividing Eq. (1.3) by Eq. (1.2), we get

$$\frac{\delta a_1}{a} = \delta A \cot A \qquad \text{... (1.5)}$$

Dividing Eq. (1.4) by Eq. (1.2), we get

$$\frac{\delta a_2}{a} = -\delta C \cot C \qquad \text{... (1.6)}$$

If $\delta A = \delta C = \pm\alpha$, is the probable error in the angles, then the probable errors in the side *a* are

$$\frac{\delta a}{a} = \pm\alpha\sqrt{\cot^2 A + \cot^2 C}$$

But $C = 180° - (A + B)$

or $= 180° - 2A$, *A* being equal to *B*.

Therefore

$$\frac{\delta a}{a} = \pm\alpha\sqrt{\cot^2 A + \cot^2 2A} \qquad \text{... (1.7)}$$

From Eq. (1.7), we find that, if $\frac{\delta a}{a}$ is to be minimum, $(\cot^2 A + \cot^2 2A)$ should be a minimum.

Differentiating $\cot^2 A + \cos^2 2A$ with respect to A, and equating to zero, we have

$$4\cos^4 A + 2\cos^2 A - 1 = 0 \quad \text{...(1.8)}$$

Solving Eq. (1.8), for $\cos A$, we get

$$A = 56°14' \ (\textit{approximately})$$

Hence, the best shape of an isoceles triangle is that triangle whose base angles are 56°14′ each. However, from practical considerations, an equilateral triangle may be treated as a well-conditional triangle. In actual practice, the triangles having an angle less than 30° or more than 120° should not be considered.

1.9 STRENGTH OF FIGURE

The strength of figure is a factor to be considered in establishing a triangulation system to maintain the computations within a desired degree of precision. It plays also an important role in deciding the layout of a triangulation system.

The U.S. Coast and Geodetic Surveys has developed a convenient method of evaluating the strength of a triangulation figure. It is based on the fact that computations in triangulation involve use of angles of triangle and length of one known side. The other two sides are computed by sine law. For a given change in the angles, the sine of small angles change more rapidly than those of large angles. This suggests that smaller angles less than 30° should not be used in the computation of triangulation. If, due to unavoidable circumstances, angles less than 30° are used, then it must be ensured that this is not opposite the side whose length is required to be computed for carrying forward the triangulation series.

The expression given by the U.S. Coast and Geodetic Surveys for evaluation of the strength of figure, is for the square of the probable error (L^2) that would occur in the sixth place of the logarithm of any side, if the computations are carried from a known side through a single chain of triangles after the net has been adjusted for the side and angle conditions. The expression for L^2 is

$$L^2 = \frac{4}{3} d^2 R \quad \text{... (1.9)}$$

where d is the probable error of an observed direction in seconds of arc, and R is a term which represents the shape of figure. It is given by

$$R = \frac{D - C}{D} \Sigma(\delta_A^2 + \delta_A \delta_B + \delta_B^2) \quad \text{... (1.10)}$$

where

D = the number of directions observed excluding the known side of the figure,

$\delta_A, \delta_B, \delta_C$ = the difference per second in the sixth place of logarithm of the sine of the distance angles A, B and C, respectively. (Distance angle is the angle in a triangle opposite to a side), and

C = the number of geometric conditions for side and angle to be satisfied in each figure. It is given by

$$C = (n' - S' + 1) + (n - 2S + 3) \quad \text{... (1.11)}$$

where

n = the total number of lines including the known side in a figure,

n' = the number of lines observed in both directions including the known side,

S = the total number of stations, and

S' = the number of stations occupied.

For the computation of the quantity $\Sigma(\delta_A^2 + \delta_A \delta_B + \delta_B^2)$ in Eq. (1.10) , Table 1.2 may be used.

In any triangulation system more than one routes are possible for various stations. The strength of figure decided by the factor R alone determines the most appropriate route to adopt the best shaped triangulation net route. If the computed value of R is less, the strength of figure is more and *vice versa.*

Table 1.2 Values of $\delta_A^2 + \delta_A\,\delta_B + \delta_B^2$

	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
0																							
10	428	359																					
12	359	295	253																				
14	315	253	214	187																			
16	284	225	187	162	143																		
18	262	204	168	143	126	113																	
20	245	189	153	130	113	100	91																
22	232	177	142	119	103	91	81	74															
24	221	167	134	111	95	83	74	67	61														
26	213	160	126	104	89	77	68	61	56	51													
28	206	153	120	99	83	72	63	57	51	47	43												
30	199	148	115	94	79	68	59	53	48	43	40	33											
35	188	137	106	85	71	60	52	46	41	37	33	27	23										
40	179	129	99	79	65	54	47	41	36	32	29	23	19	16									
45	172	124	93	74	60	50	43	37	32	28	25	20	16	13	11								
50	167	119	89	70	57	47	39	34	29	26	23	18	14	11	9	8							
55	162	115	86	67	54	44	37	32	27	24	21	16	12	10	8	7	5						
60	159	112	83	64	51	42	35	30	25	22	19	14	11	9	7	5	4	4					
65	155	109	80	62	49	40	33	28	24	21	18	13	10	7	6	5	4	3	2				
70	152	106	78	60	48	38	32	27	23	19	17	12	9	7	5	4	3	2	2	1			
75	150	104	76	58	46	37	30	25	21	18	16	11	8	6	4	3	2	2	1	1	1		
80	147	102	74	57	45	36	29	24	20	17	15	10	7	5	4	3	2	1	1	1	0	0	
85	145	100	73	55	43	34	28	23	19	16	14	10	7	5	3	2	2	1	1	0	0	0	0
90	143	98	71	54	42	33	27	22	19	16	13	9	6	4	3	2	1	1	1	0	0	0	0
95	140	96	70	53	41	32	26	22	18	15	13	9	6	4	3	2	1	1	0	0	0	0	
100	138	95	68	51	40	31	25	21	17	14	12	8	6	4	3	2	1	1	0	0	0		
105	136	93	67	50	39	30	25	20	17	14	12	8	5	4	2	2	1	1	0	0			
110	134	91	65	49	38	30	24	19	16	13	11	7	5	3	2	2	1	1	1				
115	132	89	64	48	37	29	23	19	15	13	11	7	5	3	2	2	1	1					
120	129	88	62	46	36	28	22	18	15	12	10	7	5	3	2	2	1						
125	127	86	61	45	35	27	22	18	14	12	10	7	5	4	3	2							
130	125	84	59	44	34	26	21	17	14	12	10	7	5	4	3								
135	122	82	58	43	33	26	21	17	14	12	10	7	5	4									
140	119	80	56	42	32	26	20	17	14	12	10	8	6										
145	116	77	55	41	32	25	21	17	15	13	11	9											
150	112	75	54	40	32	26	21	18	16	15	13												
152	111	75	53	40	32	26	22	19	17	16													
154	110	74	53	41	33	27	23	21	19														
156	108	74	54	42	34	28	25	22															
158	107	74	54	43	35	30	27																
160	107	74	56	45	38	33																	
162	107	76	59	48	42																		
164	109	79	63	54																			
166	113	86	71																				
168	122	98																					
170	143																						

1.10 ACCURACY OF TRIANGULATION

Errors are inevitable and, therefore, inspite of all precautions the errors get accumulated. It is, therefore, essential to know the accuracy of the triangulation network achieved so that no appreciable error in plotting is introduced. The following formula for root mean square error may be used.

$$m = \sqrt{\frac{\Sigma E^2}{3n}} \qquad \text{... (1.12)}$$

where m = the root mean square error of unadjusted horizontal angles in seconds of arc as obtained from the triangular errors,

ΣE = the sum of the squares of all the triangular errors in the triangulation series, and

n = the total number of triangles in the series.

It may be noted that

(*i*) all the triangles have been included in the computations,

(*ii*) all the four triangles of a braced quadrilateral have been included in the computations, and

(*iii*) if the average triangular error of the series is 8″, probable error in latitudes and departures after a distance of 100 km, is approximately 8 m.

ILLUSTRATIVE EXAMPLES

Example 1.1 If the probable error of direction measurement is 1.20″, compute the maximum value of R for the desired maximum probable error of (*i*) 1 in 20,000 and (*ii*) 1 in 10,000.

Solution: (*i*) L being the probable error of a logarithm, it represents the logarithm of the ratio of the true value and a value containing the probable error.

In this case L = the 6th place in log $\left(1 \pm \frac{1}{20000}\right)$

= the 6th place in log ((1 ± 0.00005)

log (1 + 0.00005) = 0.0000217

The 6th place in the log value = 21

Hence $L = \pm 21$

It is given that $d = 1.20''$

From Eq. (1.9), we have

$$L^2 = \frac{4}{3} d^2 R$$

$$R_{max} = \frac{3}{4} \frac{L^2}{d^2}$$

$$= \frac{3}{4} \times \frac{21^2}{1.20^2} = \mathbf{230}.$$

(*ii*) L = the 6th place in log $\left(1 \pm \frac{1}{10000}\right)$

log (1 + 0.0001) = 0.0000434

The 6th place in the log value = 43

Hence $L = \pm 43$

$$R_{max} = \frac{3}{4} \times \frac{43^2}{1.20^2} = \mathbf{963}.$$

Example 1.2 The probable error of direction measurement is 1″. Compute the maximum value of R if the maximum probable error is

(*i*) 1 in 25000
(*ii*) 1 in 5000.

Solution:

(*i*) $$\log\left(1+\frac{1}{25000}\right) = 0.0000174$$

The 6th place in the log value $= 17$

Hence $L = \pm 17$

From Eq. (1.9), we get

$$R_{\max} = \frac{3L^2}{4d^2}$$

The value of d is given as 1″

$$R_{\max} = \frac{3\times 17^2}{4\times 1^2} = \mathbf{217}.$$

(*ii*) $$\log\left(1+\frac{1}{50000}\right) = 0.0000086$$

The 6th place in the log value $= 9$

Hence $L = \pm 9$

$$R_{\max} = \frac{3\times 9^2}{4\times 1^2} = \mathbf{61}.$$

Example 1.3 Compute the value of $\frac{D-C}{D}$ for the following triangulation figures if all the stations have been occupied and all the lines have been observed in both directions :

(*i*) A single triangle
(*ii*) A braced quadrilateral
(*iii*) A four-sided central-point figure without diagonals
(*iv*) A four-sided central-point figure with one diagonal.

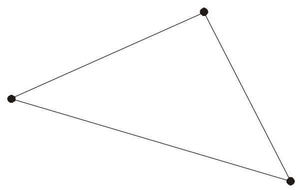

Fig. 1.11

Solution: (*i*) Single triangle (Fig. 1.11)

From Eq. (1.11), we have

$$C = (n' - S' + 1) + (n - 2S + 3)$$
$$n' = 3$$
$$n = 3$$
$$S = 3$$
$$S' = 3$$
$$C = (3 - 3 + 1) + (3 - 2 \times 3 + 3) = 1$$

and D = the number of directions observed excluding the known side.

= 2 (total number of lines – 1)

= 2 × (3 – 1) = 4

$$\frac{D-C}{D} = \frac{4-1}{4} = \mathbf{0.75}.$$

(*ii*) Braced quadrilateral (Fig. 1.12)

$$\begin{aligned} n &= 6 \\ n' &= 6 \\ S &= 4 \\ S' &= 4 \\ C' &= (6-4+1)+(6-2\times4+3) = 4 \\ D &= 2\times(6-1) = 10 \\ \frac{D-C}{D} &= \frac{10-4}{10} = \mathbf{0.6}. \end{aligned}$$

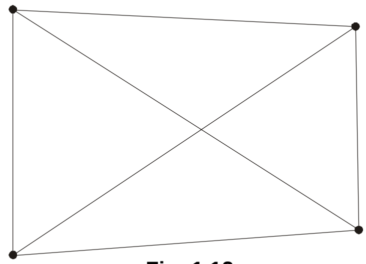

Fig. 1.12

(*iii*) Four-sided central-point figures without diagonals (Fig. 1.13)

$$\begin{aligned} n &= 8 \\ n' &= 8 \\ S &= 5 \\ S' &= 5 \\ C &= (8-5+1)+(8-2\times5+3) = 5 \\ D &= 2\times(8-1) = 14 \end{aligned}$$

Therefore $$\frac{D-C}{D} = \frac{14-5}{14} = \mathbf{0.64}.$$

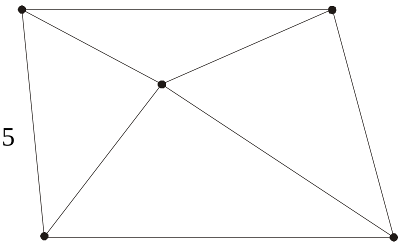

Fig. 1.13

(*iv*) Four-sided central-point figure with one diagonal. (Fig. 1.14)

$$\begin{aligned} n &= 9 \\ n' &= 9 \\ S &= 5 \\ S' &= 5 \\ C &= (9-5+1)+(9-2\times5+3) = 7 \\ D &= 2\times(9-1) = 16 \end{aligned}$$

Therefore $$\frac{D-C}{D} = \frac{16-7}{16} = \mathbf{0.56}.$$

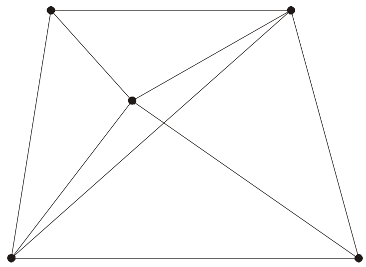

Fig. 1.14

Example 1.4 Compute the value of $\frac{D-C}{D}$ for the triangulation nets shown in Fig. 1.15 (*a* – *d*). The directions observed are shown by arrows.

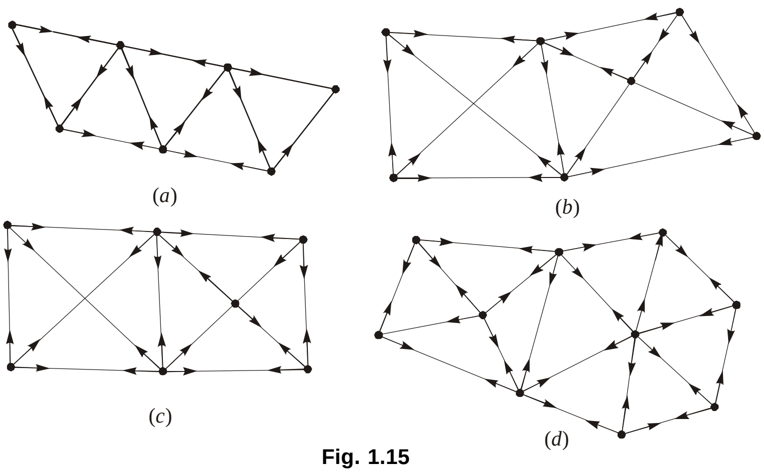

Fig. 1.15

Solution: (*i*) Fig. 1.15*a*

From Eq. (1.11), we have

$$C = (n' - S' + 1) + (n - 2S + 3)$$

n = the total number of lines
= 11

n' = the total number of lines observed in both directions
= 9

S = the total number of stations
= 7

S' = the total number of stations occupied
= 6

$$C = (9 - 6 + 1) + (11 - 2 \times 7 + 3) = 4$$

and D = the total number of directions observed excluding the known side
= $2 \times (n' - 1)$ + number of lines observed in one direction
= $2 \times (9 - 1) + 2 = 18$

Therefore $$\frac{D-C}{D} = \frac{18-4}{18} = \mathbf{0.78}.$$

(*ii*) Fig. 1.15*b*

$$n = 13$$
$$n' = 11$$
$$S = 7$$
$$S' = 7$$
$$C = (11 - 7 + 1) + (13 - 2 \times 7 + 3) = 7$$
$$D = 2 \times (11 - 1) + 2 = 22$$

Therefore $$\frac{D-C}{D} = \frac{22-7}{22} = \mathbf{0.68}.$$

(*iii*) Fig. 1.15*c*

$$n = 13$$
$$n' = 11$$
$$S = 7$$
$$S' = 7$$
$$C = (11 - 7 + 1) + (13 - 2 \times 7 + 3) = 7$$
$$D = 2 \times (11 - 1) + 2 = 22$$

Therefore $$\frac{D-C}{D} = \frac{22-7}{22} = \mathbf{0.68}.$$

(*iv*) Fig. 1.15*d*

$$n = 19$$
$$n' = 19$$
$$S = 10$$
$$S' = 10$$
$$C = (19 - 10 + 1) + (19 - 2 \times 10 + 3) = 12$$
$$D = 2\,(19 - 1) + 0 = 36$$

Therefore $$\frac{D-C}{D} = \frac{36-12}{36} = \mathbf{0.67}.$$

Example 1.5 Compute the strength of the figure *ABCD* for all the routes by which the length *CD* can be computed from the known side *AB*. Assume that all the stations were occupied.

Solution:

From Eq. (1.10), we have

$$R = \frac{D-C}{D}\Sigma(\delta_A^2 + \delta_A\delta_B + \delta_C^2)$$

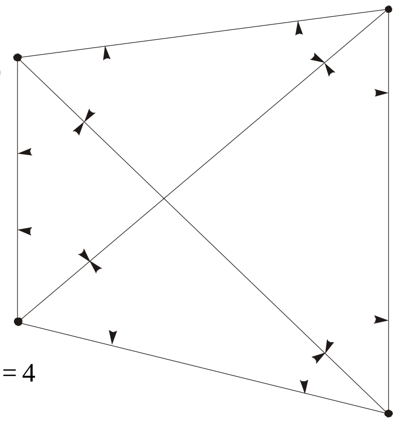

Fig. 1.16

For the given figure in Fig. 1.16, we have

$$n = 6$$
$$n' = 6$$
$$S = 4$$
$$S' = 4$$
$$D = 2 \times (n-1) = 2 \times (6-1) = 10$$

Hence $C = (n' - S' + 1) + (n - 2S + 3) = (6-4+1) + (6-2\times4+3) = 4$

and $\dfrac{D-C}{D} = \dfrac{10-4}{10} = \mathbf{0.60}$.

(*a*) Route-1, using $\Delta^s ABC$ and *ADC* with common side *AC*

For ΔABC the distance angles of *AB* and *AC* are 26° and 100° = 44° + 56°, respectively.

From Table 1.2,

$$\delta_{100}^2 + \delta_{100}\delta_{26} + \delta_{26}^2 = 17$$

For ΔADC, the distance angles of *AC* and *DC* are 112° = (44° + 68°) and 38°, respectively,

$$\delta_{112}^2 + \delta_{112}\delta_{38} + \delta_{38}^2 = 6$$

$$R_1 = 0.6 \times (17+6) = 13.8 \simeq 14$$

(*b*) Route-2, using $\Delta^s ABC$ and *BCD* with common side *BC*

For ΔABC the distance angles of *AB* and *BC* are 26° and 54°, respectively,

$$\delta_{54}^2 + \delta_{54}\delta_{26} + \delta_{26}^2 = 27$$

For ΔBCD, the distance angle of *BC* and *CD* are 68° and 56°, respectively,

$$\delta_{68}^2 + \delta_{68}\delta_{56} + \delta_{56}^2 = 4$$

$$R_2 = 0.6 \times (27+4) = 18.6 \simeq 19$$

(*c*) Route-3, using $\Delta^s ABD$ and *ACD* with common side *AD*

From ΔABC the distance for both the sides *AB* and *AD* is 44°.

$$\delta_{44}^2 + \delta_{44}\delta_{44} + \delta_{44}^2 = 13$$

From ΔACD, the distance angles of *AD* and *CD* and 30° and 38°, respectively,

$$\delta_{38}^2 + \delta_{38}\delta_{30} + \delta_{30}^2 = 31$$

$$R_3 = 0.6 \times (13+31) = 26.4 \simeq 26$$

(*d*) Route-4, using $\Delta^s ABD$ and *BCD* with common side *BD*.

From ΔABD, the distance angles of *AB* and *DB* are 44° and 92° = (38° + 54°), respectively,

$$\delta_{92}^2 + \delta_{92}\delta_{38} + \delta_{38}^2 = 7$$

From ΔBCD, the distance angles of *BD* and *CD* are 56° = (30° + 26°) and 56°, respectively,

$$\delta_{56}^2\delta_{56}\delta_{56} + \delta_{56}^2 = 7$$

$$R_4 = 0.6 \times (7 + 7) = 8.4 \simeq 8$$

Since the lowest value of R represents the highest strength, the best route to compute the length of CD is Route-4, having $R_4 = 8$.

1.11 ROUTINE OF TRIANGULATION SURVEY

The routine of triangulation survey, broadly consists of

(*a*) field work, and (*b*) computations.

The field work of triangulation is divided into the following operations :

(*i*) Reconnaissance
(*ii*) Erection of signals and towers
(*iii*) Measurement of base line
(*iv*) Measurement of horizontal angles
(*v*) Measurement of vertical angles
(*vi*) Astronomical observations to determine the azimuth of the lines.

1.12 RECONNAISSANCE

Reconnaissance is the preliminary field inspection of the entire area to be covered by triangulation, and collection of relevant data. Since the basic principle of survey is working from whole to the part, reconnaissance is very important in all types of surveys. It requires great skill, experience and judgement. The accuracy and economy of triangulation greatly depends upon proper reconnaissance survey. It includes the following operations:

1. Examination of terrain to be surveyed.
2. Selection of suitable sites for measurement of base lines.
3. Selection of suitable positions for triangulation stations.
4. Determination of intervisibility of triangulation stations.
5. Selection of conspicuous well-defined natural points to be used as intersected points.
6. Collection of miscellaneous information regarding:
   (*a*) Access to various triangulation stations
   (*b*) Transport facilities
   (*c*) Availability of food, water, etc.
   (*d*) Availability of labour
   (*e*) Camping ground.

Reconnaissance may be effectively carried out if accurate topographical maps of the area are available. Help of aerial photographs and mosaics, if available, is also taken. If maps and aerial photographs are not available, a rapid preliminary reconnaissance is undertaken to ascertain the general location of possible schemes of triangulation suitable for the topography. Later on, main reconnaissance is done to examine these schemes. The main reconnaissance is a very rough triangulation. The plotting of the rough triangulation may be done by protracting the angles. The essential features of the topography are also sketched in. The final scheme is selected by studying the relative strengths and cost to various schemes.

For reconnaissance the following instruments are generally employed:

1. Small theodolite and sextant for measurement of angles.
2. Prismatic compass for measurement of bearings.
3. Steel tape.
4. Aneroid barometer for ascertaining elevations.
5. Heliotropes for ascertaining intervisibility.
6. Binocular.
7. Drawing instruments and material.
8. Guyed ladders, creepers, ropes, etc., for climbing trees.

1.12.1 Erection of signals and towers

A *signal* is a device erected to define the exact position of a triangulation station so that it can be observed from other stations whereas a *tower* is a structure over a station to support the instrument and the observer, and is provided when the station or the signal, or both are to be elevated.

Before deciding the type of signal to be used, the triangulation stations are selected. The selection of triangulation stations is based upon the following criteria.

Criteria for selection of triangulation stations

1. Triangulation stations should be intervisible. For this purpose the station points should be on the highest ground such as hill tops, house tops, etc.
2. Stations should be easily accessible with instruments.
3. Station should form well-conditioned triangles.
4. Stations should be so located that the lengths of sights are neither too small nor too long. Small sights cause errors of bisection and centering. Long sights too cause direction error as the signals become too indistinct for accurate bisection.
5. Stations should be at commanding positions so as to serve as control for subsidiary triangulation, and for possible extension of the main triangulation scheme.
6. Stations should be useful for providing intersected points and also for detail survey.
7. In wooded country, the stations should be selected such that the cost of clearing and cutting, and building towers, is minimum.
8. Grazing line of sights should be avoided, and no line of sight should pass over the industrial areas to avoid irregular atmospheric refraction.

Determination of intervisibility of triangulation stations

As stated above, triangulations stations should be chosen on high ground so that all relevant stations are intervisible. For small distances, intervisibility can be ascertained during reconnaissance by direct observation with the aid of binocular, contoured map of the area, plane mirrors or heliotropes using reflected sun rays from either station.

However, if the distance between stations is large, the intervisibility is ascertained by knowing the horizontal distance between the stations as under.

Case-I Invervisibility not obstructed by intervening ground

If the intervening ground does not obstruct the intervisibility, the distance of visible horizon from the station of known elevation is calculated from the following formula:

$$h = \frac{D^2}{2R}(1-2m) \qquad \text{... (1.13)}$$

where h = height of the station above datum,

D = distance of visible horizon,

R = earth's mean radius, and

m = mean coefficient of refraction taken as 0.07 for sights over land, and. 0.08 for sights over sea.

Substituting the values of m as 0.071 and R as 6370 km in Eq. (1.13), the value of h in metres is given by

$$h = 0.06735\ D^2 \qquad \text{... (1.14)}$$

where D is in kilometres.

In Fig. 1.17, the distance between two stations *A* and *B* of heights h_A and h_B, respectively, is *D*. If D_A and D_B are the distances of visible horizon from *A* and *B*, respectively, we have

$$D_A = \sqrt{\frac{h_A}{0.06735}} = 3.853\sqrt{h_A} \quad \text{...(1.15)}$$

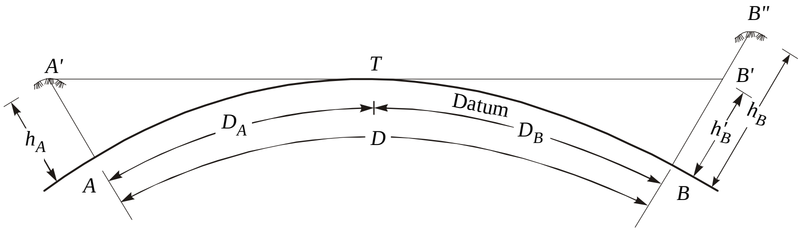

Fig. 1.17 Intervisibility not obstructed by intervening ground

We have $D = D_A + D_B$

or $D_B = D - D_A$

For the known distance of visible horizon D_B as above, the height of station *B* is computed. If the computed value is h'_B, then

$$h'_B = 0.06735\, D_B^2 \quad \text{...(1.16)}$$

The computed value of height h'_B is compared with the known value h_B as below :

If $h_B \geq h'_B$, the station *B* will be visible from A, and

if $h_B < h'_B$, the station *B* will not be visible from *A*.

If *B* is not visible from *A*, $(h'_B - h_B)$ is the required amount of height of signal to be erected at *B*. While deciding the intervisibility of various stations, the line of sight should be taken at least 3 m above the point of tangency *T* of the earth's surface to avoid grazing rays.

Case-II Intervisibility obstructed by intervening ground

In Fig. 1.18, the intervening ground at *C* is obstructing the intervisibility between the stations *A* and *B*. From Eq. (1.15), we have

$$D_A = 3.853\sqrt{h_A} \quad \text{...(1.17)}$$

The distance D_T of the peak *C* from the point of tangency *T*, is given by

$$D_T = D_A - D_C \quad \text{...(1.18)}$$

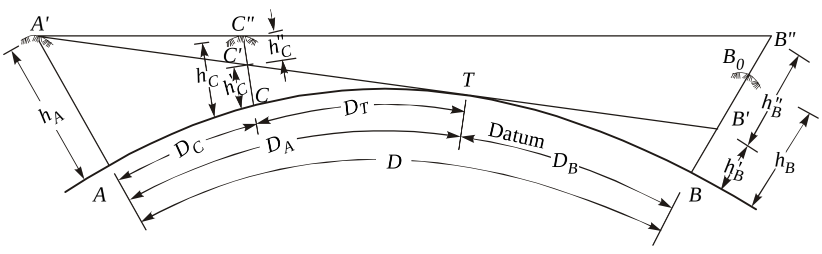

Fig. 1.18 Intervisibility obstructed by intervening ground

and $$h'_C = 0.06735\, D_T^{\ 2} \qquad \text{... (1.19)}$$

$$h'_B = 0.06735\, D_B^{\ 2} \qquad \text{... (1.20)}$$

If $h'_C > h_C$, the line of sight is clear of the obstruction, and it becomes Case-I discussed above. If $h'_C < h_C$ then the signal at *B* is to be raised. The amount of raising required at *B* is computed as below.

From similar $\Delta^s\, A'C'C''$ and $A'B'B''$ in Fig. 1.19, we get

$$\frac{h''_C}{D_C} = \frac{h''_B}{D}$$

or $$h''_B = \frac{D}{D_C}\, h''_C \qquad \text{... (1.21)}$$

where $$h''_C = h_C - h'_C\,.$$

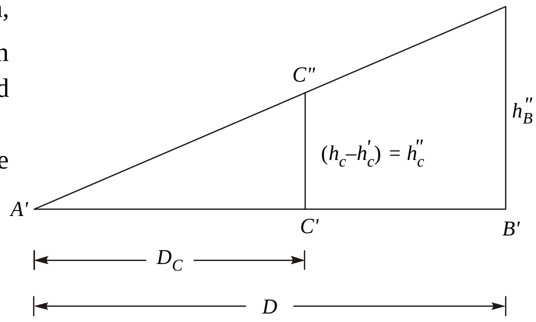

Fig. 1.19

The required height of signal above station B_0 is

$$B_0B'' = (BB' + B'B'') - BB_0$$

$$= (h'_B + h''_C) - h_B \qquad \text{... (1.22)}$$

Alternate method (Captain G.T. McCaw's method)

A comparison of elevations of the stations *A* and *B* (Fig. 1.20) decides whether the triangulation stations are intervisible or not. A direct solution suggested by Captain McCaw is known as Captain McCaw's method.

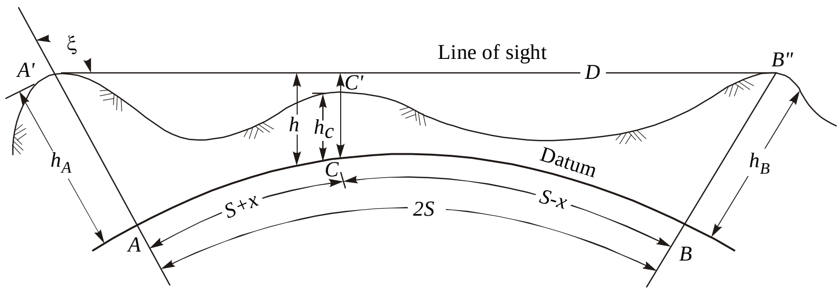

Fig. 1.20 Captain McCaw's method of ascertaining intervisibility

Let h_A = elevation of station *A*

h_B = elevation of station *B*

h_C = elevation of station *C*.

$2S$ = distance between *A* and *B*

$(S + x)$ = distance between *A* and *C*

$(S - x)$ = distance between *C* and *B*

h = elevation of the line of sight at *C*

ξ = zenith distance from *A* to *B*

= (90°-vertical angle).

From Captain McCaw's formula

$$h = \frac{1}{2}(h_3 + h_A) + \frac{1}{2}(h_B - h_A)\frac{x}{S} - (S^2 - x^2)\,\text{cosec}^2\,\xi\,\frac{(1-2m)}{2R} \qquad \text{... (1.23)}$$

Practically in most of the cases, the zenith distance is very nearly equal to 90° and, therefore, the value of $\text{cosec}^2\,\xi$ may be taken approximately equal to unity.

However, for accurate calculations,

$$\text{cosec}^2\,\xi = 1 + \frac{(h_B - h_A)^2}{4S^2} \qquad \text{... (1.24)}$$

In Eq. (1.23), the value of $\left(\frac{1-2m}{2R}\right)$ is usually taken as 0.06735.

Therefore

$$h = \frac{1}{2}(h_B + h_A) + \frac{1}{2}(h_B - h_A)\frac{x}{S} - (S^2 - x^2) \times 0.06735 \qquad \text{... (1.25)}$$

If $h > h_C$, the line of sight is free of obstruction. In case $h < h_C$, the height of tower to raise the signal at B, is computed from Eqs. (1.21) and (1.22).

ILLUSTRATIVE EXAMPLES

Example 1.6 Two stations A and B, 80 km apart, have elevations 15 m and 270 m above mean sea level, respectively. Calculate the minimum height of the signal at B.

Solution: (Fig. 1.21)

It is given that

$$h_A = 15\text{ m}$$
$$h_B = 270\text{ m}$$
$$D = 80\text{ km}$$

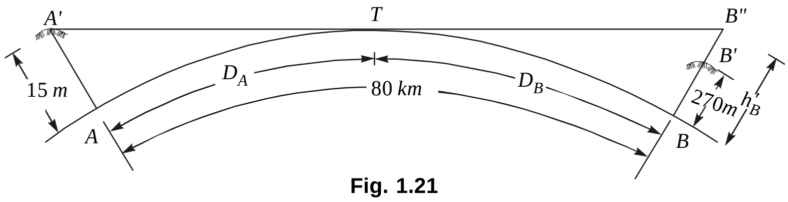

Fig. 1.21

From Eq. (1.15), we get

$$D_A = 3.853\sqrt{h_A} = 3.853 \times \sqrt{15} = 14.92\text{ km}$$

We have

$$D_B = D - D_A$$

or

$$= 80 - 14.92$$
$$= 65.08\text{ km}$$

Therefore

$$h'_B = 0.06735\ D_B^2$$
$$= 0.06735 \times 65.08^2 = 285.25\text{ m}$$

Hence, since the elevation of B is 270 m, the height of signal required at B, is

$$= 285.25 - 270 = 15.25 \simeq \mathbf{15.5\ m}.$$

Example 1.7 There are two stations *P* and *Q* at elevations of 200 m and 995 m, respectively. The distance of *Q* from *P* is 105 km. If the elevation of a peak *M* at a distance of *38* km from *P* is 301 m, determine whether *Q* is visible from *P* or not. If not, what would be the height of scaffolding required at *Q* so that *Q* becomes visible from *P* ?

Solution: (Fig. 1.22)

From Eq. (1.15), we get

$$PT = 3.853 \times \sqrt{200} = 54.45 \text{ km}$$

Therefore $$MT = PT - PM$$

$$= 54.45 - 38 = 16.45 \text{ km}$$

Using Eq. (1.14) and the value of *MT*, we get

$$MM' = 0.06735 \times 16.45^2 = 18.23 \text{ m}$$

The distance of *Q* from the point of tangency *T* is

$$QT = 105 - 54.45 = 50.55 \text{ km}$$

Therefore $$QQ' = 0.06735 \times 50.55^2 = 172.10 \text{ m}$$

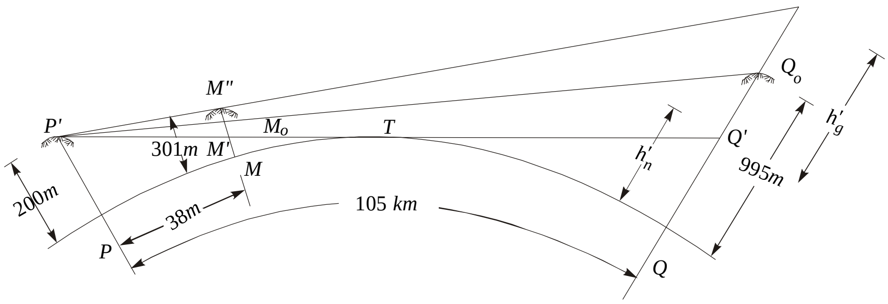

Fig. 1.22

From similar $\Delta^s P'M'M''$ and $P'Q'Q''$, we have

$$\frac{M'M''}{PM} = \frac{Q'Q''}{PQ}$$

$$Q'Q'' = \frac{PQ}{PM} M'M''$$

$$= \frac{PQ}{PM} (MM'' - MM')$$

$$= \frac{105}{38} \times (301 - 18.23) = 781.34 \text{ m}$$

We have $$QQ'' = QQ' + Q'Q''$$

$$= 172.10 + 781.34 = 953.44 \text{ m}$$

As the elevation 995 m of *Q* is more than 953.44 m, the peak at *M* does not obstruct the line of sight.

Alternatively, from the similar $\Delta^s\ P'M'M_o$ and $PQ'Q_o$, we have

$$\frac{M'M_o}{PM} = \frac{Q'Q_o}{PQ}$$

or $$M'M_o = \frac{PM}{PQ} Q'Q_o$$

$$= \frac{PM}{PQ}(QQ_o - QQ')$$

$$= \frac{38}{105} \times (995 - 172.10) = 297.81$$

The elevation of line of sight $P'Q_o$ at M is

$$MM_o = MM' + M'M_o$$

$$= 18.23 + 297.81 = 316.04.$$

Since the elevation of peak at M is 301 m, the line of sight is not obstructed by the peak and, therefore, no scaffolding is required at Q.

Example 1.8 Solve the problem given in Example 1.7 by Capt. McCaw's method.

Solution: (Fig. 1.22)

From Eq. (1.25), the elevation of line of sight at M joining the two stations is

$$h = \frac{1}{2}(h_Q + h_P) + \frac{1}{2}(h_Q - h_P)\frac{x}{S} - (S^2 - x^2) \times 0.06735$$

It is given that

$$h_P = 200 \text{ m}$$
$$h_Q = 995 \text{ m}$$
$$h_M = 301 \text{ m}$$
$$2S = 105 \text{ km or } S = 52.5 \text{ km}$$
$$S + x = 38 \text{ km or } x = -14.5 \text{ km}$$

Therefore

$$h = \frac{1}{2} \times (995 + 200) + \frac{1}{2} \times (995 - 200) \times \frac{(-14.5)}{52.5}$$
$$- (52.5^2 - 14.5^2) \times 0.06735$$
$$= \mathbf{316.24\ m}.$$

The elevation of the line of sight $p'Q_0$ at M is 316.24 m, and the elevation of the peak is 301 m, therefore, the line of sight is clear of obstruction.

Example 1.9 In a triangulation survey, the altitudes of two proposed stations A and B, 100 km apart, are respectively 425 m and 750 m. The intervening ground situated at C, 60 km from A, has an elevation of 435 m. Ascertain if A and B are intervisible, and if necessary find by how much B should be raised so that the line of sight must nowhere be less than 3 m above the surface of the ground. Take R = 6400 km and m = 0.07.

Solution: (Fig. 1.20)

From the given data we have

$$h_A = 425 \text{ m}, \; h_B = 750 \text{ m}, \; h_C = 435 \text{ m}, R = 6400 \text{ km}, m = 0.07$$
$$2S = 100 \text{ km, or } S = 50 \text{ km}$$
$$S + x = 60 \text{ km or } x = 10 \text{ km}$$

Eq. (1.23) gives

$$h'_C = \frac{1}{2}(h_B + h_A) + \frac{1}{2}(h_B - h_A)\frac{x}{S} - (S^2 - x^2)\operatorname{cosec}^2 \xi \, \frac{(1-2m)}{2R}$$

Taking $\operatorname{cosec}^2 \xi$ = 1, and substituting the values of the given data in the above equation, we have

$$h = \frac{1}{2} \times (705 + 425) + \frac{1}{2} \times (705 - 425) \times \frac{10}{50} - (50^2 - 10^2)$$
$$\times \; 1 \times \frac{(1 - 2 \times 0.07)}{2 \times 6400} \times 1000 \; = 431.75 \text{ m}$$

As the elevation of the line of sight at C is less than the elevation of C, the line of sight fails to clear C by

$$435 - 431.75 = 3.25\,\text{m}$$

To avoid grazing rays, the line of should be at least 3m above the ground. Therefore, the line of sight should be raised to 3.25 + 3 = 6.25 m at C.

Hence, the minimum height of signal to be erected at B

$$= \frac{6.25}{60} \times 100 = \mathbf{10.42\ m}.$$

Station Mark

The triangulation stations should be permanently marked on the ground so that the theodolite and signal may be centered accurately over them. The following points should be considered while marking the exact position of a triangulation station :

(*i*) The station should be marked on perfectly stable foundation or rock. The station mark on a large size rock is generally preferred so that the theodolite and observer can stand on it. Generally, a hole 10 to 15 cm deep is made in the rock and a copper or iron bolt is fixed with cement.

(*ii*) If no rock is available, a large stone is embeded about 1 m deep into the ground with a circle, and dot cut on it. A second stone with a circle and dot is placed vertically above the first stone.

(*iii*) A G.I. pipe of about 25 cm diameter driven vertically into ground up to a depth of one metre, also served as a good station mark.

(*iv*) The mark may be set on a concrete monument. The station should be marked with a copper or bronze tablet. The name of the station and the date on which it was set, should be stamped on the tablet.

(*v*) In earth, generally two marks are set, one about 75 cm below the surface of the ground, and the other extending a few centimeters above the surface of the ground. The underground mark may consist of a stone with a copper bolt in the centre, or a concrete monument with a tablet mark set on it (Fig. 1.23).

(*vi*) The station mark with a vertical pole placed centrally, should be covered with a conical heap of stones placed symmetrically. This arrangement of marking station, is known as placing a cairn (Fig. 1.27).

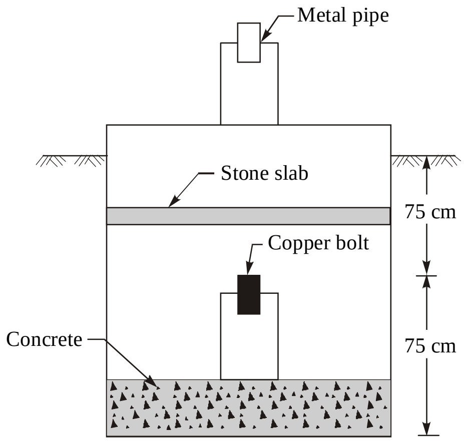

Fig. 1.23 Station mark

(*vii*) Three reference marks at some distances on fairly permanent features, should be established to locate the station mark, if it is disturbed or removed.

(*viii*) Surrounding the station mark a platform 3 m × 3 m × 0.5 m should be built up of earth.

1.13 SIGNALS

Signals are centered vertically over the station mark, and the observations are made to these signals from other stations. The accuracy of triangulation is entirely dependent on the degree of accuracy of centering the signals. Therefore, it is very essential that the signals are truly vertical, and centered over the station mark. Greatest care of centering the transit over the station mark will be useless, unless some degree of care in centering the signal is impressed upon.

A signal should fulfil the following requirements :

(*i*) It should be conspicuous and clearly visible against any background. To make the signal conspicuous, it should be kept at least 75 cm above the station mark.

(*ii*) It should be capable of being accurately centered over the station mark.

(*iii*) It should be suitable for accurate bisection from other stations.

(*iv*) It should be free from phase, or should exhibit little phase (*cf*., Sec. 1.15).

1.13.1 Classification of signals

The signals may be classified as under :

(*i*) Non-luminous, opaque or daylight signals

(*ii*) Luminous signals.

(*i*) Non-luminous signals

Non-luminous signals are used during day time and for short distances. These are of various types, and the most commonly used are of following types.

(*a*) **Pole signal** (Fig. 1.24) : It consists of a round pole painted black and white in alternate strips, and is supported vertically over the station mark, generally on a tripod. Pole signals are suitable upto a distance of about 6 km.

(*b*) **Target signal** (Fig. 1.25): It consists of a pole carrying two squares or rectangular targets placed at right angles to each other. The targets are generally made of cloth stretched on wooden frames. Target signals are suitable upto a distance of 30 km.

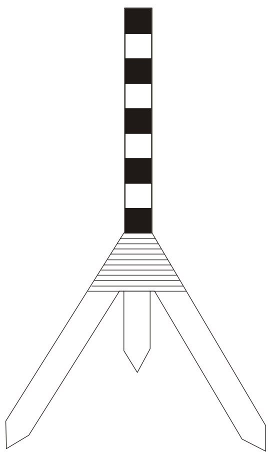

Fig. 1.24 Pole signal

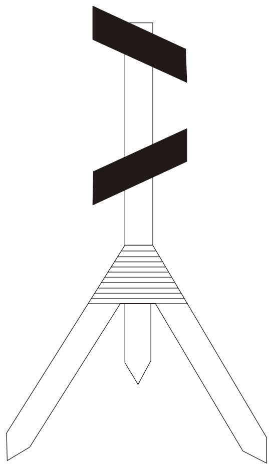

Fig. 1.25 Target signal

(*c*) **Pole and brush signal** (Fig. 1.26): It consists of a straight pole about 2.5 m long with a bunch of long grass tied symmetrically round the top making a cross. The signal is erected vertically over the station mark by heaping a pile of stones, upto 1.7 m round the pole. A rough coat of white wash is given to make it more conspicuous to be seen against black background. These signals are very useful, and must be erected over every station of observation during reconnaissance.

(*d*) **Stone cairn** (Fig. 1.27): A pile of stone heaped in a conical shape about 3 m high with a cross shape signal erected over the stone heap, is stone cairn. This white washed opaque signal is very useful if the background is dark.

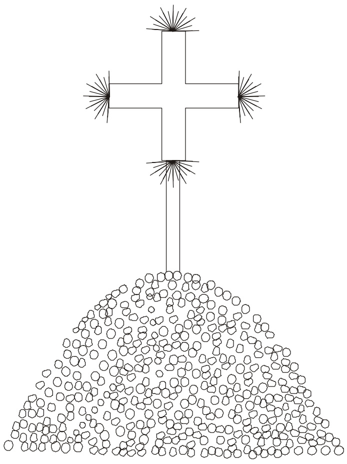

Fig. 1.26 Pole and brush signal

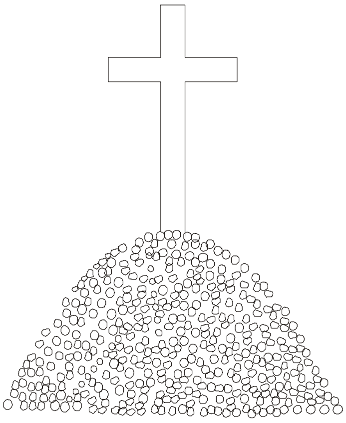

Fig. 1.27 Stone cairn

(*e*) **Beacons** (Fig. 1.28): It consists of red and white cloth tied round the three straight poles. The beacon can easily be centered over the station mark. It is very useful for making simultaneous observations.

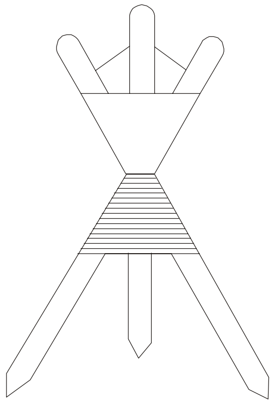

Fig. 1.28 Beacon

(*ii*) Luminous signals

Luminous signals may be classified into two types :

(*i*) Sun signals

(*ii*) Night signals.

(*a*) **Sun signals** (Fig. 1.29): Sun signals reflect the rays of the sun towards the station of observation, and are also known as heliotropes. Such signals can be used only in day time in clear weather.

Heliotrope : It consists of a circular plane mirror with a small hole at its centre to reflect the sun rays, and a sight vane with an aperture carrying a cross-hairs. The circular mirror can be rotated horizontally as well as vertically through 360°. The heliotrope is centered over the station mark, and the line of sight is directed towards the station of observation. The sight vane is adjusted looking through the hole till the flashes given from the station of observation fall at the centre of the cross of the sight vane. Once this is achieved, the heliotrope is disturbed. Now the heliotrope frame carrying the mirror is rotated in such a way that the black shadow of the small central hole of the plane mirror falls exactly at the cross of the sight vane. By doing so, the reflected beam of rays will be seen at the station of observation. Due to motion of the sun, this small shadow also moves, and it should be constantly ensured that the shadow always remains at the cross till the observations are over.

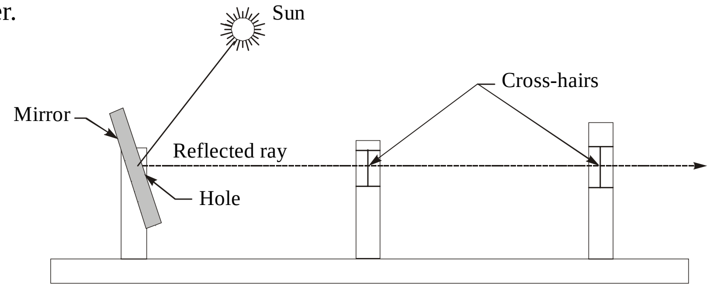

Fig. 1.29 Heliotrope

The heliotropes do not give better results compared to signals. These are useful when the signal station is in flat plane, and the station of observation is on elevated ground. When the distance between the stations exceed 30 km, the heliotropes become very useful.

(b) Night signals: When the observations are required to be made at night, the night signals of following types may be used.

1. Various forms of oil lamps with parabolic reflectors for sights less than 80 km.
2. Acetylene lamp designed by Capt. McCaw for sights more than 80 km.
3. Magnesium lamp with parabolic reflectors for long sights.
4. Drummond's light consisting of a small ball of lime placed at the focus of the parabolic reflector, and raised to a very high temperature by impinging on it a stream of oxygen.
5. Electric lamps.

1.14 TOWERS

A tower is erected at the triangulation station when the station or the signal or both are to be elevated to make the observations possible form other stations in case of problem of intervisibility. The height of tower depends upon the character of the terrain and the length of the sight.

The towers generally have two independent structures. The outer structure is for supporting the observer and the signal whereas the inner one is for supporting the instrument only. The two structures are made entirely independent of each other so that the movement of the observer does not disturb the instrument setting. The two towers may be made of masonary, timber or steel. For small heights, masonary towers are most suitable. Timber scaffolds are most commonly used, and have been constructed to heights over 50 m. Steel towers made of light sections are very portable, and can be easily erected and dismantled. Bilby towers patented by J.S. Bilby of the U.S. Coast and Geodetic Survey, are popular for heights ranging from 30 to 40 m. This tower weighing about 3 tonnes, can be easily erected by five persons in just 5 hrs. A schematic of such a tower is shown in Fig. 1.30.

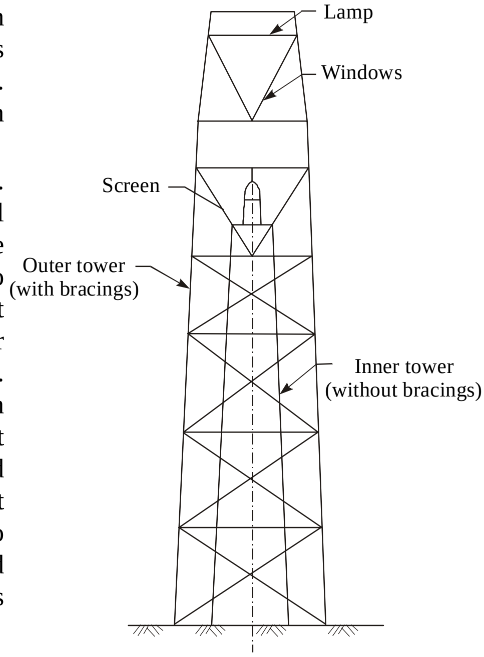

Fig. 1.30 Bilby tower

1.15 PHASE OF A SIGNAL

When cylindrical opaque signals are used, they require a correction in the observed horizontal angles due an error known as the *phase*. The cylindrical signal is partly illuminated by the sun, and the other part remains in shadow, and becomes invisible to the observer. While making the observations, the observer may bisect the bright portion or the bright line. Thus the signal is not bisected at the centre, and an error due to wrong bisection is introduced. It is, thus, the apparent displacement of the signal. The phase correction is thus necessary so that the observed horizontal angles may be reduced to that corresponding to the centre of the signal.

Depending upon the method of observation, phase correction is computed under the following two conditions.

(*i*) Observation made on bright portion

In Fig. 1.31, a cylindrical signal of radius *r*, is centered over the station *P*. The illuminated portion of the signal which the observer from *O* is able to see, is *AB*. The observer from the station *O*, makes two observations at *A* and *B* of the bright portion, *AB*. Let *C* be the midpoint of *AB*.

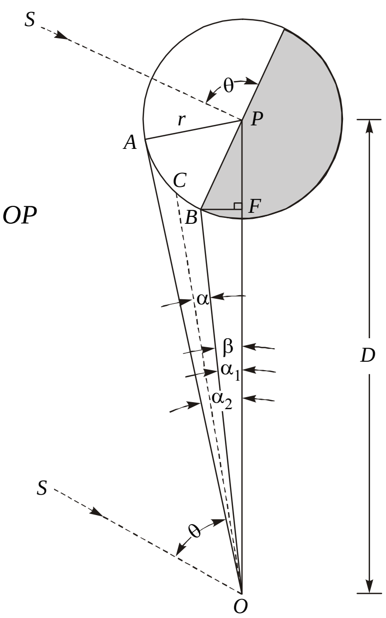

Fig. 1.31 Phase correction when observation made on the bright portion

Let θ = the angle between the sun and the line *OP*

α_1 and α_2 = the angles *BOP* and *AOP*, respectively

D = the horizontal distance *OP*

α = half of the angle *AOB*

$$= \frac{1}{2}(\alpha_2 - \alpha_1)$$

β = the phase correction

$$= \alpha_1 + \alpha = \alpha_1 + \frac{1}{2}(\alpha_2 - \alpha_1)$$

or

$$= \frac{1}{2}(\alpha_1 + \alpha_2) \qquad \text{... (1.26)}$$

From ΔOAP we get

$$\tan\alpha_2 = \frac{r}{D}$$

α_2 being small, we can write

$$\alpha_2 = \frac{r}{D} \text{ radians} \qquad \text{... (1.27)}$$

As the distance *PF* is very small compared to *OP*, *OF* may be taken as *OP*. Thus, from right angle ΔBFO, we get

$$\tan\alpha_1 = \frac{BF}{OF} = \frac{BF}{OP} = \frac{BF}{D} \qquad \text{... (1.28)}$$

From ΔPFB, we get

$$BF = r\sin(90 - \theta) = r\cos\theta$$

Substituting the value of *BF* in Eq. (1.28), we get

$$\tan\alpha_1 = \frac{r\cos\theta}{D}$$

α_1 being small, we can write

$$\alpha_1 = \frac{r\cos\theta}{D} \text{ radians} \qquad \text{... (1.29)}$$

Substituting the values of α_1 and α_2 in Eq. (1.26), we have

$$\beta = \frac{1}{2}\left(\frac{r}{D} + \frac{r\cos\theta}{D}\right) = \frac{r}{D}\left(\frac{1+\cos\theta}{2}\right)$$

$$= \frac{r}{D}\cos^2\frac{\theta}{2} \text{ radians} \qquad \text{... (1.30)}$$

$$= \frac{r}{D\sin 1''}\cos^2\frac{\theta}{2} \text{ seconds}$$

$$\beta = \frac{206265r}{D}\cos^2\frac{\theta}{2} \text{ seconds} \qquad \text{... (1.31)}$$

(*ii*) Observations made on the bright line

In this case, the bright line at C on the cylindrical signal of radius r, is sighted from O (Fig. 1.32).

Let CO = the reflected ray of the sun from the bright line at C

β = the phase correction

θ = the angle between the sun and the line OP

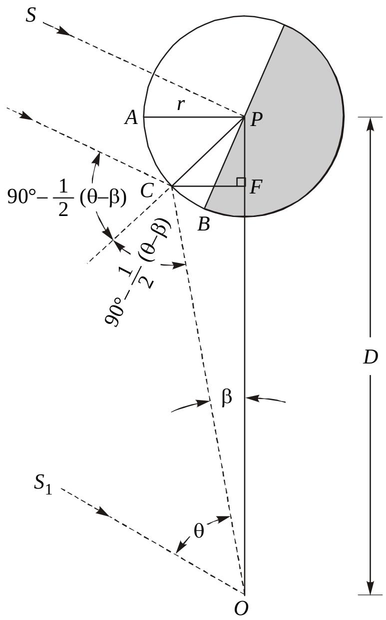

Fig. 1.32 Phase correction when observation made on the bright line

The rays of the sun are always parallel to each other, therefore, SC is parallel to S_1O.

$$\angle SCO = 180° - (\theta - \beta)$$

$$\angle PCO = 180° - \frac{1}{2}\angle SCO$$

or

$$= 180° - \frac{1}{2}[180° - (\theta - \beta)]$$

$$= 90° + \frac{1}{2}(\theta - \beta) \quad \text{... (1.32)}$$

Therefore,

$$\angle CPO = 180° - (\beta + \angle PCO) \quad \text{... (1.33)}$$

Substituting the value of $\angle PCO$ from Eq. (1.32) in Eq. (1.33) and after simplification, we get

$$\angle CPO = 90° - \frac{1}{2}(\theta + \beta)$$

As β is very small compared to θ, it can be ignored, Therefore

$$\angle CPO = 90° - \frac{1}{2}\theta$$

From the right angle ΔCFP, we have

$$\frac{CF}{CP} = \sin CPO = \sin\left(90° - \frac{1}{2}\theta\right)$$

or

$$CF = r\sin\left(90° - \frac{1}{2}\theta\right) \quad \text{... (1.34)}$$

From ΔCFO, we get

$$\tan\beta = \frac{CF}{OF} \quad \text{... (1.35)}$$

PF being very small compared to *OP*, *OF* may be taken as *OP*. Substituting the value of *CF* from Eq. (1.34) and taking *OF* equal to *D*, we get the Eq. (1.35) as

$$\tan\beta = \frac{r\sin\left(90° - \frac{1}{2}\theta\right)}{D}$$

or $$\beta = \frac{r\cos\frac{\theta}{2}}{D} \text{ radians}$$

$$\beta = \frac{206265r}{D}\cos\frac{\theta}{2} \text{ seconds} \quad \text{... (1.36)}$$

The phase correction β is applied to the observed horizontal angles in the following manner.

Let there be four stations S_1, S_2, P, and O as shown in (Fig. 1.33). The observer is at O, and the angles S_1OP and POS_2 have been measured from O as θ_1' and θ_2', respectively.

If the required corrected angles are θ_1 and θ_2, then

$$\theta_1 = \theta_1' + \beta$$

and $$\theta_2 = \theta_2' - \beta$$

when β is the phase correction.

While applying the corrections the directions of the phase correction, and the observed stations with respect to the line OP, must be noted carefully.

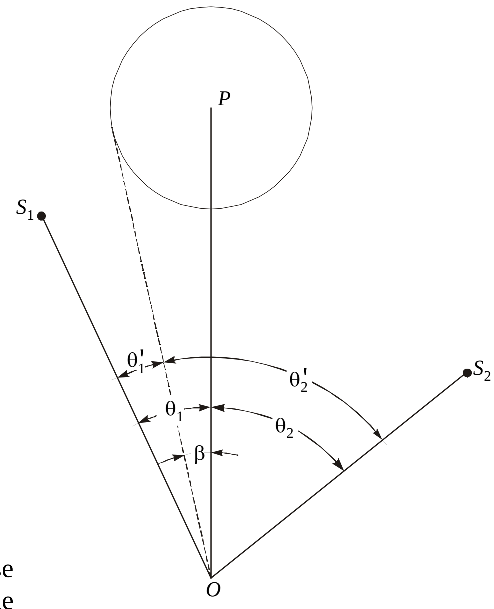

Fig. 1.33 Applying the phase correction to the measured horizontal angles

ILLUSTRATIVE EXAMPLES

Example 1.10 A cylindrical signal of diameter 4 m, was erected at station *B*. Observations were made on the signal from station *A*. Calculate the phase corrections when the observations were made

(*i*) on the bright portion, and

(*ii*) on the bright line.

Take the distance *AB* as 6950 m, and the bearings of the sun and the station *B* as 315° and 35°, respectively.

Solution: Given that

$$\theta = \text{Bearing of sun} - \text{bearing of } B$$
$$= 315° - 35° = 280°$$
$$r = \frac{\text{Diameter}}{2} = \frac{4}{2} = 2\,\text{m}$$
$$D = 6950\,\text{m}$$

(*i*) (Fig. 1.31)

From Eq. (1.31), the phase correction

$$\beta = \frac{206265r}{D}\cos^2\frac{\theta}{2} \text{ seconds}$$
$$= \frac{206265\times 2}{6950}\times\cos^2\frac{280°}{2} = \mathbf{34.83\ seconds}.$$

(*ii*) (Fig. 1.32)

From Eq. (1.36), the phase correction

$$\beta = \frac{206265r}{D}\cos\frac{\theta}{2} \text{ seconds}$$
$$= \frac{206265\times 2}{6950}\times\cos\frac{280°}{2} = \mathbf{45.47\ seconds}.$$

Example 1.11 The horizontal angle measured between two stations P and Q at station R, was 38°29′30″. The station Q is situated on the right of the line RP.

The diameter the cylindrical signal erected at station P, was 3 m and the distance between P and R was 5180 m. The bearing of the sun and the station P were measured as 60° and 15°, respectively. If the observations were made on the bright line, compute the correct horizontal angle PRQ.

Solution: (Fig. 1.34)

From the given data

$$\theta = 60° - 15° = 45°$$
$$D = 5180\,\text{m}$$
$$r = 1.5\,\text{m}$$

From Eq. (1.36), we get

$$\beta = \frac{206265r}{D}\cos\frac{\theta}{2}$$
$$= \frac{206265 \times 1.5}{5180}\cos\frac{45°}{2}$$
$$= 55.18 \text{ seconds}$$

The correct horizontal angle $PRQ = 38°\,29'\,30'' + \beta$

$= 38°29'30'' + 55.18'' =$ **38°30′25.18″**.

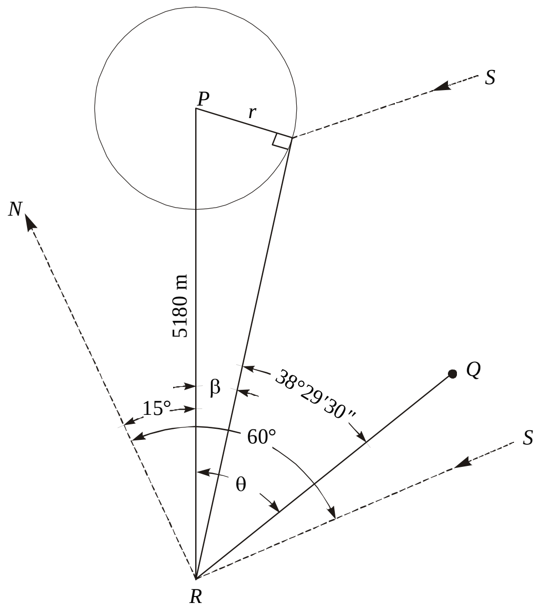

Fig. 1.34

1.16 MEASUREMENT OF BASE LINE

The accuracy of an entire triangulation system depends on that attained in the measurement of the base line and, therefore, the measurement of base line forms the most important part of the triangulation operations. As base line forms the basis for computations of triangulation system it is laid down with great accuracy in its measurement and alignment. The length of the base line depends upon the grade of the triangulation. The length of the base is also determined by the desirability of securing strong figures in the base net. Ordinarily the longer base, the easier it will be found to secure strong figures.

The base is connected to the triangulation system through a base net. This connection may be made through a simple figure as shown in Fig. 1.35, or through a much more complicated figures discussed in the base line extension (Sec. 1.16.3).

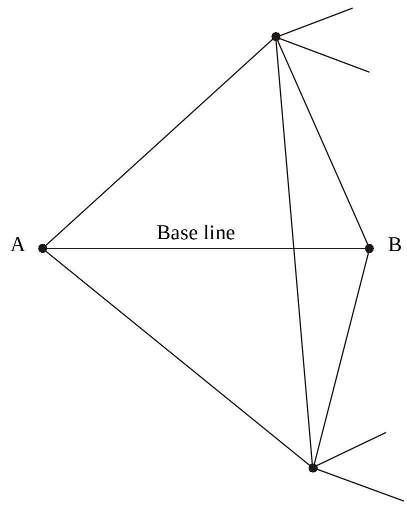

Fig. 1.35 Base net

Apart from main base line, several other check bases are also measured at some suitable intervals. In India, ten bases were measured, the length of nine bases vary from 6.4 to 7.8 miles, and that of the tenth base is 1.7 miles.

1.16.1 Selection of site for base line

Since the accuracy in the measurement of the base line depends upon the site conditions, the following points should be taken into consideration while selecting the site for a base line.

1. The site should be fairly level or gently undulating. If the ground is sloping, the slope should be uniform and gentle.
2. The site should be free from obstructions throughout the length of the base line.
3. The ground should be firm and smooth.
4. The two extremities of the base line should be intervisible.
5. The site should be such that well-conditioned triangles can be obtained while connecting extremities to the main triangulation stations.
6. The site should be such that a minimum length of the base line as specified, is available.

1.16.2 Equipment for base line measurement

Generally the following types of base measuring equipments are used :

1. Standardised tapes : These are used for measuring short bases in plain grounds.
2. Hunter's short base: It is used for measuring 80 m long base line and its extension is made by subtense method.
3. Tacheometric base measurements : It is used in undulating grounds for small bases (*cf.,* Chapter 8 of *Plane Surveying*).
4. Electronic distance measurement: This is used for fairly long distances and has been discussed in Chapter 11.

Standardised tapes : For measuring short bases in plain areas standardised tapes are generally used. After having measured the length, the correct length of the base is calculated by applying the required corrections. For details of corrections, refer to Chapter 3 of *Plane Surveying*. If the triangulation system is of extensive nature, the corrected lengths of the base is reduced to the mean sea level.

Hunter's short base : Dr. Hunter who was a Director of Survey of India, designed an equipment to measure the base line, which was named as Hunter's short base. It consists of four chains, each of 22 yards (20.117 m) linked together. There are 5 stands, three-intermediate two-legged stands, and two three-legged stands at ends (Fig. 1.36). A 1 kg weight is suspended at the end of an arm, so that the chains remain straight during observations. The correct length of the individual chains is supplied by the manufacturer or is determined in the laboratory. The lengths of the joints between two chains at intermediate supports, are measured directly with the help of a graduated scale. To obtain correct length between the centres of the targets, usual corrections such as temperature, sag, slope, etc., are applied.

To set up of the Hunter's short base the stand at the end *A* (marked in red colour) is centered on the ground mark and the target is fitted with a clip. The target *A* is made truly vertical so that the notch on its tip side is centered on the ground mark. The end of the base is hooked with the plate *A* and is spread carefully till its other end is reached. In between, at every joint of the chains, two-legged supports are fixed to carry the base. The end *B* (marked in green colour) is fixed to the *B* stand and the 1 kg weight is attached at the end of the lever. While fixing the end supports *A* and *B* it should be ensured that their third leg should face each other under the base. Approximate alignment of the base is the done by eye judgement.

For final alignment, a theodolite is set up exactly over the notch of the target *A*, levelled and centered accurately. The target at *B* is then bisected. All intermediate supports are set in line with the vertical cross-hair of the theodolite. At the end again ensure that all the intermediate supports and the target *B* are in one line.

In case the base is spread along undulating ground, slope correction is applied. To measure the slope angles of individual supports, a target is fixed to a long iron rod of such a length that it is as high above the tape at *A* as the trunion axis of the theodolite. The rod is held vertically at each support and the vertical angles for each support are read.

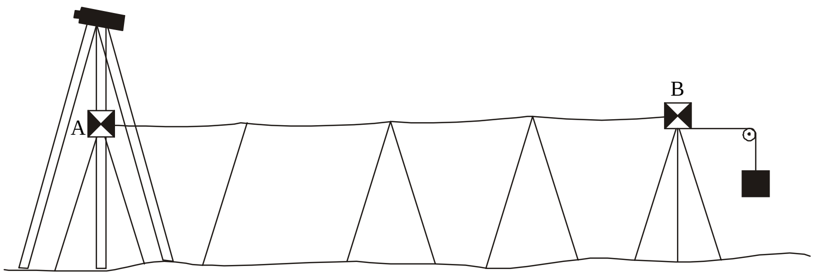

Fig. 1.36 Hunter's short base

ILLUSTRATIVE EXAMPLES

Example 1.12 A tape of standard length 20 m at 85° F was used to measure a base line. The measured distance was 882.10 m. The following being the slopes for the various segments of the line.

Segment	Slope
100 m	2°20′
150 m	4°12′
50 m	1°06′
200 m	7°45′
300 m	3°00′
82.10 m	5°10′

Find the true length of the base line if the mean temperature during measurement was 63°F. The coefficient of expansion of the tape material is 6.5×10^{-6} per °F.

Solution: (refer to Sec. 3.5 of *Plane Surveying*):

Correction for temperature

$$C_t = \alpha (t_m - t_0) L$$
$$= 6.5 \times 10^{-6} \times (63 - 65) \times 882.10$$
$$= 0.126 \text{ m (subtractive)}$$

Correction for slope

$$C_s = \Sigma[(1 - \cos\alpha) L]$$
$$= (1 - \cos 2°20') \times 100 + (1 - \cos 4°12') \times 150 + (1 - \cos 1°06') \times 50$$
$$+ (1 - \cos 7°48') \times 200 + (1 - \cos 3°00') \times 300 + (1 - \cos 5°10') \times 82.10$$
$$= 3.079 \text{ m (subtractive)}$$

$$\text{Total correction} = C_t + C_s$$
$$= 0.126 + 3.079$$
$$= 3.205 \text{ m (subtractive)}$$
$$\text{Corrected length} = 882.10 - 3.205$$
$$= \mathbf{878.895\ m}.$$

Example 1.13. A base line was measured between two points *A* and *B* at an average elevation of 224.35 m. The corrected length after applying all correction was 149.3206 m. Reduce the length to mean sea level. Take earth's mean radius as 6367 km.

Solution: (Refer Sec. 3.5 of *Plane Surveying*):

The reduced length at mean seal level is

$$L' = \frac{R}{(R + h)} L$$

$$= \frac{6367}{6367 + \left(\frac{224.35}{1000}\right)} \times 149.3205$$

$$= \mathbf{149.3152\ m}.$$

1.16.3 Extension of base line

Usually the length of the base lines is much shorter than the average length of the sides of the triangles. This is mainly due to the following reasons:

(*a*) It is often not possible to get a suitable site for a longer base.

(*b*) Measurement of a long base line is difficult and expensive.

The extension of short base is done through forming a base net consisting of well-conditioned triangles. There are a great variety of the extension layouts but the following important points should be kept in mind in selecting the one.

(*i*) Small angles opposite the known sides must be avoided.

(*ii*) The length of the base line should be as long as possible.

(*iii*) The length of the base line should be comparable with the mean side length of the triangulation net.

(*iv*) A ratio of base length to the mean side length should be at least 0.5 so as to form well-conditioned triangles.

(*v*) The net should have sufficient redundant lines to provide three or four side equations within the figure.

(*vi*) Subject to the above, it should provide the quickest extension with the fewest stations.

There are two ways of connecting the selected base to the triangulation stations. There are

(*a*) extension by prolongation, and

(*b*) extension by double sighting.

(a) Extension by prolongation

Let up suppose that *AB* is a short base line (Fig. 1.37) which is required to be extended by four times. The following steps are involved to extend *AB*.

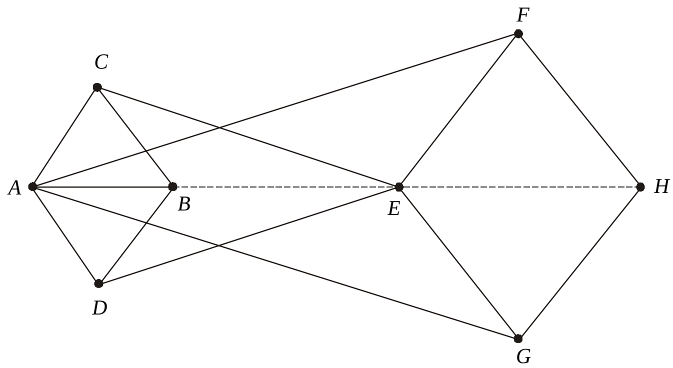

Fig. 1.37 Base extension by prolongation

(*i*) Select *C* and *D* two points on either side of *AB* such that the triangles *BAC* and *BAD* are well-conditioned.

(*ii*) Set up the theodolite over the station *A*, and prolong the line *AB* accurately to a point *E* which is visible from points *C* and *D*, ensuring that triangles *AEC* and *AED* are well-conditioned.

(*iii*) In triangle *ABC*, side *AB* is measured. The length of *AC* and *AD* are computed using the measured angles of the triangles *ABC* and *ABD*, respectively.

(*iv*) The length of *AE* is calculated using the measured angles of triangles *ACE* and *ADE*, and taking mean value.

(*v*) Length of *BE* is also computed in similar manner using the measured angles of the triangles *BEC* and *BDE*. The sum of lengths of *AB* and *BE* should agree with the length of *AE* obtained in step (*iv*).

(*vi*) If found necessary, the base can be extended to *H* in the similar way.

(*b*) Extension by double sighting

Let *AB* be the base line (Fig. 1.38). To extend the base to the length of side *EF*, following steps are involved.

(*i*) Chose intervisible points *C*, *D*, *E*, and *F*.

(*ii*) Measure all the angles marked in triangles *ABC* and *ABD*. The most probable values of these angles are found by the theory of least-squares discussed in Chapter 2.

(*iii*) Calculate the length of *CD* from these angles and the measured length *AB*, by applying the sine law to triangles *ACB* and *ADB* first, and then to triangles *ADC* and *BDC*.

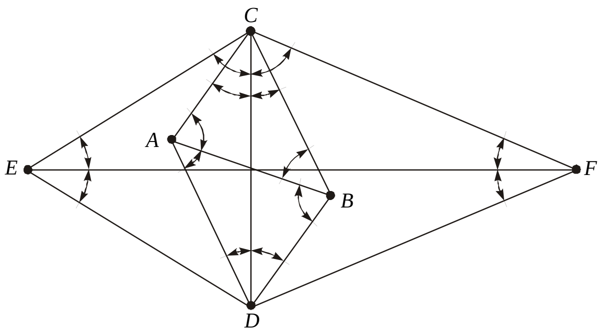

Fig. 1.38 Base extension by double sighting

(*iv*) The new base line *CD* can be further extended to the length *EF* following the same procedure as above. The line *EF* may from a side of the triangulation system.

If the base line *AB* is measured on a good site which is well located for extension and connection to the main triangulation system, the accuracy of the system is not much affected by the extension of the base line. In fact, in some cases, the accuracy may be higher than that of a longer base line measured over a poor terrain.

1.17 MEASUREMENT OF HORIZONTAL ANGLES

The instruments used for triangulation surveys, require great degree of precision. Horizontal angles are generally measured with an optical or electronic theodolite in primary and secondary triangulation. For tertiary triangulation generally transit or Engineer's transit having least count of 20" is used.

Various types of theodolities have been discussed in Sec. 4.4.5 of *Plane Surveying*. The salient features of the modern theodolities are as follow:

(*i*) These are small in dimension, and light in weight.

(*ii*) The graduations are engraved on glass circles, and are much finer.

(*iii*) The mean of two readings on the opposite sides of the circles can be read directly through an eyepiece, saving the observation time.

(*iv*) There is no necessity to adjust the micrometers.

(*v*) These are provided with optical plummet which makes possible accurate centering of the instrument even in high winds.

(*vi*) These are water proof and dust proof.

(*vii*) These are provided with electrical arrangement for illumination during nights if necessary.

(*viii*) Electronic theodolites directly display the value of the angle on *LCD* or *LED*.

1.17.1 Methods of observation of horizontal angles

The horizontal angles of a triangulation system can be observed by the following methods:

(*i*) Repetition method

(*ii*) Reiteration method.

The procedure of observation of the horizontal angles by the above methods has been discussed in Sec. 4.5 of *Plane Surveying.*

(*i*) Repetition method

For measuring an angle to the highest degree of precision, several sets of repetitions are usually taken. There are following two methods of taking a single set.

(*a*) In the first method, the angle is measured clockwise by 6 repetitions keeping the telescope normal. The first value of the angle is obtained by dividing the final reading by 6. The telescope is inverted, and the angle is measured again in anticlockwise direction by 6 repetitions. The second value of the angle is obtained by dividing the final reading by 6. The mean of the first and second values of the angle is the average value of the angle by first set.

For first-order work, five or six sets are usually required. The final value of the angle is the mean of the values obtained by different sets.

(*b*) In the second method, the angle is measured clockwise by six repetitions, the first three with telescope normal and the last three with telescope inverted. The first value of the angle is obtained by dividing the final reading by 6. Now without altering the reading obtained in the sixth repetition, the explement angle (i.e., 360°– the angle), is measured clockwise by six repetitions, the first three with telescope inverted and the last three with telescope normal. The final reading should theoretically be zero. If the final reading is not zero, the error is noted, and half of the error is distributed to the first value of the angle. The result is the corrected value of the angle by the first set. As many sets as desired are taken, and the mean of all the value of various sets, is the average value of the angle. For more accurate work and to eliminate the errors due to inaccurate graduations of the horizontal circle, the initial reading at the beginning of each set may not be set to zero but to different values. If n sets are required, the initial setting should be sucessively increased by 180°/n. For example, for 6 sets the initial readings would be 0°, 30°, 60°, 90°, 120° and 150°, respectively.

(*ii*) Reiteration method or direction method

In the reiteration method, the triangulation signals are bisected successively, and a value is obtained for each direction in each of several rounds of observations. One of the triangulation stations which is likely to be always clearly visible may be selected as the initial station or reference station. The theodolites used for the measurement of angles for triangulation surveys, have more than one micrometer. One of the micrometer is set to 0° and with telescope normal, the initial station is bisected, and all the micrometers are read. Each of the successive stations are then bisected, and all the micrometers are read. The stations are then again bisected in the reverse direction, and all the micrometers are read after each bisection. Thus, two values are obtained for each angle when the telescope is normal. The telescope is then inverted, and the observations are repeated. This constitutes one set in which four value of each angle are obtained. The micrometer originally at 0° is now brought to a new reading equal to 360°/mn (where m is the number of micrometers and n is the number of sets), and a second set is observed in the same manner. The number of sets depends on the accuracy required. For first-order triangulation, sixteen such sets are required with a 1″ direction theodolite, while for second-order triangulation four, and for third-order triangulation two. With more refined instrument having finer graduations, however, six to eight sets are sufficient for the geodetic work.

1.18 MEASUREMENT OF VERTICAL ANGLES

Measurement of vertical angles is required to compute the elevation of the triangulation stations. The method of measurement of vertical angles is discussed in Sec. 4.5.4 of *Plane Surveying.*

1.19 ASTRONOMICAL OBSERVATIONS

To determine the azimuth of the initial side, intermediate sides, and the last side of the triangulation net, astronomical observations are made. For detailed procedure and methods of observation, refer to Chapter 7.

1.20 SOME EXTRA PRECAUTIONS IN TAKING OBSERVATIONS

To satisfy first-second, and third-order specifications as given in Table 1.1, care must be exercised. Observer must ensure the following:

1. The instrument and signals have been centred very carefully.
2. Phase in signals has been eliminated.
3. The instrument is protected form the heating effects of the sun and vibrations caused by wind.
4. The support for the instrument is adequately stable.
5. In case of adverse horizontal refraction, observations should be rescheduled to the time when the horizontal refraction is minimum.

Horizontal angles should be measured when the air is the clearest, and the lateral refraction is minimum. If the observations are planned for day hours, the best time in clear weather is from 6 AM to 9 AM and from 4 PM till sunset. In densely clouded weather satisfactory work can be done all day. The best time for measuring vertical angles is form 10 AM to 2 PM when the vertical refraction is the least variable.

First-order work is generally done at night, since observations at night using illuminated signals help in reducing bad atmospheric conditions, and optimum results can be obtained. Also working at night doubles the hours of working available during a day. Night operations are confined to period from sunset to midnight.

1.21 SATELLITE STATION AND REDUCTION TO CENTRE

To secure well-conditioned triangles or to have good visibility, objects such as chimneys, church spires, flat poles, towers, lighthouse, etc., are selected as triangulation stations. Such stations can be sighted from other stations but it is not possible to occupy the station directly below such excellent targets for making the observations by setting up the instrument over the station point. Also, signals are frequently blown out of position, and angles read on them have to be corrected to the true position of the triangulation station. Thus, there are two types of problems:

1. When the instrument is not set up over the true station, and
2. When the target is out of position.

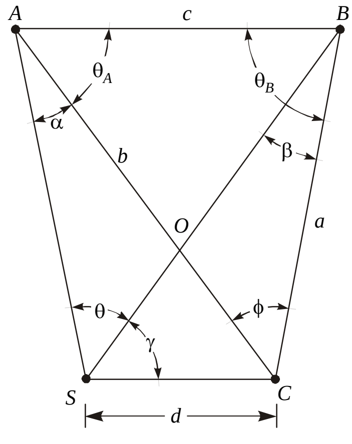

Fig. 1.39 Reduction to centre

In Fig. 1.39, *A*, *B*, and *C* are the three triangulation stations. It is not possible to place instrument at *C*. To solve this problem another station *S*, in the vicinity of *C,* is selected where the instrument can be set up, and from where all the three stations are visible for making the angle observations. Such station is known as *satellite station.* As the observations from *C* are not possible, the observations form *S* are made on *A*, *B*, and, *C* from *A* and *B* on *C*. From the observations made, the required angle *ACB* is calculated. This is known as *reduction to centre.*

In the other case, S is treated as the true station point, and the signal is considered to be shifted to the position C. This case may also be looked upon as a case of *eccentricity of signal*. Thus, the observations from S are made to the triangulation stations A and B, but from A and B the observations are made on the signal at the shifted position C. This causes errors in the measured values of the angles BAC and ABC.

Both the problems discussed above are solved by reduction to centre.

Let the measured

$$\angle BAC = \theta_A$$
$$\angle ABC = \theta_B$$
$$\angle ASB = \theta$$
$$\angle BSC = \gamma$$
$$\text{Eccentric distance } SC = d$$

The distance AB is known by computations form preceding triangle of the triangular net. Further, let

$$\angle SAC = \alpha$$
$$\angle SBC = \beta$$
$$\angle ACB = \phi$$
$$AB = c$$
$$AC = b$$
$$BC = a$$

As a first approximation in ΔABC the $\angle ACB$ may be taken as

$$= 180° - (\angle BAC + \angle ABC)$$

or
$$\phi = 180° - (\theta_A + \theta_B) \qquad \text{...(1.37)}$$

In the triangle ABC we have

$$\frac{c}{\sin\phi} = \frac{a}{\sin\theta_A} = \frac{b}{\sin\theta_B}$$

$$a = \frac{c.\sin\theta_A}{\sin\phi} \qquad \text{...(1.38)}$$

and
$$b = \frac{c.\sin\theta_B}{\sin\phi} \qquad \text{...(1.39)}$$

Compute the values of a and b by substituting the value of ϕ obtained from Eq. (1.37) in Eqs. (1.38) and (1.39), respectively.

Now, from $\Delta^s SAC$ and SBC we have

$$\frac{d}{\sin\alpha} = \frac{b}{\sin(\theta + \gamma)}$$

$$\frac{d}{\sin\beta} = \frac{a}{\sin\gamma}$$

$$\sin\alpha = \frac{d\ \sin(\theta + \gamma)}{b}$$

$$\sin\beta = \frac{d\sin\gamma}{a}$$

As the satellite station S is chosen very close to the main station C, the angles α and β are extremely small. Therefore, taking $\sin\alpha = \alpha$, and $\sin\beta = \beta$ in radians, we get.

$$\alpha = \frac{d\sin(\theta + \gamma)}{b\ \sin 1''}$$

or $= \dfrac{d \sin(\theta + \gamma)}{b} \times 206265$ seconds ...(1.40)

and $\beta = \dfrac{d \sin \gamma}{a} \times 206265$ seconds ...(1.41)

In Eqs. (1.40) and (1.41), θ, γ, d, b and a are known quantities, therefore, the values of α and β can be computed. Now a more correct value of the angle $\angle ACB$ can be found.

We have

$$\angle AOB = \theta + \alpha = \phi + \beta$$

or $\phi = \theta + \alpha - \beta$ ...(1.42)

Eq. (1.42) gives the value of ϕ when the satellite station *S* is to the left of the main station *C*. In the general, the following four cases as shown in Fig. 1.40*a*, can occur depending on the field conditions.

Case I: *S* towards the left of *C* (Fig. 1.39)

$$\phi = \theta + \alpha - \beta$$

Case II: *S* towards the right of *C* (Fig. 1.40*b*), the position S_2.

$\phi = \theta - \alpha + \beta$ ...(1.43)

Case III: *S* inside the triangle *ABC* (Fig. 1.40*c*), the position S_3.

$\phi = \theta - \alpha - \beta$ ...(1.44)

Case IV: *S* outside the triangle *ABC* (Fig. 1.40*d*), the position S_4.

$\phi = \theta + \alpha + \beta$ ...(1.45)

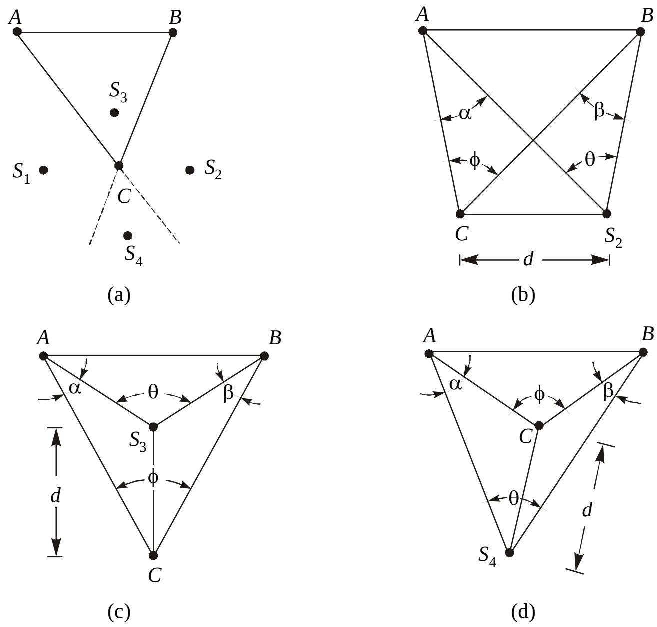

Fig. 1.40 Locations of satellite station with reference to triangulation stations *C*

1.22 ECCENTRICITY OF SIGNAL

When the signal is found shifted from its true position, the distance between the shifted signal and the station point *d* is measured. The corrections α and β to the observed angles *BAC* and *ABC*, respectively, are computed from Eqs. (1.40) and (1.41), and the corrected values of the angles are obtained as under (Fig. 1.39):

Correct $\angle BAS = \theta_A + \alpha$ ...(1.46)

Correct $\angle ABS = \theta_B - \beta$ ...(1.47)

For other cases shown in Fig. 1.40, one can easily find out the correct angles.