2.1 Traffic Stream Characteristics

- Time Space Diagram and Measurement Procedures
- Variables of Interest
- Traffic Stream Models
\( k \) density of a traffic stream in a specified length of road

\( L \) length of vehicles of uniform length

\( c_k \) constant of proportionality between occupancy and density, under certain simplifying assumptions

\( k_i \) the (average) density of vehicles in substream \( I \)

\( q_i \) the average rate of flow of vehicles in substream \( I \)

\( \bar{u} \) average speed of a set of vehicles

\( A \) \( A(x,t) \) the cumulative vehicle arrival function over space and time

\( k_j \) jam density, i.e. the density when traffic is so heavy that it is at a complete standstill

\( u_f \) free-flow speed, i.e. the speed when there are no constraints placed on a driver by other vehicles on the road
2.1 Traffic Stream Characteristics

Time Space Diagram

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>7.2</td>
</tr>
<tr>
<td>400</td>
<td>14.4</td>
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<td>600</td>
<td>21.6</td>
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<td>800</td>
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<td>36</td>
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<td>1200</td>
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<td>1400</td>
<td>50.4</td>
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<tr>
<td>1600</td>
<td>57.6</td>
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<td>1800</td>
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<td>2200</td>
<td>79.2</td>
</tr>
<tr>
<td>2400</td>
<td>86.4</td>
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</table>
Time Space Diagram

Time distance diagram

- $V = 100\, \text{km/hr}$
- $V = 150\, \text{km/hr}$
- $V = 50\, \text{km/hr}$

Distance (m)

Time (s)

Freeway lane
**Time Space Diagram**

**Speed**: 150 km/h = 41.67 m/sec

<table>
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<tbody>
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<tr>
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<td>208</td>
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<td>2083</td>
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<tr>
<td>2292</td>
<td>60</td>
</tr>
<tr>
<td>2500</td>
<td>65</td>
</tr>
</tbody>
</table>

**Time distance diagram**

![Time distance diagram](image)
**Time Space Diagram**

**Speed =** 150 km/h = 41.67 m/sec

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</tr>
<tr>
<td>2500</td>
<td>70</td>
</tr>
</tbody>
</table>

The Time distance diagram shows the relationship between time and distance for a vehicle traveling at a constant speed of 150 km/h. The speed is equivalent to 41.67 m/sec. The diagram illustrates how distance changes with time, with different lines representing different scenarios or speeds on the freeway.
Time-space diagram (Single vehicle)

(a) Linear increase in distance over time.
(b) Curve and peak, indicating some event or slowdown.
(c) Stepped increase in distance, possibly indicating stops or pauses in movement.
Time-space diagram (Multiple vehicle)
Assignment

Question 1

Assuming the distance of a free way is 2400m, draw the time space diagram given the following conditions:

- A vehicle in lane one starts at time 0 and moves with a speed of 60 km/hr.
- A vehicle at lane 2 starts at time 15 sec and moves with a speed of 110 km/hr.
Assignment

Question 2

When a driver (moving with 55 km/hr) was at point A (100 m from the intersection), as shown in the figure, he saw a yield sign, so he decided to decelerate with a constant deceleration rate 1.5 m/s². When he was 40 m from the intersection he realized that the intersection is clear, so he decided to re-accelerate with a constant acceleration 2 m/s². Knowing that the maximum allowable speed in the road is 55 km/hr, draw the time distance diagram.
Measurement Procedures

- Measurement at a point;
- Measurement over a short section (by which is meant less than about 10 meters);
- Measurement over a length of road (usually at least 0.5 kilometers);
- The use of an observer moving in the traffic stream; and
- Wide-area samples obtained simultaneously from a number of vehicles, as part of Intelligent Transportation Systems (ITS);
Measurement at a point

Measurement at a time

**Figure 2.1**
*Four Methods of Obtaining Traffic Data (Modified from Drew 1968, Figure 12.9).*
Measurement at a Point

- Provide **volume counts** and therefore **flow rates** directly, and with care can also provide **time headways**.
- **Speeds at a ‘point’** can be obtained only by radar or microwave detectors. Otherwise, a second observation location is necessary to obtain speeds (measurements over a short section).
Measurement at a Point

Density, which is defined as vehicles per unit length, does not make sense for a point measurement, because no length is involved.

Hence volume (or flow rate), headways, and speeds are the only direct measurements at a point.
Measurement Over a Short Section

- Occupancy is defined as the percentage of time that the detection zone of the instrument is occupied by a vehicle.

- As with point measurements, short-section data acquisition does not permit direct measurement of density. Where studies based on short-section measurements have used density, it has been calculated.
It is suggested that at least 0.5 km of road be observed.

On the basis of a single frame from such sources, only density can be measured.

The single frame gives no sense of time, so neither volumes nor speed can be measured.
Measurement Along a Length of Road

Once several frames are available, speeds can be measured, often over a distance approximating the entire section length over which densities have been calculated.

Flow and density refer to different measurement frameworks: flow over time at a point in space; density over space at a point in time.
Moving Observer Method

Two approaches:

- The first is a simple floating car procedure in which speed and travel times are recorded as a function of time and location along the road.

- The other approach was developed by Wardrop and Charlesworth (1954) for urban traffic measurements and is meant to obtain both speed and volume measurement simultaneously.
**Wardrop and Charlesworth method**

Based on a survey vehicle that travels in both directions on the road.

\[
q = \frac{(x+y)}{(t_a + t_w)} \quad (2.1)
\]

where,

- \( q \) is the estimated flow on the road in the direction of interest,
- \( x \) is the number of vehicles traveling in the direction of interest, which are met by the survey vehicle while traveling in the opposite direction,
- \( y \) is the net number of vehicles that overtake the survey vehicle while traveling in the direction of interest (i.e. those passing minus those overtaken),
- \( t_a \) is the travel time taken for the trip against the stream,
- \( t_w \) is the travel time for the trip with the stream, and
- \( \bar{t} \) is the estimate of mean travel time in the direction of interest.

\[
\bar{t} = t_w - \frac{y}{q} \quad (2.2)
\]
ITS Wide-Area Measurements

- Involve the use of communications from specially-equipped vehicles to a central system. All of them provide for transmission of information on the vehicles’ speeds.

- The major difficulty with implementing this approach is that of establishing location precisely. Global positioning systems have almost achieved the capability for doing this well, but they would add considerably to the expense of this approach.
Variables of Interest

- In general, traffic streams are not uniform, but vary over both space and time. Because of that, measurement of the variables of interest for traffic flow theory is in fact the sampling of a random variable.

- In reality, the traffic characteristics that are labeled as flow, speed, and concentration are parameters of statistical distributions, not absolute numbers.
**Variables of Interest**

- Time headway between vehicles (time per vehicles)
- Spacing, or space headway between vehicles (distance per vehicle); and
- Rates of flow (vehicles per unit time)
- Speeds (distance per unit time)
- Travel time over a know length of road (or sometimes the inverse of speed, “tardity”)
- Occupancy (percent of time a point on the road is occupied by vehicles)
- Density (vehicles per unit distance)
- Concentration (measured by density or occupancy)
Time Headway

The time headway or headway is the average time interval between the arrival of vehicles at a point.

It is the inverse of flow (q)

E.g. If flow is 1200 veh/hr, then

\[ h = \frac{3600}{1200} = 3 \text{ sec} \]
Spacing

Spacing or the distance headway is the average distance between the front of one vehicle and the front of the next.

It is the inverse of density (k).

E.g: If density is 120 veh/km then

\[ s = \frac{1000}{120} = 8.3 \text{ m} \]
Time-space diagram / Trajectory

Distance vs. Time

- \( h \)
- \( q = 6 \)
Time-space diagram / Trajectory
Flow Rates

Flow rates are collected directly through point measurements, and by definition require measurement over time.

Flow rates are usually expressed in terms of vehicles per hour,

Flow rate, $q$, is the number of vehicles counted, divided by the elapsed time, $T$ (or $\Delta T$):

$$q = \frac{N}{T}$$
Time Space Diagram
Flow Rates

-The total elapsed study time is made up of the sum of the headways recorded for each vehicle:

\[ T = \sum_{i=1}^{N} h_i \]

-If the sum of the headways is substituted in Equation for total time, \( T \), then it can be seen that the flow rate and the average headway have a reciprocal relationship with each other:

\[ q = \frac{N}{T} = \frac{N}{\sum_{i} h_i} = \frac{1}{\frac{1}{N} \sum_{i} h_i} = \frac{1}{\bar{h}} \]
Density

Density is the number of vehicles occupying a given length of a lane or roadway at a particular instant expressed as vehicles per kilometer.

It is a measure of the quality of traffic operation.

Drivers behavior significantly depends on density.

\[ k = \frac{n}{\Delta X} \]

\[ k = \frac{n}{\sum_{i=1}^{n} s_i} = \frac{1}{s} \]
**Speeds**

Measurement of the speed of an individual vehicle requires observation over both time and space. The instantaneous speed of an individual vehicle is defined as

\[ u_i = \frac{dx}{dt} = \lim_{t_2-t_1 \to 0} \frac{x_2-x_1}{t_2-t_1} \]
**Mean speeds**

- **Time mean speed**
  - average of all vehicles passing a point over a duration of time
  - It is the simple average of spot speed
  - Expression for $v_t$

\[
v_t = \frac{1}{n} \sum_{i=1}^{n} v_i,
\]

- $v_i$ spot speed of $i^{th}$ vehicle
- $n$ number of observations
### Mean speeds

- **Time mean speed**
  - Speeds may be in the form of frequency table
  - Then $v_t$

\[
v_t = \frac{\sum_{i=1}^{n} q_i v_i}{\sum_{i=1}^{n} q_i},
\]

- $q_i$ number of vehicles having speed $v_i$
- $n$ number of such speed categories
Mean speeds

- **Space mean speed**
  - average speed in a stretch at an instant
  - It also averages the spot speed
  - But spatial weightage instead of temporal
Mean speeds

Space mean speed - derivation

- Consider unit length of a road
- Let $v_i$ is the spot speed of $i^{th}$ vehicle
- Let $t_i$ is the time taken to complete unit distance
- $t_i = \frac{1}{v_i}$
- If there are $n$ such vehicles, then the average travel time $t_s$ is given by

$$t_s = \frac{\sum t_i}{n} = \frac{1}{n} \sum \frac{1}{v_i}$$
Mean speeds

Space mean speed - derivation

If average travel time is \( t_s \) then

average speed \( v_s \) is \( \frac{1}{t_s} \)

\[
t_s = \frac{\sum t_i}{n} = \frac{1}{n} \sum \frac{1}{v_i} \quad \rightarrow \quad v_s = \frac{n}{\sum_{i=1}^{m} \frac{1}{v_i}}
\]

the harmonic mean of the spot speed

If speeds are in a frequency table
Mean speeds

Space mean speed

If speeds are in a frequency table

then \( v_s \)

\[ v_s = \frac{\sum_{i=1}^{n} q_i v_i}{\sum_{i=1}^{n} q_i} \]

\( q_i \) number of vehicles having speed \( v_i \)

\( n \) number of such speed categories
Exercise

Calculate:
volume (veh/hr),
density (veh/mi).

Is the space-mean speed equal to the time-mean speed?
Mean speeds: Illustration

Calculate:
1- the time mean speed over 60 sec time period
2- space mean speed over a 1km road length?
### Mean speeds: Illustration

**Lane s**

- Speed: 10 m/s
- Time Headway: \( \frac{50}{10} = 5 \text{ sec} \)
- Vehicles in 60 s: \( \frac{60}{5} = 12 \) vehicles
- Vehicles in 1 km: \( \frac{1000}{50} = 20 \) vehicles

**Lane f**

- Speed: 20 m/s
- Time Headway: \( \frac{100}{20} = 5 \text{ sec} \)
- Vehicles in 60 s: \( \frac{60}{5} = 12 \) vehicles
- Vehicles in 1 km: \( \frac{1000}{100} = 10 \) vehicles

**Average Speed**

\[
\nu_t = \frac{12 \times 10 + 12 \times 20}{24} = 15 \text{ m/s}
\]

\[
V_s = \frac{30}{\frac{20}{10} + \frac{10}{20}} = 12 \text{ m/sec}
\]
Space-mean Speed from Spot Observations

Let’s assume that $N$ speed measurements have been collected over some period of time at some spot.

The space-mean speed is:

$$\bar{U}_s = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{\sum_{i=1}^{N} U_i}$$

The space-mean speed in the vicinity of the spot can be approximated as follows:

$$\bar{U}_s = \bar{U}_t - \frac{\text{var} \bar{U}_t}{\bar{U}_t}$$
Assignment: Mean speeds

Example 1

If the spot speeds (at one point) are 51, 41, 61, 55 and 46, then find the TMS and SMS.

Example 2

The results of a speed study (at one point) is given in the form of a frequency distribution table. Find the TMS and SMS.

<table>
<thead>
<tr>
<th>speed range</th>
<th>frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-5</td>
<td>2</td>
</tr>
<tr>
<td>6-9</td>
<td>5</td>
</tr>
<tr>
<td>10-13</td>
<td>1</td>
</tr>
<tr>
<td>14-17</td>
<td>8</td>
</tr>
</tbody>
</table>
Occupancy

Occupancy is the fraction of time that vehicles are over the detector. For each individual vehicle, the time spent over the detector is determined by the vehicle's speed, $u$, and its length, $L$, plus the length of the detector itself, $d$.

\[
\text{occupancy} = \frac{T}{\sum_i \frac{(L_i + d)}{u_i}}
\]

\[
= \frac{1}{T} \sum_i \frac{L_i}{u_i} + \frac{d}{T} \sum_i \frac{1}{u_i}
\]
**Speed-Density Models**

The most interesting aspect of this particular model is that its empirical basis consisted of half a dozen points in tone cluster near free-flow speed, and a single observation under congested conditions. (1935) There are other advanced research lately.

\[ u = u_f (1 - \frac{k}{k_j}) \]
\[ q = k_j(u - \frac{u^2}{u_f}) \]

where \( u \) is the free-flow speed, and \( k \) is the jam density.
Fundamental Diagram
$q, v, k – Greenshield’s$

\[ v = v_f - \left( \frac{v_f}{k_j} \right) k \]
$q, v, k – Greenshield’s$

$q = k_j v - \left( \frac{k_j}{v_f} \right) v^2$
$q, \ v, \ k \ - \ Greenshield's$

$$q = v_f k - \left( \frac{v_f}{k_j} \right) k^2$$

$\tan \theta = q_m \frac{k}{k_m} = v_m$

$V_m$

$V_f$

$\theta$

$q_m$

$q_f$

$V_f$
Fundamental diagrams

- Combined
**Speed-Flow Models**

This curve has speeds remaining flat as flows increase, out to somewhere between half and two-thirds of capacity values, and a very small decrease in speeds at capacity from those values.

![Image of Speed-Flow Curve](image-url)

**Figure 2.3**

*Speed-Flow Curves Accepted for 1994 HCM.*
Figure 2.4
Generalized Shape of Speed-Flow Curve
Proposed by Hall, Hurdle, & Banks
(Hall et al. 1992).
Assignment

Describe at least 2 other speed density relationships?

Draw the fundamental diagram of these relationships?