Optimizing the thulium doped fiber amplifier (TDFA) gain and noise figure for S-band 16 × 10 Gb/s WDM systems

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Abstract

This paper aims to evaluate a comprehensive numerical model based on solving rate equations of a thulium-doped silica-based fiber amplifier. The pump power and thulium-doped fiber (TDF) length for single-pass thulium-doped fiber amplifiers (TDFA) are theoretically optimized to achieve the optimum gain and noise figure (NF) at the center of S-band region. The 1064 nm pump is used to provide both ground-state and excited state absorptions for amplification in the S-band region. The theoretical result is in agreement with the published experimental result.

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1. Introduction

Increasing demands on the capacity of WDM transmission system now require newly developed transmission windows beyond the amplification bandwidth supported by erbium-doped fiber amplifiers (EDFA’s). Due to the tremendous increase in communication traffic in recent years, more and more efforts in research have been directed toward developing highly efficient broad-band fiber amplifier that fully exploit the low-loss band of silica fiber at 1450–1525 nm range which has a loss of only 0.25 db/km in order to increase the transmission capacity of wavelength division multiplexing (WDM) networks [1,2].

Thulium-doped fiber amplifier (TDFA) provides high-power optical amplification in the S+ (1450–1480 nm) and S-bands (1480–1530 nm) [3], hence the TDFA is expected to complement C-(1530–1560 nm) and L-band (1560–1580 nm) amplification based on EDFA’s in high-capacity dense wavelength division multiplexed (DWDM) systems [4,5]. The additional bandwidth, modularity, inherent higher pumping efficiency, and lower nonlinear signal degradation (compared with alternatives such as S-band Raman amplification [6,7]) offered by TDFA enables applications such as coarse wavelength-division multiplexing (CWDM) and fiber to the home (FTTH).

The TDFA length and Pump power are the important parameters that determine the achievable gain and NF in TDFA [8]. In this paper, we detail the observation and modeling of TDFA where TDFA gain and NF are optimized by solving the rate equations.

2. Configuration of the TDFA

The basic architecture used to model TDFA in the WDM system is shown in Fig. 1. The system consists of 16 input signals (channels), an ideal multiplexer, a pump laser, pump coupler, thulium-doped fiber (TDF), optical spectrum analyzer and dual port WDM analyzer.

The structural design shows the setup of the single pass TDFA. The input of the system is 16 equalized wavelength multiplexed signals (channels) in the wavelength region of 80 nm (1450–1530 nm) with 5 nm channels spacing. The power of each channel is −20 dbm. The pumping at 1064 nm is used to excite the doped atoms to a higher energy level. The TDF used is a glass based one with thulium density of 15.6 × 10−24 m³, core radius is 1.3 μm, doping radius is 1.3 μm and numerical aperture (NA) is 0.3. The simulation done with maximum number of iterations is 150 and relative error is 5 × 10−4.

3. Theory of the TDFA

The rate equations describe the interaction between signal, pump, and ASE light in the TDFA. The rate equations are used to analyze theoretically the populations in the energy levels of Tm³⁺ ions under 1064 nm pump and signal power conditions. The absorption and stimulated emission cross sections define the absorption coefficient for pump light and gain coefficient for signal light [9]. The transition cross-sections of thulium are shown in Fig. 2 [8]. The transition cross-sections were calculated in fluoride based TDF [10]. The Judd–Ofelt analysis shows that the transition strengths obtained were consistent with those for silica.

An analysis of a six energy levels system is shown in Fig. 3, where the energy levels of trivalent thulium ion in fluoride glass
are displayed. The absorption and emission transitions are shown in Fig. 3(a) and (b), respectively for the T DFA with 1064 nm pump wavelength. For S-band amplification, the main transition is from $^{3}H_{4} \rightarrow ^{3}F_{4}$ energy levels. Pumping at 1064 nm range takes benefit of the excited state absorption (ESA) $^{3}F_{4}$ at the level to excite electrons to the upper energy state. On the other hand, as 1064 nm is the main source of excitation, ground state absorption (GSA) of 1064 nm and/or WDM signals at the $^{3}H_{6}$ ground state must be nonzero in order to populate $^{3}H_{5}$ energy level and then relaxed to the $^{3}F_{4}$ energy level by non-radiative decay [11]. By exciting the TDFA at a fixed level (at 1064 nm), increasing the input WDM signals power further populates the lower energy state ($^{3}F_{2}$), from which the excited ions are raised to the upper energy state ($^{3}H_{4}$) because of excess pump power [11]. The pumping transition $^{3}F_{4} \rightarrow ^{3}G_{4}$ is (ESA). The energy level of the $^{3}F_{2}$ and $^{3}F_{3}$ are very close nearly the same and can be regarded as one level for simplicity. So the $^{3}F_{2}$ energy level ions are re-excited to the $^{3}F_{4}$ energy level and experience non-radiative decay to the $^{3}H_{4}$ energy level via exited state absorption [12,13].

The thulium doped fiber ions can be considered homogeneously broadened in amplification system and also characterized by the variables $N_{0}$, $N_{1}$, $N_{2}$, $N_{3}$, $N_{4}$ and $N_{5}$ which are used to represent population ions in the $^{3}H_{6}$, $^{3}F_{4}$, $^{3}H_{4}$, $^{3}F_{2}$, and $^{1}G_{4}$ energy levels, respectively. For simplicity, $\gamma_{31}$ and $\gamma_{32}$ are ignored because they are very small compared with $\gamma_{50}$, $\gamma_{51}$ and $\gamma_{53}$ are also ignored because they are small compared with $\gamma_{50}$ and $\gamma_{52}, \gamma_{20}$ and $\gamma_{4j}$ ($j=0, 1, 2$) are very small and can be disregarded because $\gamma_{21}$ and $\gamma_{43}$ are multiphonon decay. On the basis of the energy level diagram as in Fig. 3. The rate equation for Tm$^{3+}$ population density can be written as follows [14]:

$$\frac{dN_{0}}{dt} = -W_{p1}N_{0} + \gamma_{10}N_{1} + \gamma_{30}N_{3} + \gamma_{50}N_{5} \tag{1}$$
$$\frac{dN_{1}}{dt} = -(\gamma_{10} + W_{p2} + W_{s})N_{1} + \gamma_{21}N_{2} + W_{s}N_{3} \tag{2}$$
$$\frac{dN_{2}}{dt} = W_{p1}N_{0} - \gamma_{21}N_{2} + \gamma_{52}N_{5} \tag{3}$$
$$\frac{dN_{3}}{dt} = W_{p1}N_{1} - (\gamma_{30} + W_{p3} + W_{s})N_{3} + \gamma_{43}N_{4} \tag{4}$$
$$\frac{dN_{4}}{dt} = W_{p2}N_{2} - \gamma_{43}N_{4} \tag{5}$$
$$\frac{dN_{5}}{dt} = W_{p3}N_{3} - (\gamma_{50} + \gamma_{52})N_{5} \tag{6}$$

$$N_{i} = \sum_{i=1}^{5} N_{i} \tag{7}$$

where $W_{p1}$, $W_{p2}$, and $W_{p3}$ are transition rates of $^{3}H_{6} \rightarrow ^{3}H_{5}$, $^{3}H_{4} \rightarrow ^{3}F_{4}$, and $^{3}F_{4} \rightarrow ^{1}G_{4}$ pumping transition. The signal of the central S-band is 1470 nm as signal stimulated absorption and emission is described by transition rate $W_{s}$. The non-radiative transition rate from $^{3}F_{2} \rightarrow ^{3}F_{4}$ and from $^{3}H_{5} \rightarrow ^{3}F_{4}$ energy levels are defined as $\gamma_{21}^{nr}$ and $\gamma_{43}^{nr}$, respectively. $\gamma_{ij}$ is the radiative rate from level i to level j. Others radiative transitions are not included in the rate equations because they have an ignorable effect on the S-band amplification. For simplicity, $\gamma_{31}$ and $\gamma_{32}$ are ignored because they are very small compared with $\gamma_{50}$, $\gamma_{51}$ and $\gamma_{53}$ are also ignored because they are small compared with $\gamma_{50}$ and $\gamma_{52}, \gamma_{20}$ and $\gamma_{4j}$ ($j=0, 1, 2$) are very small and can be disregarded because $\gamma_{21}$ and $\gamma_{43}$ are multiphonon decay [15,16]. Rate equations can be solved by considering the steady state regime where the populations are time independent, $dN_{i}/dt=0$ ($i=0, 1, 2, \ldots, 5$). The average thulium ion concentration in the core $N_{i}$ is calculated by [17]

$$N_{i} = \frac{2}{b^{2}} \int_{0}^{\infty} N(r)r \, dr \tag{8}$$

where $b$ is the doping radius, i.e. the half of the concentration profile FWHM. In general, the variable $N_{i}$ is functions of position r, z and time t. $N(r)$ is the thulium ions concentration profile. $N_{2}$ and $N_{4}$ are very small compared to other $N_{i}$ values. Therefore the total population density $N_{i}$ is expressed as:

$$N_{i} = N_{0} + N_{1} + N_{3} + N_{5} \tag{9}$$

The transition rates, which describe the interaction of the electromagnetic field with the Tm$^{3+}$ ions for a TDFA can be written as [14]:

$$W_{p1} = \frac{p_{p}^{}a_{1}^{}p_{1}^{}a_{1}^{2}}{h_{p}^{2}} \tag{10}$$

$$W_{p2} = \frac{p_{p}^{}a_{2}^{}p_{2}^{}a_{2}^{2}}{h_{p}^{2}} \tag{11}$$
\[ W_{p1} = \frac{P_p \sigma_{p1}^a}{h \nu_p} \]  \hspace{1cm} (12)

\[ W_s = \frac{P_s \sigma_{s}^a}{h \nu_s} \]  \hspace{1cm} (13)

where \( P_p \) is the pump power intensity and \( P_s \) is the signal power intensity. \( \sigma_{p1}^a \), \( \sigma_{s}^a \), and \( \sigma_{p3}^a \) are \( ^{3}H_6 \rightarrow ^{3}H_5 \), \( ^{3}H_4 \rightarrow ^{3}F_2 \), and \( ^{3}F_4 \rightarrow ^{1}G_4 \) stimulation absorption cross sections where the Tm\textsuperscript{3+} ions are excited homogeneously across the fiber cross-section. So,

\[ \gamma_{30} = 1 \Gamma_{3} \]  \hspace{1cm} (14)

\[ \gamma_{10} = 1 \Gamma_{1} \]  \hspace{1cm} (15)

where \( \gamma_{3} \) and \( \gamma_{1} \) are the lifetimes of the \( ^{3}F_4 \) and \( ^{3}H_4 \) levels, respectively. \( b \) is the Planck constant, \( v_p \) is pump light frequency and \( v_s \) is signal light frequency. The light wave propagation equations along the thulium fiber in the z-direction can be recognized as follows \([8]:\)

\[ \frac{dp}{dz} = -\Gamma_{p}(\sigma_{p1} N_0 - \sigma_{p2} N_1 - \sigma_{p3} N_3) P_p - \alpha P_p \]  \hspace{1cm} (16)

\[ \frac{dp}{dz} = \Gamma_{s}(\sigma_{s}^a N_3 - \sigma_{s}^a N_1 - \sigma_{01} N_0) P_s - \alpha P_s \]  \hspace{1cm} (17)

\[ \frac{dP_{ASE}}{dz} = \pm \Gamma_{ASE}(\sigma_{s}^a N_3 - \sigma_{s}^a N_1 - \sigma_{01} N_0) P_{ASE} \pm \Gamma_{ASE} 2h \nu \Delta \nu \sigma_{s}^a N_1 \pm \alpha P_{ASE} \]  \hspace{1cm} (18)

where \( \alpha \) is the background scattering loss which assumed to constant for all wavelength. \( P_{ASE} \) is the amplified spontaneous emission (ASE) at S-band in forward (+) and backward (−) directions a along the fiber. \( \sigma_{s} \) is transition cross-section from background level \( N_0 \) to the first level \( N_1 \) for 1800 nm wavelength. \( \Gamma_{s} P_{ASE} \) is the over-lapping factor between each radiation and the fundamental mode for the signal, the pump, and ASE respectively, \( \Gamma \) can be given by \([15,18]:\)

\[ \Gamma = 1 - e^{-2b^2/w_0^2} \]  \hspace{1cm} (19)

where \( w_0 \) is the model field radius and \( b \) is the thulium ion-dopant radius.

\[ w_0 = a \left( 0.761 + \frac{1.237}{V^{1/5}} + \frac{1.429}{V^{2}} \right) \]  \hspace{1cm} (20)

where \( a \) is the core diameter, \( V \) is the normalized frequency. In Eq. (17) the term \( \sigma_{01} N_0 \) is ignored because the \( \sigma_{01} \) is very small, so Eq. (17) becomes as:

\[ \frac{dp}{dz} = \Gamma (\sigma_{s}^a N_3 - \sigma_{s}^a N_1) P_s - \alpha P_s \]  \hspace{1cm} (21)

The gain (\( G \)) is given by integration Eq. (21) along z-direction from 0 to \( L \):

\[ G = \frac{P_s(L)}{P_s(0)} = \exp[\Gamma s(\sigma_{s}^a N_3 - \sigma_{s}^a N_1) L] - \exp(\alpha L) \]  \hspace{1cm} (22)

where \( L \) is the length of the TDFA. The gain in decibel (dB) can be written as

\[ G(\text{dB}) = 10 \log_{10} \left[ \exp[\Gamma s(\sigma_{s}^a N_3 - \sigma_{s}^a N_1) L] - \exp(\alpha L) \right] \]  \hspace{1cm} (23)

From a practical point of view, the noise figure (NF) characteristic is very important in an optical amplifier's performance. The rate equation analysis predicts a low-noise characteristic in the optical amplification. Therefore, the NF was calculated using fiber by an optical method \([19]:\)

\[ NF = \frac{1}{G} + \frac{p_{ASE}^\text{aux}(\lambda_s)}{G h \nu \Delta \nu} \]  \hspace{1cm} (24)

where \( p_{ASE}^\text{aux}(\lambda_s) \) is the output ASE spectral density (W/Hz) at the signal wavelength. For each signal wavelength, the NF in dB is given by:

\[ NF(\text{dB}) = 10 \times \log_{10} \left[ \frac{1}{G} + \frac{p_{ASE}^\text{aux}(\lambda_s)}{G h \nu \Delta \nu} \right] \]  \hspace{1cm} (25)

where \( G \) is the total gain of the signal, \( \nu \) is the frequency of the signal and \( \Delta \nu \) is the optical bandwidth.

### 4. Result and discussion

The proposed system amplifies a set of 16 channels in the S-band going from 1450 nm to 1525 nm. The parameters used in the simulation are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameters used in the simulation ([12]:)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thulium ion density</td>
<td>1.68e+025 1/m³</td>
</tr>
<tr>
<td>Numerical aperture</td>
<td>0.4</td>
</tr>
<tr>
<td>Fiber length</td>
<td>2.5, 7.5, 10 m</td>
</tr>
<tr>
<td>Core radius</td>
<td>1.3 µm</td>
</tr>
<tr>
<td>Background loss type</td>
<td>Constant</td>
</tr>
<tr>
<td>Nonradiative lifetime 1 ((1/\gamma_{10}^s))</td>
<td>430 × 10(^{-6}) s</td>
</tr>
<tr>
<td>Nonradiative lifetime 2 ((1/\gamma_{20}^s))</td>
<td>45 × 10(^{-6}) s</td>
</tr>
<tr>
<td>Nonradiative lifetime 3 ((1/\gamma_{30}^s))</td>
<td>784 × 10(^{-6}) s</td>
</tr>
<tr>
<td>Radiative transition rate from level 1 to level 0 (\gamma_{10}^s)</td>
<td>29577 s(^{-1})</td>
</tr>
<tr>
<td>Radiative transition rate from level 3 to level 0 (\gamma_{30}^s)</td>
<td>1353.83 s(^{-1})</td>
</tr>
<tr>
<td>Radiative transition rate from level 5 to level 0 (\gamma_{50}^s)</td>
<td>5814.8 s(^{-1})</td>
</tr>
<tr>
<td>Radiative transition rate from level 5 to level 2 (\gamma_{52}^s)</td>
<td>384.84 s(^{-1})</td>
</tr>
</tbody>
</table>

**Fig. 4.** Gain and noise figure (NF) versus pump power at different lengths, \( L = 2.5 \text{ m}, 7.5 \text{ m} \) and 10 m.
Optimization of the length of the thulium-doped fiber (TDF) is one of the most important issues for optical networks that need to be considered for designing a TDFA in order to obtain the best gain with the lowest noise figure. The gain and noise figure of the TDFA are dependent on the TDF length and the operating pump power. The TDF length is selected carefully, when the TDF length is too short, the TDFA will be saturated at a low pump power and this does not provide a high gain. For a short TDF, the total population is very low and therefore the TDF is fully inverted by a low amount of pump power. When this low amount of pump power is used then the optimized TDF length is short. The length of the TDF is optimized by calculating the gain as a function of TDF length for various operating pump powers. The input signal power and wavelength is fixed at $-20 \text{ dBm}$ and $1470 \text{ nm}$, respectively and the pump power is varied from $1000 \text{ mW}$ to $2000 \text{ mW}$.

Three different amplifier lengths are simulated ($2.5 \text{ m}$, $7.5 \text{ m}$, $10 \text{ m}$) and the gain and NF curves are plotted in Fig. 4. It is clear from the results that the gain increases with increasing the pump power for $L = 7.5 \text{ m}$ and $10 \text{ m}$ and it stays almost constant at $L = 2.5 \text{ m}$. However, the best gain is achieved at $L = 7.5 \text{ m}$. For the NF results, it is clear that increasing the pump power and the fiber length has a little impact on the NF.

Although the $10 \text{ m}$ long TDFA design provides the highest gain (16.7 dB) at pump power of $2000 \text{ mW}$. However the use of a high pump power is in conflict with the main objective of the TDFA design which requires a smaller pump power especially for long haul applications. For this reason, a very long TDF is not
recommended to be considered as a reference TDF length during the design of single-pass TDFAs. In the optical network, an amplifier is mainly designed to obtain a gain as high as possible with a low noise figure using a minimum pump power. So the optimum length is 7.5 m with optimum pump power of 1500 mw, where a gain of 15.4 dB and NF of 2.9 dB are achieved.

Fig. 5 shows the gain versus signal wavelength at different lengths 2.5 m, 7.5 m, and 10 m with different pump powers of 1000 mW, 1500 mW and 2000 mW respectively. At 10 m length of TDFAs with pump power 1000 mW, 1500 mW and 2000 mW the maximum value of the gain is found 10.98 dB, 13.64 dB, and 17 dB, respectively. At the pump power 1000 mW, 1500 mW and 2000 mW the gain of TDFAs at length of amplifier 7.5 m is archived and found 11.18 dB, 15.41 dB, and 16.7 dB, respectively. At the length 2.5 m the maximum gain is found 7.87 dB, 7.37 dB, and 6.6 dB, respectively. Fig. 5 shows that the maximum gain is achieved at the center of S-band (1470 nm) channel 12. It is clear from the figure that the gain increases with increasing signal wavelength from 1450 nm to 1470 nm then decreases to the minimum at 1530 nm. Moreover, it is noted from the figure that the maximum gain at the center wavelength (1470 nm) is achieved at length of 7.5 m for different pump powers. However at pump power of 2000 mW the maximum gain at the center wavelength is approximately the same at 7.5 m and 10 m.

Fig. 6 shows the noise figures against input signal wavelength with power –20 dBm at different pump power at different TDFAs lengths. At 2.5 m length the value of NF is considered high with the gain dB and is found at the center of S-band 1470 nm (channel 12), 3.95 dB, 3.51 dB, and 3.12 for 1000 mW, 1500 mW and 2000 mW pump powers, respectively as shown in Fig. 6(a)–(c). It is noted that at the center of S-band with TDFA length of 7.5 m the value of NF is 6.39 dB, 7.6 dB and 7.4 dB for 1000 mW, 1500 mW and 2000 mW pump powers, respectively. While the NF values at 10 m TDFAs length and center S-band 1470 nm channel 12 are 12.9 dB, 10.18 dB and 9.27 dB for 1000 mW, 1500 mW and 2000 mW pump powers, respectively. The NF fluctuates with increasing the signal wavelength as shown in Fig. 6.

The output signal wavelength spectrum is shown in Fig. 7 for different input pump power with different length. The power is increasing form 1450 nm to reach the maximum value at the center of S-band 1470 nm then decreasing to reach the minimum value at channel 16. At the center channel 12, the signal power at 1500 mw and 2000 mw nearly the same for different lengths of TDFAs amplifiers.

5. Conclusion

This paper has described in detail the relation between the operating 1064 nm pump power and TDFAs length for single-pass TDFAs. The simulation results are based on the rate equations to determine the gain and noise figures for TDFAs. The simulated model was also used for optimizing of the TDFAs parameters: fiber length, pump wavelength and pump power. The theoretical results obtained here is in agreement with the published experimental result. It is found that the optimum TDFA length is 7.5 m with optimum pump power of 1500 mW. The results show that silica-based TDFAs amplifiers are interesting comparing to its competitors within the S-band optical amplifiers, namely the fluoride-fiber based TDFAs and the Raman amplifiers.

References

[8] S.D. Emami a nd, S.W. Harun, Optimization of the 1050 nm pump power and fiber length in single-pass and double-pass...


