L2. Sewer Hydraulics

Based on Dr. Fahid Rabah lecture notes
Sewer Hydraulics

Gravity flow: full flow

$S = \frac{HGL}{L} = \text{slope of the sewer}$

$R = \frac{A_f}{P_f} = \frac{D}{4}$

Gravity flow: Partial flow

$S = \frac{HGL}{L} = \text{slope of the sewer}$

$R = \frac{A_p}{P_p}$

Pressure flow: full flow

$S = \frac{h}{L} = \text{slope of the HGL}$

$R = \frac{D}{4}$
Manning equation

Many formulas are used to solve the flow parameters in sewers were discussed in the hydraulic course. The most used formula for sanitary sewers is Manning equation:

\[ V = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \]  
\[ Q = \frac{0.312}{n} D^{\frac{8}{3}} S^{\frac{1}{2}} \]

\( R \) = Hydraulic radius (Area/ wetted parameter)  
\( S \) = slope.  
\( n \) = manning coefficient.  
\( D \) = pipe diameter.  
\( Q \) = flow rate.  

**Note:** Equation 1 is used for calculating the velocity in pipes either flowing full or partially full. Equation 2 same for flow rate.

\[ V_f = \frac{1}{n} R^{\frac{2}{3}} f^{\frac{1}{2}} S^{\frac{1}{2}} \]  
\[ V_p = \frac{1}{n} R^{\frac{2}{3}} p^{\frac{1}{2}} S^{\frac{1}{2}} \]

\( V_f \) = velocity flowing full  
\( V_p \) = velocity flowing partially  

**Note:** Equations 3 and 4 are the same as Equation 1, but they are written using the subscript \((f)\) and \((P)\), to indicate flowing full and partially full, respectively:
In sanitary sewers the flow is not constant; consequently the depth of flow is varying as mentioned above. In this case it is difficult to find the hydraulic radius to apply Manning’s equation. For partially full pipe the following relations are applied:

\[
\frac{d}{D} = \frac{1}{2} \left( 1 - \cos \frac{\theta}{2} \right) \quad \text{....................... (5)}
\]

\[
\frac{A_p}{A_f} = \left( \frac{\theta}{360} - \frac{\sin \frac{\theta}{2 \pi}}{2 \pi} \right) \quad \text{....................... (6)}
\]

\[
\frac{R_p}{R_f} = \left( 1 - \frac{360}{2 \pi} \sin \frac{\theta}{2 \pi} \right) \quad \text{....................... (7)}
\]

\[
\frac{V_p}{V_f} = \left( \frac{R_p}{R_f} \right)^{\frac{2}{3}} \quad \text{....................... (8)}
\]

\[
\frac{Q_p}{Q_f} = \left( \frac{A_p V_p}{A_f V_f} \right) \quad \text{....................... (9)}
\]

\(d\) = partial flow depth.
\(R\) = Hydraulic radius (\(P\) = partial, \(f\) = full)
\(\Theta\) = flow angle in degrees.

Maximum capacity of the pipe when \(d/D = 0.95\)
\(A\) = Flow area.
Maximum velocity in the pipe occurs at \(d/D=0.81\)
Example 1

Find the diameter of the pipe required to carry a design flow of 0.186 m3/s when flowing partially, d/D = 0.67, slope = 0.4%, n = 0.013 use the relations of partial flow.

Solution

1. Find the flow angle: \( \theta \)

\[
\frac{d}{D} = \frac{1}{2} \left( 1 - \cos \left( \frac{\theta}{2} \right) \right) = 0.67
\]

From this relation \( \theta = 219.75^\circ \)

2. Find \( Q_p/Q_f \):

\[
\frac{R_p}{R_f} = \left( 1 - \frac{360}{2 \pi} \sin \left( \frac{\theta}{2} \right) \right) = \left( 1 - \frac{360 \sin 219.75}{2 \pi (219.75)} \right) = 1.167
\]

\[
\frac{A_p}{A_f} = \left( \frac{\theta}{360} - \frac{\sin \left( \frac{\theta}{2} \right)}{2 \pi} \right) = \left( \frac{219.75}{360} - \frac{\sin 219.75}{2 \pi} \right) = 0.712
\]

\[
\frac{V_p}{V_f} = \left( \frac{R_p}{R_f} \right)^{\frac{2}{3}} = (1.167)^{2/3} = 1.1088
\]

\[
\frac{Q_p}{Q_f} = \left( \frac{A_p V_p}{A_f V_f} \right) = 0.712 \times 1.1088 = 0.789
\]
3. Calculate $Q_f$:

$$Q_f = \frac{Q}{P} = \frac{0.186}{0.789} = 0.2355 \text{ m}^3/\text{s}$$

4. Find the diameter of the pipe ($D$):

$$Q_f = \frac{0.312}{n} D^{\frac{8}{3}} S^{\frac{1}{2}} = \frac{0.312}{0.013} D^{\frac{8}{3}} (0.004)^{\frac{1}{2}} = 0.2355$$

$$D = (0.15517)^{\frac{3}{8}} = 0.497 \approx 0.50 \text{ m} \approx 20'' \text{ (design pipe diameter)}$$

5. Find the partial flow velocity ($V_p$):

$$V_f = \frac{1}{n} R_f^{\frac{2}{3}} S^{\frac{1}{2}} = V_f = \frac{1}{0.013} \left( \frac{0.497}{4} \right)^{\frac{2}{3}} (0.004)^{\frac{1}{2}} = 1.203 \text{ m/s}$$

$$A_f = \frac{Q_f}{V_f} = \frac{0.2355}{1.203} = 0.196 \text{ m}^2 \text{ (or } A_f = \frac{\pi D^2}{4} \text{ )}$$

$$A_p = A_f \times 0.712 = 0.196 \times 0.712 = 0.139 \text{ m}^2$$

$$V_p = V_f \times 1.109 = 1.203 \times 1.109 = 1.33 \text{ m/s}$$

It is noticed that it is quite long procedure to go through the above calculations for each pipe in the system of large numbers of pipes. The alternative procedure is to use the nomographs of Mannings equation and the partial flow curves.
Example 2
Find the diameter of the pipe required to carry a design flow of 0.186 m$^3$/s when flowing partially, $d/D = 0.67$, slope = 0.4%, $n = 0.013$ using the nomographs and partial flow curves.

Solution

From partial flow curves:

- Start from the Y-axis with $\frac{d}{D} = 0.67$, and draw a horizontal line until you intersect the Q curve (for $n =$ constant, the dashed line), then draw a vertical line to intersect the X-axis at $\frac{Q_P}{Q_f} = 0.78$.

- Extend the horizontal line until it intersects the velocity curve, then draw a vertical line to intersect the X-axis (for $n =$ constant, the dashed line) at $\frac{V_P}{V_f} = 1.12$.

- Calculate $Q_f$: $Q_f = \frac{Q_P}{0.78} = \frac{0.186}{0.78} = 0.238$ m$^3$/s
Use the nomographs for pipes flowing full to find D:

- Locate the slope (0.004) on the “S” axis.
- Locate the manning coefficient “n” (0.013) on the “n” axis.
- Draw a line connecting “S” and “n” and extended it until it intersects the Turning Line.
- Locate the $Q_f$ (0.238 m$^3$/s) on the “Q” axis.
- Draw a line connecting “$Q_f$” and the point of intersection on the Turning Line and find the diameter “D” by reading the value that this line intersect the D axis at. $D = 500$ mm = 20”.
- find the Velocity “$V_f$” by reading the value that this line intersect the V axis at. $V_f = 1.2$ m/s.
- Calculate $V_p = V_f * 1.12 = 1.2*1.12 = 1.34$ m/s.
Partial Flow Curves

Discharge vs. Water Depth

Velocity vs. Water Depth

$Q_p/Q_f$

$V_p/V_f$

$d/D$

$d$
Partial Flow Curves Q & V

\[ \frac{Q}{Q_{\text{full}}} \quad \frac{v}{v_{\text{full}}} \]
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