Design of Laterally Restrained Beams
3.1 **Beam Design Limit States:**

1. Bending strength at ULS, including
   a) Local buckling of flange or web.
   b) LTB if the beam is not fully restrained.
   c) Plastic moment capacity.

2. Shear strength at ULS.

3. Resistance to transverse forces at ULS (web bearing and web buckling).

4. Deflection SLS.

3.2 **Laterally Restrained Beams**

3.2.1 **Bending Resistance:**

Laterally restrained beams are not subject to LTB, so their bending strength is determined from the properties of the cross-section and the strength of the steel.

3.2.2 **Uniaxial Bending:**

\[ M_{Ed} \leq M_{c,Rd} \]

- For class 1 or 2 cross-sections: \[ M_{c,Rd} = M_{pI,Rd} = W_{pl} f_y / Y_{M0} \]
- For class 3 cross-sections: \[ M_{c,Rd} = M_{el,Rd} = W_{el,min} f_y / Y_{M0} \]
- For class 4 cross-sections: \[ M_{c,Rd} = W_{eff,min} f_y / Y_{M0} \]

Where: \( W_{pl} \) is the plastic section modulus.

\( W_{el,min} \) is the minimum elastic section modulus.

\( W_{eff,min} \) is the effective minimum elastic section modulus.

3.2.3 **Bi-axial Bending:**

- For class 1 or 2 cross-sections:
  \[
  \left[ \frac{M_{y,Ed}}{M_{pl,y,Rd}} \right]^\alpha + \left[ \frac{M_{z,Ed}}{M_{pl,z,Rd}} \right]^\beta \leq 1.0
  \]

Where:

**I or H Sections:** \( \alpha = 2 \) & \( \beta = 1 \)

**CHS Sections:** \( \alpha = \beta = 2 \)

**RHS Sections:** \( \alpha = \beta = 1.66 \)
3.2.4 Shear Resistance: \( V_{Ed} \leq V_{c,Rd} \)

For plastic design

\[ V_{c,Rd} = V_{PL,Rd} = \frac{A_v (f_y / \sqrt{3})}{\gamma M_0} \]

The shear area \( A_v \) may be taken as follows:

- a) rolled I and H sections, load parallel to web
  \[ A - 2bt + (t_w + 2r) t_r \text{ but not less than } \eta h_w t_w \]
- b) rolled channel sections, load parallel to web
  \[ A - 2bt + (t_w - r) t_r \]
- c) rolled T-section, load parallel to web
  \[ 0.9 (A - bt_r) \]
- d) welded I, H and box sections, load parallel to web
  \[ \eta \sum (h_w t_w) \]
- e) welded I, H, channel and box sections, load parallel to flanges
  \[ A - \sum (h_w t_w) \]
- f) rolled rectangular hollow sections of uniform thickness:
  - load parallel to depth
    \[ Ah/(b+h) \]
  - load parallel to width
    \[ Ab/(b+h) \]
- g) circular hollow sections and tubes of uniform thickness
  \[ 2A/\pi \]

where \( A \) is the crosssectional area;
\( b \) is the overall breadth;
\( h \) is the overall depth;
\( h_w \) is the depth of the web;
\( r \) is the root radius;
\( t_r \) is the flange thickness;
\( t_w \) is the web thickness (If the web thickness in not constant, \( t_w \) should be taken as the minimum thickness.).
\( \eta \) see EN 1993-1-5.

NOTE \( \eta \) may be conservatively taken equal 1.0.

3.2.5 Bending and Shear Interaction:

\( V_{Ed} < 50\% V_{PL,Rd} \rightarrow No Reduction \)

\( V_{Ed} \geq 50\% V_{PL,Rd} \rightarrow Reduction Moment \)

In I or H sections with equal flanges, under major axis bending, the reduced design plastic moment resistance \( M_{y,V,Rd} \) may be obtained from:

\[ M_{y,V,Rd} = \left( W_{pl,y} - \frac{\rho A_w^2}{4 t_w} \right) \frac{f_y}{\gamma M_0}, \text{ but } M_{y,V,Rd} \leq M_{y,c,Rd} \]

Where:
\[ \rho = \left( 2V_{Ed}/V_{PL,Rd} - 1 \right)^2 \]
\[ A_w = h_w \times t_w \]
3.2.6 Deflection Resistance:

Deflections of flexural members must be limited to avoid damage to finishes, ceilings and partitions, and should be calculated under SLS loads.

EC3 states that limits for vertical deflections should be specified for each project and agreed with the client. The UK National Annex to EC3 suggests:

### NA.2.23 Vertical deflections [BS EN 1993-1-1:2005, 7.2.1(1)B]

The following table gives suggested limits for calculated vertical deflections of certain members under the characteristic load combination due to variable loads and should not include permanent loads. Circumstances may arise where greater or lesser values would be more appropriate. Other members may also need deflection limits.

<table>
<thead>
<tr>
<th>Vertical deflection</th>
<th>Cantilevers</th>
<th>Beams carrying plaster or other brittle finish</th>
<th>Other beams (except purlins and sheeting rails)</th>
<th>Purlins and sheeting rails</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Length/180</td>
<td>Span/360</td>
<td>Span/200</td>
<td>To suit the characteristics of particular cladding</td>
</tr>
</tbody>
</table>

- Standard rules for maximum deflection:

### BEAM BENDING

\[
L = \text{overall length} \\
W = \text{point load} \\
M = \text{moment} \\
w = \text{load per unit length}
\]

<table>
<thead>
<tr>
<th>End Slope</th>
<th>Max Deflection</th>
<th>Max bending moment</th>
</tr>
</thead>
</table>
| \[
\frac{ML}{EI}
\] | \[
\frac{ML^2}{2EI}
\] | \[
M
\] |
| \[
\frac{WL^2}{2EI}
\] | \[
\frac{WL^3}{3EI}
\] | \[
WL
\] |
| \[
\frac{wL^2}{EI}
\] | \[
\frac{wL^4}{8EI}
\] | \[
\frac{wL^2}{2}
\] |

\[
\theta_b = \frac{Wac^2}{2LEI} \\
\theta_l = \frac{Wc^3}{3LEI} \\
\theta_a = \frac{Wab}{L}
\]

(at position \(c\)) (under load)
Note:

Steel beam with web stiffeners:
3.3 Solved Problems

Problem (1)

Verify a S275 IPE300 (Parallel Flange I Section) simply supported beam in accordance with EC3, which is subjected to a uniform dead load of 5.0 kN/m and a uniform live load of 10.0 kN/m. The beam supports plastered partition walls. Lateral displacement and torsion are prevented at both ends.

Solution:

- Combination of actions at ULS:
  \[ F_{Ed} = \gamma_G \cdot G_K + \gamma_Q \cdot Q_K \rightarrow 1 \]
  \[ F_{Ed} = 1.35 \times 5 + 1.5 \times 10 = 21.75 \text{ KN/m} \]
  \[ F_{Ed} = \gamma_G \cdot G_K + \zeta \cdot \gamma_Q \cdot Q_K \rightarrow 2 \]
  \[ F_{Ed} = 1.35 \times 5 + 0.925 \times 1.5 \times 10 = 20.625 \text{ KN/m} \]
  \[ F_{Ed} = \text{max.} (21.75, 20.625) = 21.75 \text{ KN/m} \]

- Shear force and Bending Moment Diagrams:
  \[ M_{Ed} = 67.97 \text{ KN.m} \text{ and } V_{Ed} = 54.38 \text{ KN} \]

- Geometrical properties:
  \[ h = 300 \text{ mm}; b_f = 150 \text{ mm}; t_f = 10.7 \text{ mm}; t_w = 7.1 \text{ mm}; A = 5380 \text{ mm}^2 \]
  \[ r = 15 \text{ mm}; W_{P_L,Y} = 628.4 \text{ cm}^3; W_{eL,Y} = 557.1 \text{ cm}^3; I_y = 8356 \text{ mm}^4 \]

- X-Section Classification:
  For flange:
  \[ \left( \frac{c}{t} \right) = \frac{150 - 7.1 - 2 \times 15}{2 \times 10.7} = 5.30 \]
  \[ 9\epsilon = 9 \times 0.92 = 8.28 \]
  \[ \rightarrow 5.3 < 8.28 \rightarrow \text{ class 1} \]

  For web:
  \[ \left( \frac{c}{t} \right) = \frac{300 - 2 \times 10.7 - 2 \times 15}{7.1} = 35.00 \]
  \[ 72\epsilon = 72 \times 0.92 = 66.24 \]
  \[ \rightarrow 35 < 66.24 \rightarrow \text{ class 1} \]
  \[ \rightarrow \text{ the x-section is class 1(塑料)} \]
- Flexural Strength Verification:
  \[ M_{Ed} = 67.97 \text{ KN.m} \]
  \[ M_{c,Rd} = M_{PL,Rd} = \frac{W_{pl} f_y}{\gamma_{M0}} = \frac{628.4 \times 10^3 \times 275}{1.0} \times 10^{-6} = 172.81 \text{ KN.m} \]
  \[ > 67.97 \text{ KN.m} \rightarrow O.K \]

- Shear Strength Verification:
  \[ V_{Ed} = 54.38 \text{ KN} \]
  Shear area:
  \[ A_v = A - 2bt_f + (t_w + 2r)t_f \geq \eta h_w t_w \]
  \[ = 5380 - 2 \times 150 \times 10.7 + (7.1 + 2 \times 15) \times 10.7 = 2567 \text{ mm}^2 \]
  \[ > 1.0 \times (300 - 2 \times 10.7) \times 7.1 = 278.6 \text{ mm}^2 \]
  \[ V_{PL,Rd} = \frac{A_v (f_y / \sqrt{3})}{\gamma_{M0}} = \frac{2567 \times 275}{\sqrt{3} \times 1.0} \times 10^{-3} = 407.57 \text{ KN} > 54.375 \text{ KN} \]
  \[ V_{Ed} = 54.375 \text{ KN} < 50\% V_{PL,Rd} = 203.78 \text{ KN} \]
  \[ \rightarrow \text{No influence on design resistance for bending} \]

- Deflection Verification:
  SLS unfactored vertical action:

  \[ \delta_{max,mid\,span} = \frac{5}{384} \times \frac{q_k l^4}{E I_y} = \frac{5}{384} \times \frac{10 \times 5000^4}{210000 \times 8356 \times 10^4} = 4.64 \text{ mm} \]
  Vertical deflection limit:
  \[ \frac{5000}{360} = 13.89 \text{ mm} \]
  \[ \rightarrow 4.64 \text{ mm} < 13.89 \text{ mm} \rightarrow O.K \]
Problem (2)

Select the lightest suitable section in S275 steel required for a simply supported steel beam loaded as shown below. Assume that the beam is fully laterally restrained. Choose a suitable UB section.

Solution:
- Combination of actions at ULS:
  \[ F_{Ed(UDL)} = \gamma_G G_K + \gamma_Q Q_K \]
  \[ F_{Ed(UDL)} = 1.35 \times 9 + 1.5 \times 7.5 = 23.4 \text{ KN/m} \]
  \[ F_{Ed(PL)} = \gamma_G G_K + \gamma_Q Q_K \]
  \[ F_{Ed(PL)} = 1.35 \times 10 + 1.5 \times 15 = 36 \text{ KN} \]

- Design Shear forces and Bending moments:
  \[ V_{Ed} = \frac{F_{Ed(UDL)}L}{2} + \frac{F_{Ed(PL)}L}{2} = \frac{23.4 \times 8}{2} + \frac{36}{2} = 111.6 \text{ KN} \]
  \[ M_{Ed} = \frac{F_{Ed(UDL)}L^2}{8} + \frac{F_{Ed(PL)}L}{4} = \frac{23.4 \times 8^2}{8} + \frac{36 \times 8}{4} = 259.2 \text{ KN.m} \]

- Choose a suitable section size:
  Assume class 1 x-section, calculate the minimum plastic modulus \( W_{PL,y} \) about y-y axis.
  \[ W_{PL,y} = \frac{M_{Ed}Y_{MO}}{f_y} = \frac{259.2 \times 10^6 \times 1.0}{275} \times 10^{-3} = 942.55 \text{ cm}^3 \]
  From blue book try UB356x171x57 where \( W_{PL,y} = 1010 \text{ cm}^3 \).

- Geometrical properties:
  \[ h = 358 \text{ mm}; b_f = 172.2 \text{ mm}; t_f = 13 \text{ mm}; t_w = 8.1 \text{ mm}; A = 72.6 \text{ mm}^2 \]
  \[ r = 10.2 \text{ mm}; W_{PL,y} = 1010 \text{ cm}^3; W_{el,y} = 896 \text{ cm}^3; I_y = 16000 \text{ cm}^4 \]
• X-Section Classification:
  
  **For flange:**
  \[
  \left( \frac{c}{t} \right) = 5.33 \\
  9\varepsilon = 9 \times 0.92 = 8.28 \\
  \rightarrow 5.33 < 8.28 \rightarrow class \ 1 
  \]
  
  **For web:**
  \[
  \left( \frac{c}{t} \right) = 38.50 \\
  72\varepsilon = 72 \times 0.92 = 66.24 \\
  \rightarrow 38.50 < 66.24 \rightarrow class \ 1 
  \]
  
  \[\rightarrow the \ x - section \ is \ class \ 1(Plastic)\]

• Flexural Strength Verification:

\[M_{Ed} = 259.2 \ K N.m\]

\[M_{c,Rd} = M_{PLRd} = \frac{W_{Pl} f_y}{Y_{M0}} = \frac{1010 \times 10^3 \times 275}{1.0} \times 10^{-6} = 277.75 \ K N.m\]

\[> 259.2 \ K N.m \rightarrow 0.K\]

• Shear Strength Verification:

\[V_{Ed} = 111.6 \ K N\]

Shear area:

\[A_v = A - 2bt_f + (t_w + 2\tau)t_f \geq \eta h_w t_w\]

\[= 7260 - 2 \times 172.2 \times 13 + (8.1 + 2 \times 10.2) \times 13 = 3153.3 \ mm^2\]

\[> 1.0 \times (358 - 2 \times 13) \times 8.1 = 2689.2 \ mm^2\]

\[V_{PL,Rd} = \frac{A_v (f_y)}{\sqrt{3}} = \frac{3153.3 \times 275}{\sqrt{3} \times 1.0} \times 10^{-3} = 500.65 \ K N > 111.6 \ K N\]

\[V_{Ed} = 111.6 \ K N < 50\% V_{PL,Rd} = 250.33 \ K N\]

\[\rightarrow No \ influence \ on \ design \ resistance \ for \ bending\]

Shear buckling:

Shear buckling of the unstiffened web need not be considered provided:

\[\frac{h_w}{t_w} \leq \frac{\varepsilon}{\eta}\]

\[h_w = 332 \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ t_w = 8.1 = 40.99\]

\[72\varepsilon = 72 \times 0.92 = 66.24\]

\[40.99 < 66.24 \rightarrow Therefore, \ shear \ buckling \ check \ need \ not \ be \ considered\]
Deflection Verification:
SLS unfactored vertical action:

\[
\delta_{\text{max, mid span}} = \frac{5}{384} \times \frac{q_k l^4}{EI_y} + \frac{Q_k l^3}{48EI_y}
\]

\[
= \frac{5}{384} \times \frac{7.5 \times 8000^4}{210000 \times 16000 \times 10^4} + \frac{15 \times 10^3 \times 8000^3}{48 \times 210000 \times 16000 \times 10^4}
\]

\[
= 16.67 \text{mm}
\]

Vertical deflection limit:

\[
\frac{\text{span}}{360} = \frac{8000}{360} = 22.22 \text{mm}
\]

\[
\rightarrow 16.67 \text{mm} < 22.22 \text{mm} \rightarrow O.K
\]

*Adopt 356x171x57 UKB in S275 steel.*