CHAPTER 11

EELE 3332 – Electromagnetic II
Chapter 11
Transmission Lines

Prof. Hala J. EL-Khozondar
Islamic University of Gaza
Electrical Engineering Department

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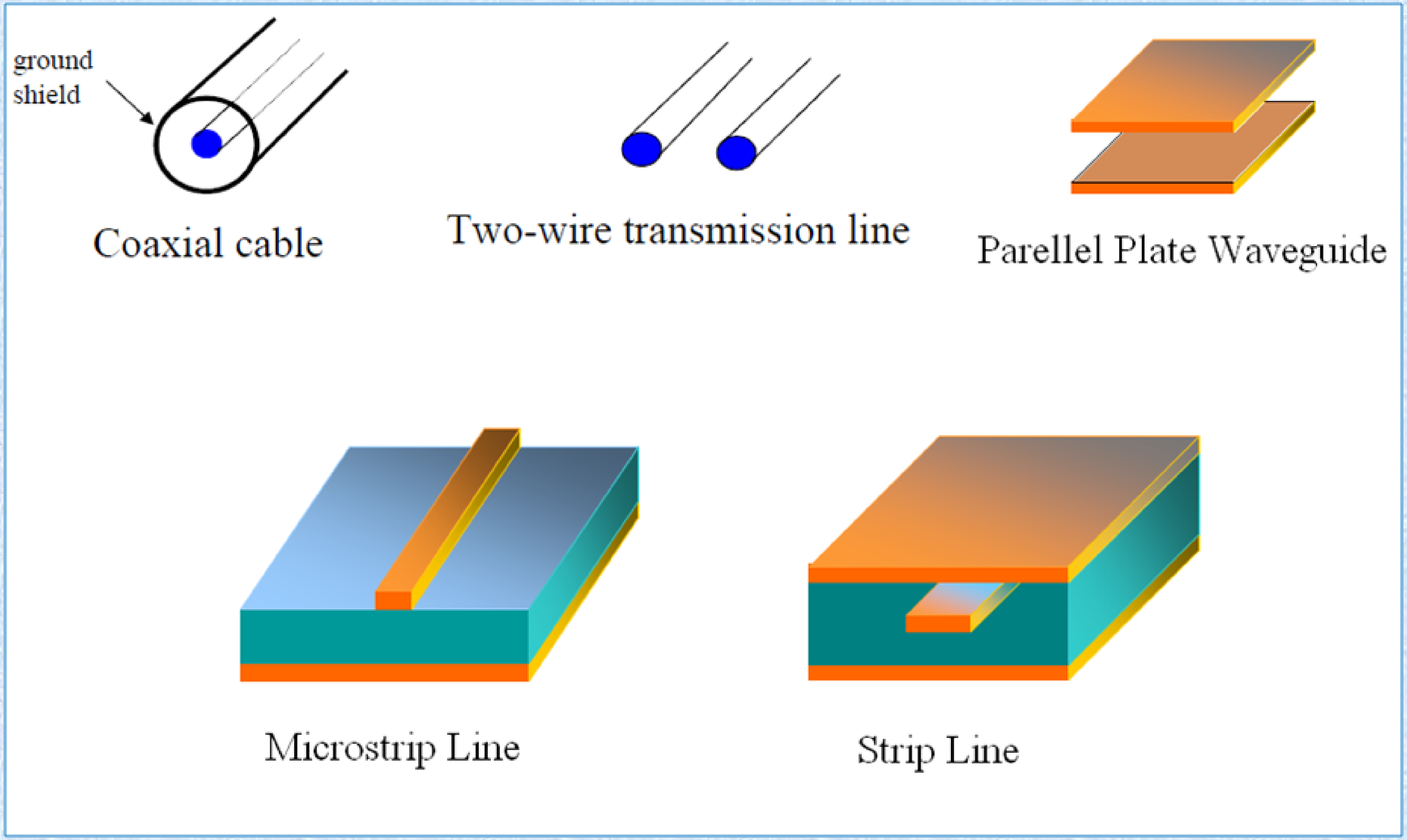
11.1 Introduction

- Wave propagation in unbounded media is used in radio or TV broadcasting, where the information being transmitted is meant for everyone who may be interested.
- Another means of transmitting power or information is by guided structures. Guided structures serve to guide (or direct) the propagation of energy from the source to the load.
- Typical examples of such structures are ***transmission lines*** and ***waveguides***.
- Waveguides are discussed in the next chapter; transmission lines are considered in this chapter.

Introduction

- Transmission lines are commonly used in power distribution (at low frequencies) and in communications (at high frequencies).
- A transmission line basically consists of two or more parallel conductors used to connect a source to a load.
- Typical transmission lines include **coaxial cable**, **a two-wire line, a parallel-plate line**, and **a microstrip line**.
- Coaxial cables are used in connecting TV sets to TV antennas. Microstrip lines are particularly important in integrated circuits.
- Transmission line problems are usually solved using EM field theory and electric circuit theory, the two major theories on which electrical engineering is based.

Introduction



11.2 Transmission Line Parameters

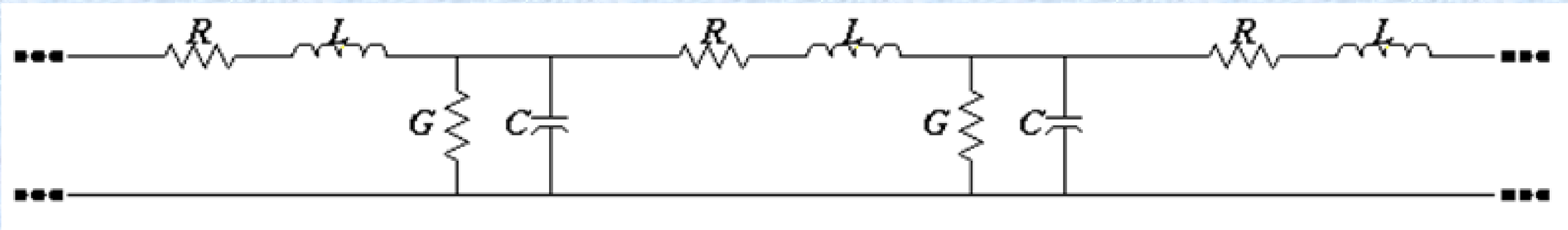
Transmission line parameters are:

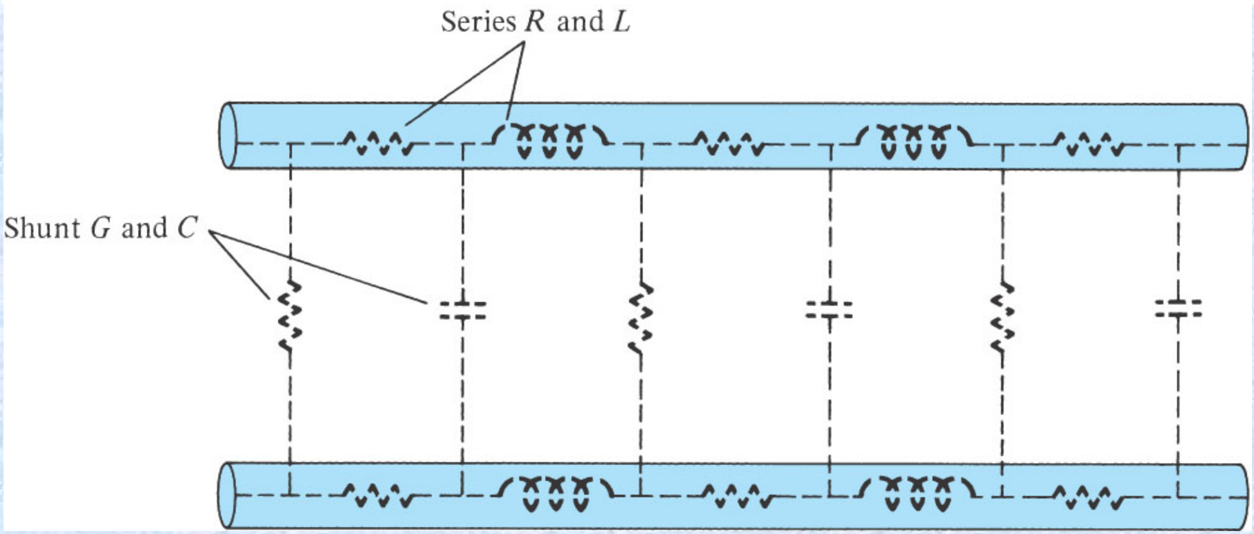
R: Resistance per unit length. (Ω/m)

L: Inductance per unit length. (H/m)

G: Conductance per unit length. (S/m)

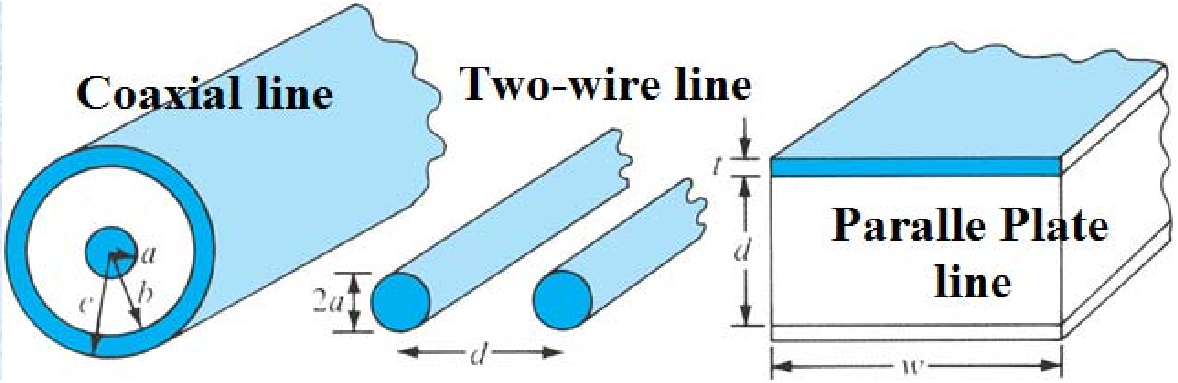
C: Capacitance per unit length. (F/m)





Distributed parameters of a two-conductor transmission line

Transmission Line Parameters



Parameters	Coaxial Line	Two-Wire Line	Planar Line
R (Ω/m)	$\frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}+\frac{1}{b}\right]$	$\frac{1}{\pi a\delta\sigma_c}$	$\frac{2}{w\delta\sigma_c}$
L (H/m)	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	$\frac{\mu}{\pi}\cosh^{-1}\frac{d}{2a}$	$\frac{\mu d}{w}$
G (S/m)	$\frac{2\pi\sigma}{\ln\frac{b}{a}}$	$\frac{\pi\sigma}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\sigma w}{d}$
C (F/m)	$\frac{2\pi\varepsilon}{\ln\frac{b}{a}}$	$\frac{\pi\varepsilon}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\varepsilon w}{d}$ $(w \gg d)$

$*\delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}}$ = skin depth of the conductor; $\cosh^{-1}\frac{d}{2a} \simeq \ln\frac{d}{a}$ if $\left[\frac{d}{2a}\right]^2 \gg 1$.

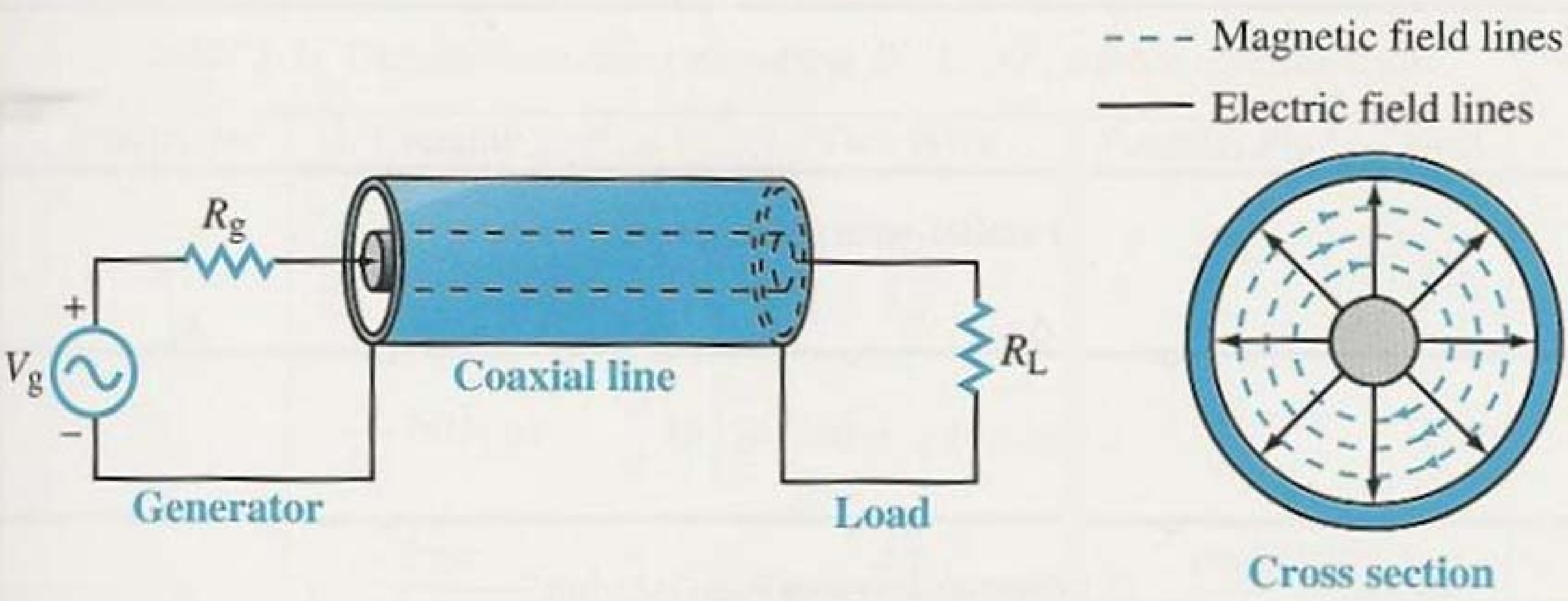
Transmission Line Parameters

- The line parameters ***R, L, G***, and ***C*** are uniformly distributed along the entire length of the line.
- For each line, the conductors are characterized by $\sigma_c, \mu_c, \varepsilon_c = \varepsilon_0$, and the homogeneous dielectric separating the conductors is characterized by σ, μ, ε.
- $G \neq 1/R$; ***R*** is the ac resistance per unit length of the conductors comprising the line and ***G*** is the conductance per unit length due to the dielectric medium separating the conductors.
- For each line:

$$LC = \mu\varepsilon \qquad \text{and} \qquad \frac{G}{C} = \frac{\sigma}{\varepsilon}$$

Fields inside transmission line

- Transmission lines transmit TEM waves.
- *V* proportional to *E*,
- *I* proportional to *H*



In a coaxial line, the electric field lines are in the radial direction between the inner and outer conductors, and the magnetic field forms circles around the inner conductor.

11.3 Transmission Line Equations

➢Two-conductor transmission lines support a **TEM** wave; E and H are perpendicular to each other and transverse to the direction of propagation.

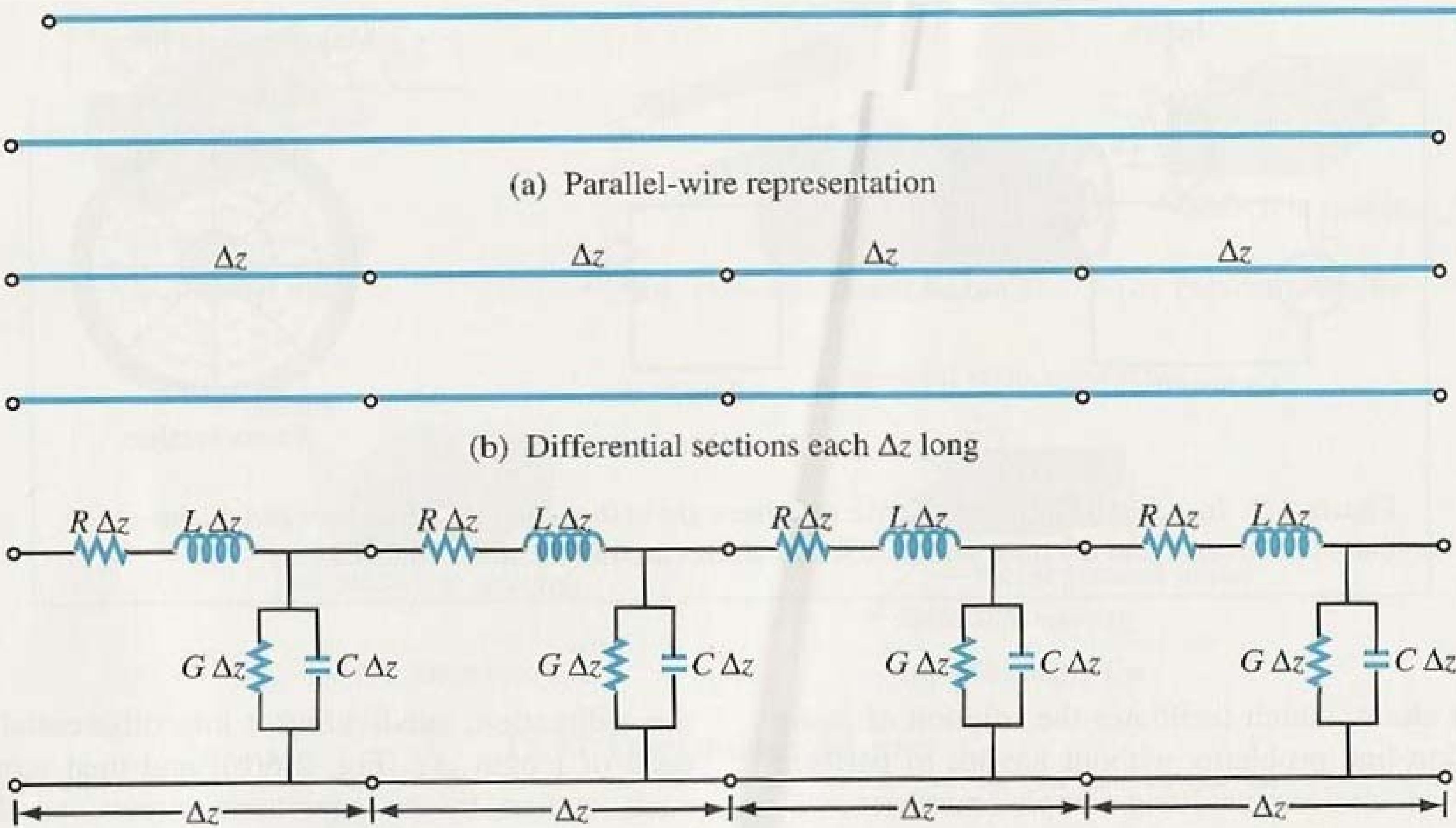
➢E and H are related to V and I:

$$V = -\int \mathrm{E}.dl, \qquad \mathrm{I} = \oint \mathrm{H}.dl$$

➢Using ***V*** and ***I*** in solving the transmission line problem is simpler than solving ***E*** and ***H*** (requires Maxwell's equations).

➢Examine an incremental portion of length Δz of a two-conductor transmission line.

Transmission Line Representation



(a) Parallel-wire representation

(b) Differential sections each Δz long

(c) Each section is represented by an equivalent circuit

Regardless of its actual shape, a TEM transmission line is represented by the parallel-wire configuration shown in (a). To analyze the voltage and current relations, the line is subdivided into small differential sections (b), each of which is then represented by an equivalent circuit (c).

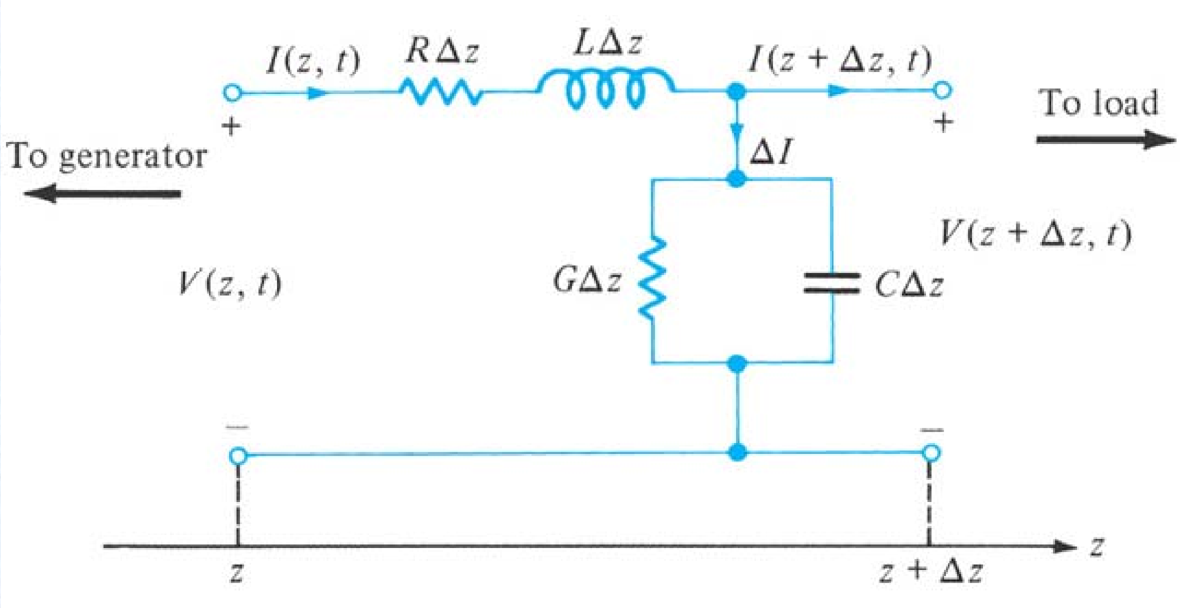
Transmission Line Equations

Using KVL:- $V(z,t) = R\Delta z I(z,t) + L\Delta z \frac{\partial I(z,t)}{\partial t} + V(z+\Delta z,t)$

or $$-\frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

Taking the limit as $\Delta z \to 0$ leads to:

$$-\frac{\partial V(z,t)}{\partial z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t}$$



equivalent circuit model of a two-conductor T.L. of differential length Δz.

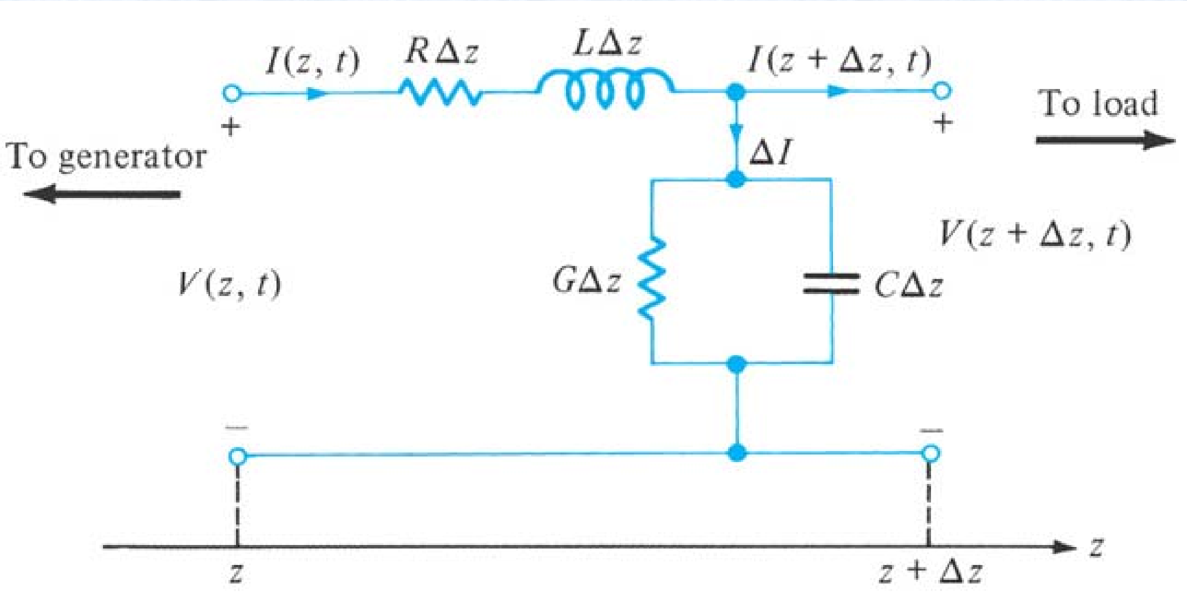
Using KCL:- $I(z,t) = I(z+\Delta z,t) + \Delta I$

$$I(z,t) = I(z+\Delta z,t) + G\Delta z V(z+\Delta z,t) + C\Delta z \frac{\partial V(z+\Delta z,t)}{\partial t}$$

or $$-\frac{I(z+\Delta z,t) - I(z,t)}{\Delta z} = GV(z+\Delta z,t) + C\frac{\partial V(z+\Delta z,t)}{\partial t}$$

Taking the limit as $\Delta z \to 0$ leads to:

$$-\frac{\partial I(z,t)}{\partial z} = GV(z,t) + C\frac{\partial V(z,t)}{\partial t}$$



Transmission Line Equations

The time domain form of the transmission line equations:

$$-\frac{\partial V(z,t)}{\partial z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

$$-\frac{\partial I(z,t)}{\partial z} = GV(z,t) + C\frac{\partial V(z,t)}{\partial t}$$

If we assume harmonic time dependence so that:

$V(z,t) = \mathrm{Re}[V_s(z)e^{j\omega t}]$

$I(z,t) = \mathrm{Re}[I_s(z)e^{j\omega t}]$

where V_s and I_s are the phasor forms of $V(z,t)$ and $I(z,t)$,

$$-\frac{dV_s}{dz} = (R + j\omega L)I_s$$

$$-\frac{dI_s}{dz} = (G + j\omega C)V_s$$

Transmission Line Equations

$$-\frac{dV_s}{dz} = (R + j\omega L)I_s \quad , \quad -\frac{dI_s}{dz} = (G + j\omega C)V_s$$

To solve the previous equations, take second derivative of V_s gives

$$\frac{d^2V_s}{dz^2} = (R + j\omega L)(G + j\omega C)V_s$$

Now take second derivative of I_s gives

$$\frac{d^2I_s}{dz^2} = (G + j\omega C)(R + j\omega L)I_s$$

Hence, the **wave equations** for voltage and current become

$$\frac{d^2V_s}{dz^2} - \gamma^2 V_s = 0$$

$$\frac{d^2I_s}{dz^2} - \gamma^2 I_s = 0$$

, where $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

γ : is the propagation constant

α : attenuation constant (Np/m or dB/m)

β : phase constant (rad/m)

wavelength is: $\lambda = \dfrac{2\pi}{\beta}$, wave velocity is: $u = \dfrac{\omega}{\beta} = f\lambda$

The solutions to the wave equations are:

$$V_s = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad , \quad I_s = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$\longrightarrow$ +z -z $\longleftarrow$ $\longrightarrow$ +z -z $\longleftarrow$

where $V_0^+, V_0^-, I_0^+, I_0^-$ are wave amplitudes.

+ *sign* $\rightarrow$ wave traveling along +z direction.

- sign $\rightarrow$ wave traveling along -z direction.

Transmission Line Equations

$$V_s = V_0^+ e^{-\gamma z} \quad + \quad V_0^- e^{\gamma z}$$

$$I_s = I_0^+ e^{-\gamma z} \quad + \quad I_0^- e^{\gamma z}$$

In time domain:

$$V(z,t) = \mathrm{Re}[V_s(z) e^{j\omega t}]$$

$$V(z,t) = V_0^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_0^- e^{+\alpha z} \cos(\omega t + \beta z)$$

Similarly for current:

$$I(z,t) = I_0^+ e^{-\alpha z} \cos(\omega t - \beta z) + I_0^- e^{\alpha z} \cos(\omega t + \beta z)$$

Characteristic Impedance, Z_0

The Characteristic Impedance Z_0 of the line is the ratio of the positively travelling voltage wave to the current wave at any point on the line.

$$V(z) = V_0^+ e^{-\gamma z}, \quad I(z) = I_0^+ e^{-\gamma z}$$

$$\text{since} \quad -\frac{dV(z)}{dz} = (R + j\omega L)I(z),$$

$$\rightarrow -(-\gamma V_0^+ e^{-\gamma z}) = (R + j\omega L)I_0^+ e^{-\gamma z}$$

$$Z_o = \frac{V_0^+}{I_0^+} = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_o + jX_o = -\frac{V_0^-}{I_0^-}$$

$$\boxed{Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_o + jX_o},$$

Characterestic Admittance $\boxed{Y_0 = \frac{1}{Z_0}}$

Lossless Line (R=G=0)

A transmission line is said to be **lossless** if the conductors of the line are perfect **($\sigma_c \approx \infty$)** and the dielectric medium separating them is lossless **($\sigma \approx 0$)**

$$LC = \mu\varepsilon \quad \text{and} \quad \frac{G}{C} = \frac{\sigma}{\varepsilon}$$

Since $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$

$\rightarrow$

$$\alpha = 0, \quad \gamma = j\beta, \quad \beta = \omega\sqrt{LC}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda, \qquad \lambda = \frac{2\pi}{\beta}$$

$$\boxed{Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_o + jX_o}$$

$$X_o = 0 \qquad Z_o = R_o = \sqrt{\frac{L}{\ }}$$

Distortionless Line

➢Any signal that carries significant information must has some non-zero bandwidth. In other words, the signal energy (as well as the information it carries) is spread across many frequencies.

➢If the different frequencies that comprise a signal travel at different velocities, that signal will arrive at the end of a transmission line ***distorted***. We call this phenomenon signal ***dispersion***.

➢Recall for lossless lines, however, the phase velocity is independent of frequency—no dispersion will occur! $u = 1/\sqrt{LC}$

➢Of course, a perfectly lossless line is impossible, but we find phase velocity is approximately constant if the line is low-loss.

Distortionless Line (R/L=G/C)

A **distortionless line** is one in which the attenuation constant **α** is **frequency independent** while the phase constant **β** is **linearly dependent on frequency**.

A distorionless line results if the line parameters are such that

$$\boxed{\frac{R}{L} = \frac{G}{C}}$$

$$Thus, \quad \gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + \frac{j\omega C}{G}\right)}$$

$$\gamma = \sqrt{RG}\left(1 + \frac{j\omega C}{G}\right) = \alpha + j\beta$$

$$or \quad \alpha = \sqrt{RG}, \quad \beta = \omega\sqrt{LC}$$

$$u = \omega / \beta = 1/\sqrt{LC} \text{ (frequency independent)}$$

α does not depend on frequency, whereas **β** is a linear function of frequency.

Distortionless Line (R/L=G/C)

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{R\left(1 + j\omega L / R\right)}{G\left(1 + j\omega C / G\right)}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} \qquad \text{(Real)}$$

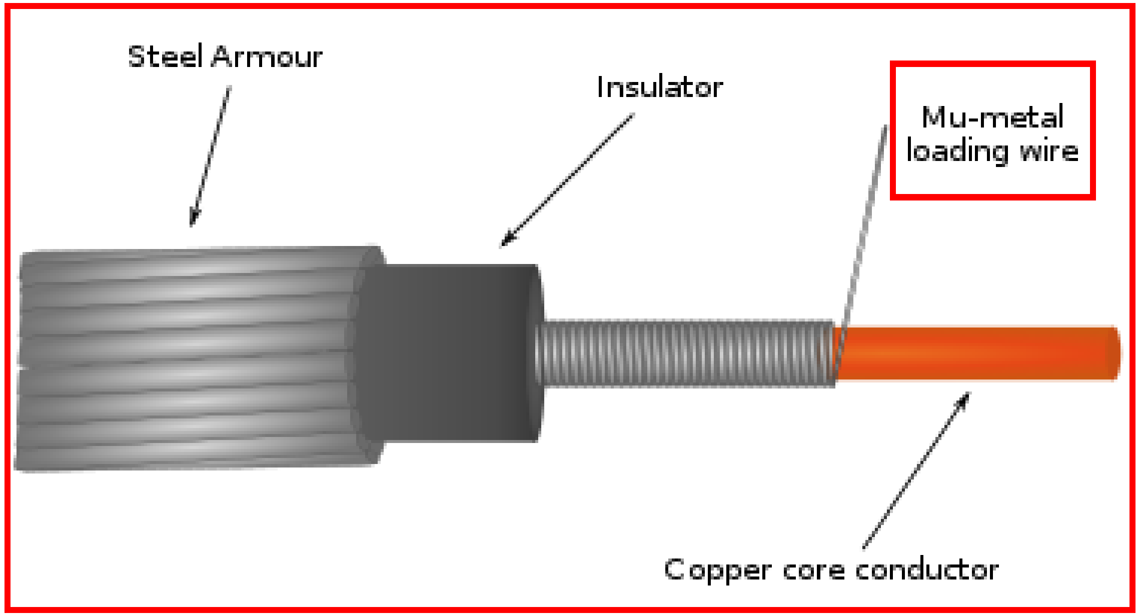
$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

Notes:

- Shape distortion of signals happen if α and u are frequency dependent.
- u and Z_0 for distortionless line are the same as lossless line.
- A lossless line is also a distortionless line, but a distortionless line is not necessarily lossless.
- Lossless lines are desirable in power transmission, and telephone lines are required to be distortionless.

Distortionless Line – Practical use

➢To achieve the required condition of R/L=G/C for a transmission line, L may be increased by loading the cable with a metal with high magnetic permeability (μ).



➢A common practice is to replace repeaters in long lines to maintain the desired shape and duration of pulses for long distance transmission.

Summary

	γ	Z_o
General	$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$	$Z_o = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$
Lossless	$\gamma = 0 + j\omega\sqrt{LC}$	$Z_o = R_o = \sqrt{\dfrac{L}{C}}$
Distortionless	$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$	$Z_o = R_o = \sqrt{\dfrac{L}{C}}$

Example 11.1

An air line has characteristic impedance of 70 Ω and a phase constant of 3 rad/m at 100 MHz. Calculate the inductance per meter and the capacitance per meter of the line.

An air line can be regarded as lossless line because $\sigma \simeq 0$

and $\sigma_c \to \infty$. *Hence* $R = G = 0$ and $\alpha = 0$

$$Z_0 = R_0 = \sqrt{\frac{L}{C}}$$

$$\beta = \omega\sqrt{LC}$$

Deviding the two equations yields: $\dfrac{R_0}{\beta} = \dfrac{1}{\omega C}$

or $C = \dfrac{\beta}{\omega R_0} = \dfrac{3}{2\pi \times 100 \times 10^6 (70)} = 68.2 \text{ pF}$

$$L = R_0^2 C = (70)^2 (68.2 \times 10^{-12}) = 334.2 \text{ nH/m}$$

Example 11.2

A distortionless line has Z_0=60 Ω, **α**=20 mNp/m, *u*=0.6c, where c is the speed of light. Find *R*,*L*,*G* , C and λ at 100 MHz.

$$\text{A distortionless line has } RC = GL \quad \text{or} \quad G = \frac{RC}{L},\ u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

$$Z_0 = \sqrt{\frac{L}{C}}\ ,\ \ \alpha = \sqrt{RG} = R\sqrt{\frac{C}{L}} = \frac{R}{Z_0}$$

$$\rightarrow R = \alpha Z_0 = (20\times10^{-3})(60) = 1.2\ \Omega/\text{m}$$

$$Since\ \ \alpha = \sqrt{RG} \rightarrow G = \frac{\alpha^2}{R} = \frac{400\times10^{-6}}{1.2} = 333\ \mu\text{S/m}$$

$$Dividing\ \ Z_0 = \sqrt{\frac{L}{C}}\ \text{ by }\ u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}\ \text{ gives}$$

$$L = \frac{Z_0}{u} = \frac{60}{0.6(3\times10^8)} = 333\ \text{nH/m}$$

Example 11.2 – solution continued

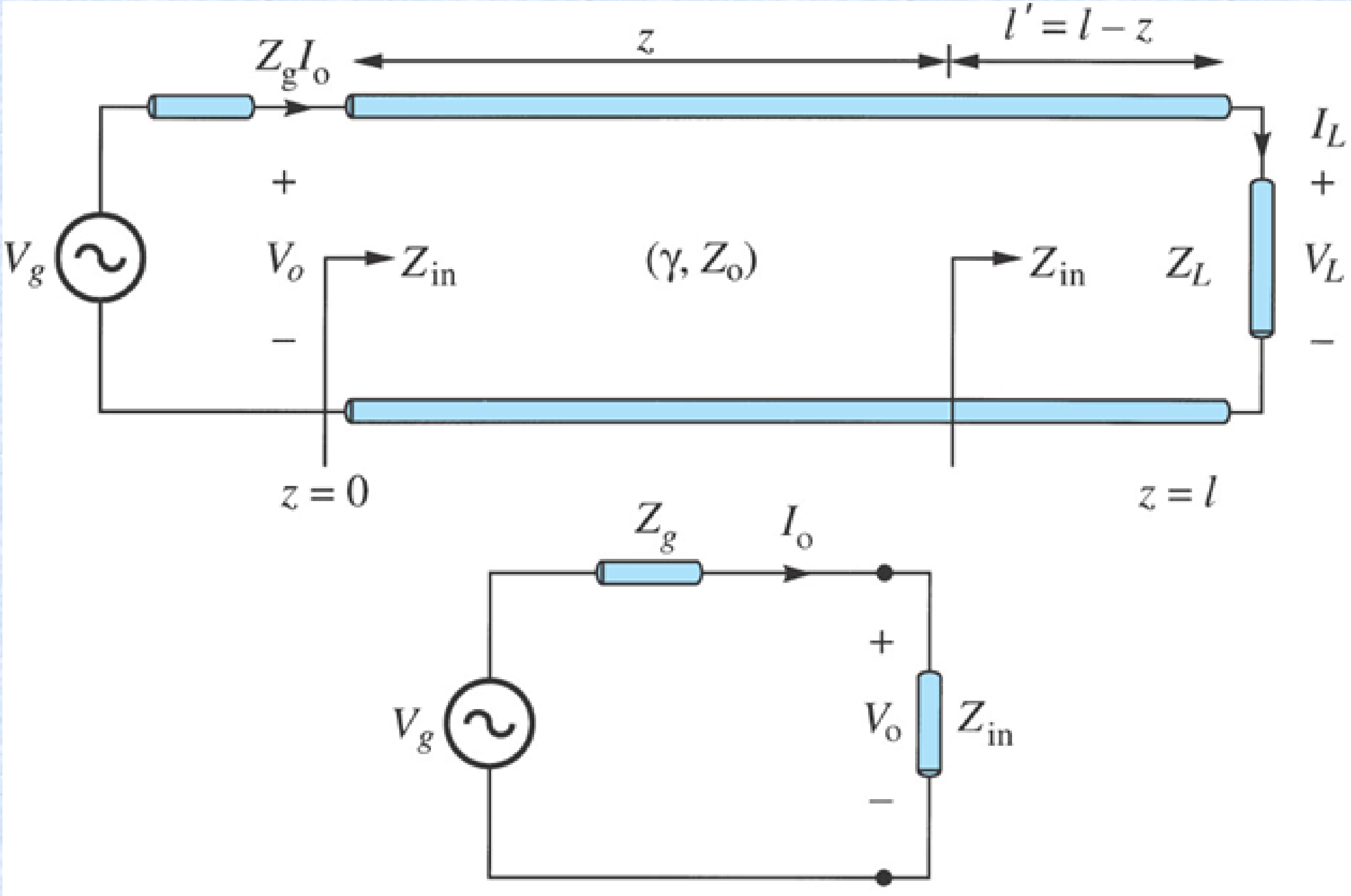
$$Multiplying \quad Z_0 = \sqrt{\frac{L}{C}} \text{ by } u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \text{ gives}$$

$$uZ_0 = \frac{1}{C} \rightarrow C = \frac{1}{uZ_0} = \frac{1}{0.6(3\times10^8)60} = 92.59 \text{ pF/m}$$

$$\lambda = \frac{u}{f} = \frac{0.6(3\times10^8)}{10^8} = 1.8 \text{ m}$$

11.4 Input impedance, standing wave ratio, power

Consider a transmission line of length ***l***, characterised by γ and $\mathbf{Z_0}$, connected to a load $\mathbf{Z_L}$. Generator sees the line with the load as an ***input impedance*** $\mathbf{Z_{in}}$.



Input impedance

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}, \qquad \left(Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} \right)$$

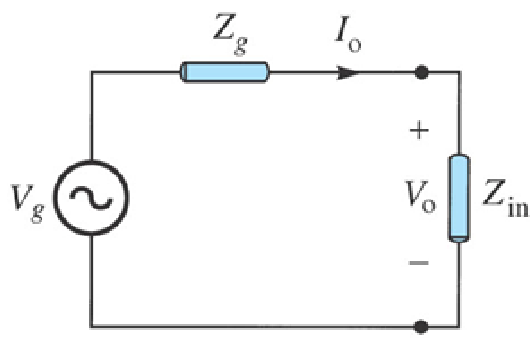
At generator terminals (sending end):

Let $V_0 = V(z=0)$, $I_0 = I(z=0)$, Substitute in prev. equs.:

$$\rightarrow \begin{aligned} V_0 &= V_0^+ + V_0^- \\ I_0 &= \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} \end{aligned} \Rightarrow \boxed{\begin{aligned} V_0^+ &= \frac{1}{2}\left(V_0 + Z_0 I_0\right) \\ V_0^- &= \frac{1}{2}\left(V_0 - Z_0 I_0\right) \end{aligned}} \quad \ldots (1)$$

If the input impedance at the terminals is Z_{in},

then $V_0 = \dfrac{Z_{in}}{Z_{in} + Z_g} V_g$, $\quad I_0 = \dfrac{V_g}{Z_{in} + Z_g}$



Input impedance

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad , \qquad I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$

At the load:

Let $V_L = V(z = l)$, $I_L = I(z = l)$, Substitute in prev. equs.

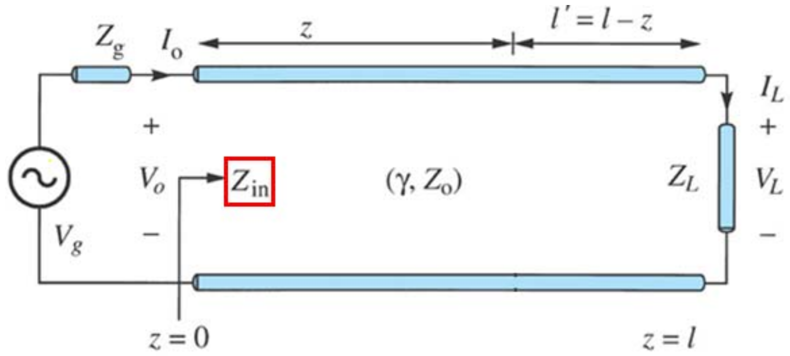
$$\rightarrow \quad \begin{aligned} V_L &= V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l} \\ I_L &= \frac{V_0^+}{Z_0} e^{-\gamma l} - \frac{V_0^-}{Z_0} e^{\gamma l} \end{aligned} \quad \Rightarrow \quad \boxed{\begin{aligned} V_0^+ &= \frac{1}{2}\left(V_L + Z_0 I_L\right) e^{\gamma l} \\ V_0^- &= \frac{1}{2}\left(V_L - Z_0 I_L\right) e^{-\gamma l} \end{aligned}} \quad \ldots (2)$$

Now determine the input impedance $Z_{in} = V_s(z) / I_s(z)$ at any point on the line.

Input impedance

first:At the generator, recall $V_0 = V_0^+ + V_0^-$, $\quad I_0 = \dfrac{V_0^+}{Z_0} - \dfrac{V_0^-}{Z_0}$, *then*

$$Z_{\text{in}} = \frac{V_s(z)}{I_s(z)} = \frac{V_0}{I_0} = \frac{Z_0\left(V_0^+ + V_0^-\right)}{V_0^+ - V_0^-}$$



Substituting eq. 2 and utilizing the fact that:

$$\frac{e^{\gamma l} + e^{-\gamma l}}{2} = \cosh \gamma l, \quad \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \sinh \gamma l \ ,$$

$$\text{or} \quad \tanh \gamma l = \frac{\sinh \gamma l}{\cosh \gamma l} = \frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}}$$

we get

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right]}$$

(General - Lossy Line)

Although this has been derived for the input impedance at the generator end, it is a general expression.

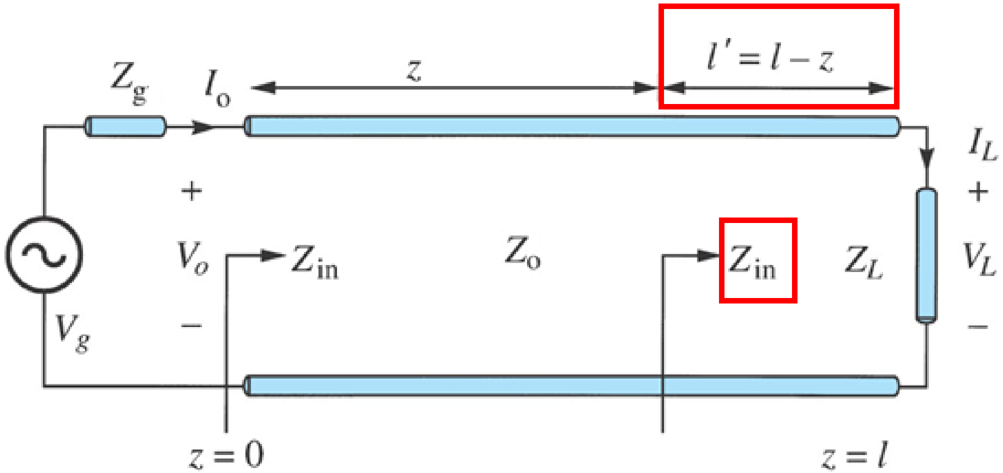
Input impedance (Lossless Line)

$$Z_{in} = Z_0\left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}\right] \quad \text{(General - Lossy Line)}$$

For a lossless line, $\gamma = j\beta$, $\tanh j\beta l = j\tan \beta l$, then

$$\boxed{Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\right]} \quad \text{(Lossless Line)}$$

βl is known as ***electrical length***, in degrees or radians



Note: To find Z_{in} at a distance l' from load, replace l by l' :-

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan \beta l'}{Z_0 + jZ_L \tan \beta l'}\right]$$

Reflection Coefficient, (at load)

Define Γ_L as the voltage reflection coefficient (at the load), as the ratio of the voltage reflection wave to the incident wave at the load,

$$\Gamma_L = \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}}$$

Since
$$V_0^+ = \frac{1}{2}\left(V_L + Z_0 I_L\right)e^{\gamma l}$$
$$V_0^- = \frac{1}{2}\left(V_L - Z_0 I_L\right)e^{-\gamma l}$$
, and $V_L = Z_L I_L$

$$\Rightarrow \quad \boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$ (Voltage Reflection coefficient at load)

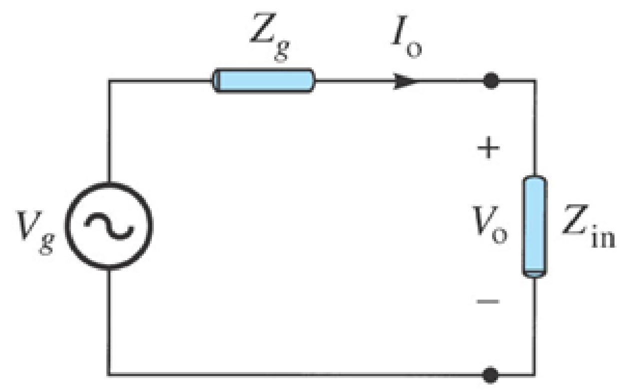
Reflection Coefficient, (at generator)

Define Γ_0 (at $z = 0$) as the voltage reflection coefficient at the source,

as the ratio of the voltage reflection wave to the incident wave at source,

$$\Gamma_0 = \frac{V_0^- e^{\gamma 0}}{V_0^+ e^{-\gamma 0}} = \frac{V_0^-}{V_0^+}$$

Since $V_0^+ = \frac{1}{2}\left(V_0 + Z_0 I_0\right)$, $V_0^- = \frac{1}{2}\left(V_0 - Z_0 I_0\right)$, and $V_0 = Z_{in} I_0$



$$\Rightarrow \quad \boxed{\Gamma_0 = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}}$$ (Voltage Reflection coefficient at source)

Reflection Coefficient

The **voltage reflection coefficient** at any point on the line is the ratio of the reflected voltage wave to that of the incident wave.

That is: $$\Gamma(z) = \frac{V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z}} = \frac{V_0^-}{V_0^+} e^{2\gamma z}$$

The **current reflection coefficient** at any point on the line is the negative of the voltage reflection coefficient at that point.

Thus the current reflection coefficient at the load is

$$I_0^- e^{\gamma l} / I_0^+ e^{-\gamma l} = -\Gamma_L$$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$

Standing Wave Ratio

Whenever there is a reflected wave, a **standing wave** will form out of the combination of incident and reflected waves.

The standing wave ratio s is defined as: (as we did for plane waves)

$$s = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

When load is *perfectly matched* $(Z_L = Z_0) \rightarrow$ Total Transmission

$$|\Gamma_L| = 0 \rightarrow s = 1$$

When load is a *short circuit* :

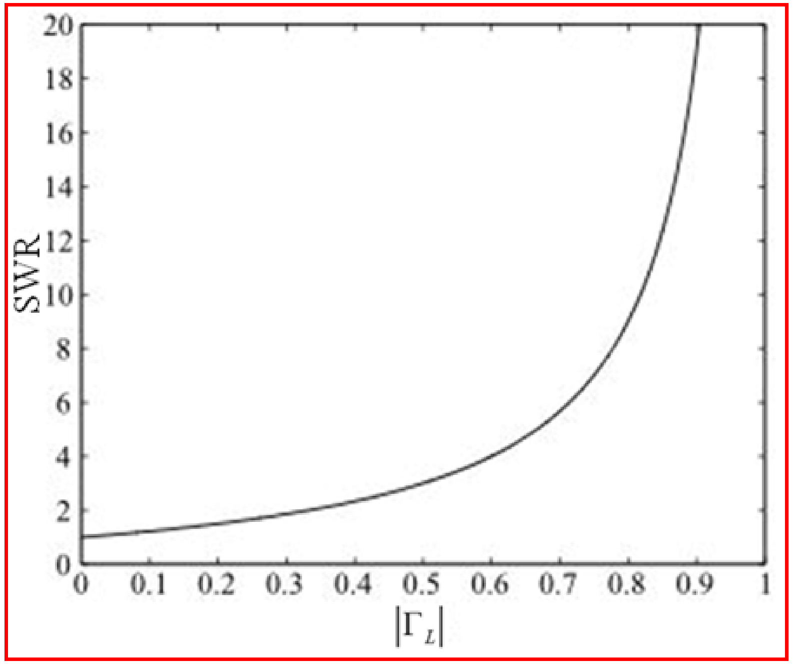
$\rightarrow$ Total Reflection

$$\Gamma_L = -1 \;\rightarrow\; |\Gamma_L| = 1 \;\Rightarrow s = \infty$$

When load is an *open circuit* :

$\rightarrow$ Total Reflection

$$\Gamma_L = +1 \;\rightarrow\; |\Gamma_L| = 1 \;\Rightarrow s = \infty$$



Power

The time-average power flow along the line at the point z is:

$P_{ave} = \frac{1}{2}\mathrm{Re}[V_s(z)I_s^*(z)].$ For a *lossless* line, this can be reduced to:

$$P_{ave} = \frac{1}{2}\mathrm{Re}[V_0^+(e^{j\beta l} + \Gamma e^{-j\beta l})\left(\frac{V_0^{+*}}{Z_0}(e^{-j\beta l} - \Gamma^* e^{j\beta l})\right)]$$

$$\boxed{P_{ave} = \frac{|V_0^+|^2}{2Z_0}\left(1 - |\Gamma|^2\right)},\qquad P_{ave} = |V_0^+|^2/2Z_0 - |\Gamma|^2|V_0^+|^2/2Z_0$$

Incident Power (Pi)

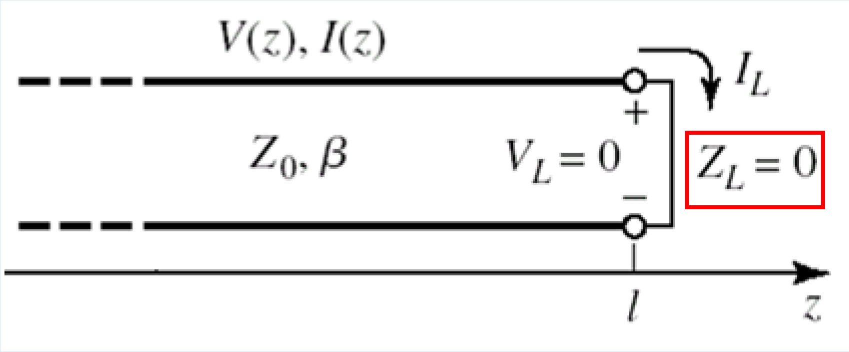
Reflected Power (Pr)

* The average power flow is constant at any point on the *lossless* line.
* The total power delivered to the load (P_{av}) is equal to the incident power ($|V_0^+|^2/2Z_0$) minus the reflected power ($|V_0^+|^2|\Gamma|^2/2Z_0$)
* If $\Gamma = 0$, maximum power is delivered to the load, while no power is delivered for $|\Gamma| = 1$.
* The above discussion assumes that the generator is matched.

Special Cases , $Z_L=0$, $Z_L=\infty$, $Z_L=Z_0$

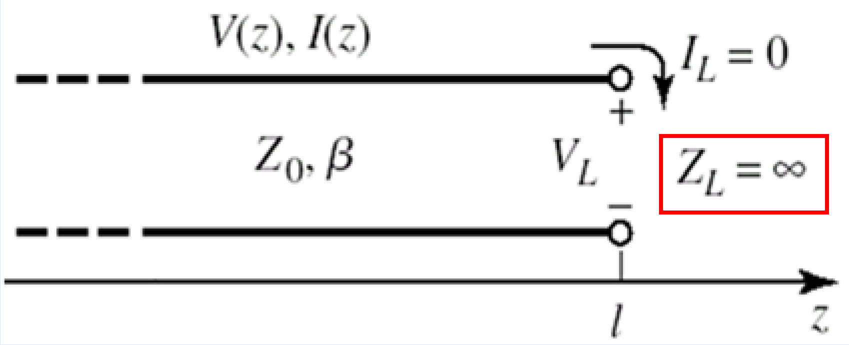
Short Circuited Line

($Z_L=0$)



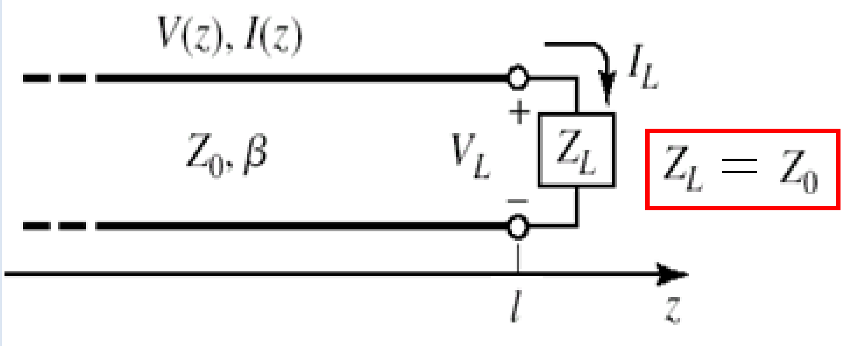
Open Circuited Line

($Z_L=\infty$)

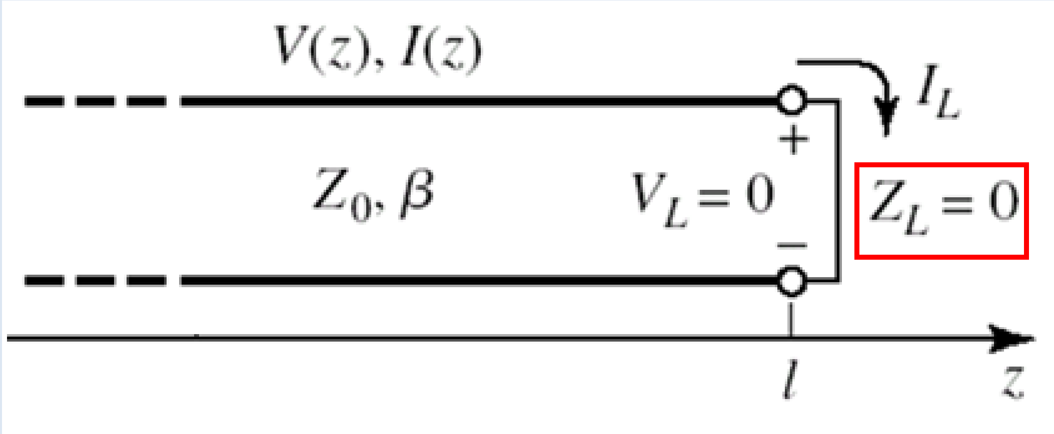


Matched Line

($Z_L=Z_0$)



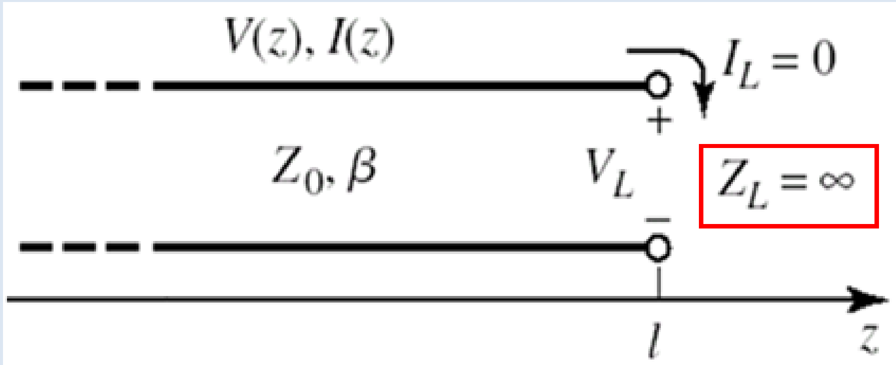
Shorted Line (Z_L=0)



$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = jZ_0 \tan \beta l$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -1, \quad s = \infty \qquad \text{(Total Reflection)}$$

Open-Circuited Line ($Z_L=\infty$)

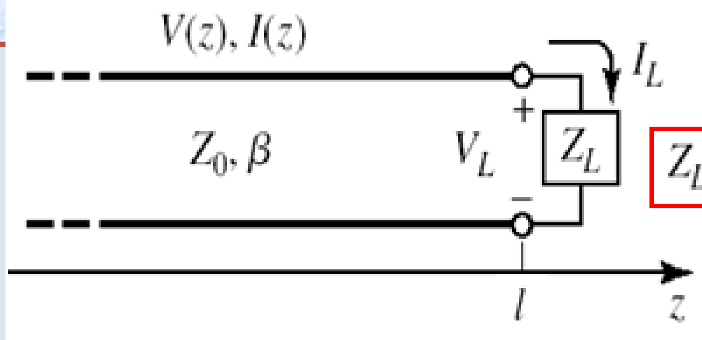


$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}, \qquad (Z_L = \infty)$$

$$Z_{in} = Z_0 \frac{\left(1 + j\dfrac{Z_0}{Z_L}\tan \beta l\right)}{\left(\dfrac{Z_0}{Z_L} + j\tan \beta l\right)} = \frac{Z_0}{j\tan \beta l} = -jZ_0 \cot \beta l$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 1, \quad s = \infty \qquad \text{(Total Reflection)}$$

Matched Line $(Z_L=Z_0)$



$Z_L = Z_0$

Most desired case from practical point of view.

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l},$$

$$Since \quad Z_L = Z_0 \;\rightarrow\; Z_{in} = Z_0$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0, \quad s = 1$$

The whole wave is transmitted, and there is no reflection.

The incident power is fully absorbed by the load.

$\Rightarrow$ (Maximum power transfer)

Example 11.3

A certain transmission line 2 m long operating at $\omega=10^6$ rad/s has $\alpha=8$ dB/m, $\beta=1$ rad/m, and $Z_0= 60+j40\ \Omega$. If the line is connected to a source of $10\angle 0^0$ V, $Z_g=40\ \Omega$ and transmitted by a load of $20+j50\ \Omega$, determine

(a) The input impedance

(b) The sending end current

(c) The current at the middle of the line.

Solution

(*a*) Since 1 Np=20 log (e)=8.686 dB

$$\alpha=\frac{8}{8.686}=0.921\ Np/m$$

$$\gamma=\alpha+j\beta=0.921+j1$$

$$\gamma l=2(0.921+j1)=1.84+j2$$

Example 11.3 – Solution continued

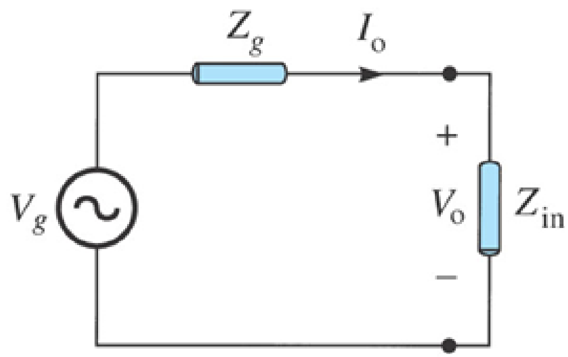
$$\tanh\gamma l = \tanh(1.84 + j2) = 1.033 - j0.03929$$

$$\tanh(x + jy) = \frac{\sinh 2x}{\cosh 2x + \cos 2y} + j\frac{\sin 2y}{\cosh 2x + \cos 2y}$$

$$Z_{in} = Z_0\left[\frac{Z_L + Z_0\tanh\gamma l}{Z_0 + Z_L\tanh\gamma l}\right]$$

$$Z_{in} = (60 + j40)\left[\frac{20 + j50 + (60 + j40)(1.033 - j0.03929)}{60 + j40 + (20 + j50)(1.033 - j0.03929)}\right]$$

$$Z_{in} = 60.25 + j38.79\ \Omega$$



(b) The sending end current is $I(z = 0) = I_0$

$$I_0 = \frac{V_g}{Z_{in} + Z_g} = \frac{10}{60.25 + j38.79 + 40} = 93.03\angle -21.15\ mA$$

Example 11.3 – Solution continued

(c) To find the current at any point, we need V_0^+ and V_0^-. *But*

$$I_0 = 93.03\angle -21.15\ mA$$

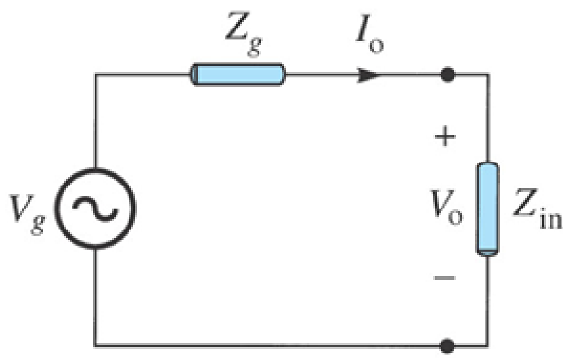
$$V_0 = Z_{in}I_0 = (71.66\angle 32.77^0)(0.09303\angle -21.15^0) = 6.667\angle 11.62^0$$

$$V_0^+ = \frac{1}{2}\left(V_0 + Z_0 I_0\right) = \frac{1}{2}\left[6.667\angle 11.62 + (60 + j40)(0.09303\angle -21.15)\right]$$

$$= 6.687\angle 12.08^0$$

$$V_0^- = \frac{1}{2}\left(V_0 - Z_0 I_0\right) = \frac{1}{2}\left[6.667\angle 11.62 - (60 + j40)(0.09303\angle -21.15)\right]$$

$$= 0.0518\angle 260^0$$



Example 11.3 – Solution continued

At the middle of the line, $z = l/2$, $\gamma z = \gamma l/2 = 0.921 + j1$, Hence the current at this point is:

$$I_s(z = l/2) = \frac{V_0^+}{Z_0} e^{-\gamma l/2} - \frac{V_0^-}{Z_0} e^{\gamma l/2}$$

$$= \frac{(6.687e^{j12.08^0})e^{-0.921-j1}}{60 + j40} - \frac{(0.0518e^{j260^0})e^{0.921+j1}}{60 + j40}$$

Note that $j1$ is in radians and is equivalent to $j57.3^0$,$(j1 \times 180/\pi)$:-

$$I_s(z = l/2) = \frac{(6.687e^{j12.08^0})e^{-0.921}e^{-j57.3^0}}{72.1e^{j33.69^0}} - \frac{(0.0518e^{j260^0})e^{0.921}e^{j57.3^0}}{72.1e^{j33.69^0}}$$

$$= 0.0369e^{-j78.91^0} - 0.001805e^{j283.61^0}$$

$$= 6.673 - j34.456 \text{ mA}$$

$$= 35.10\angle 281^0 \quad \text{mA}$$