Mechanical Vibrations

Lecture 10

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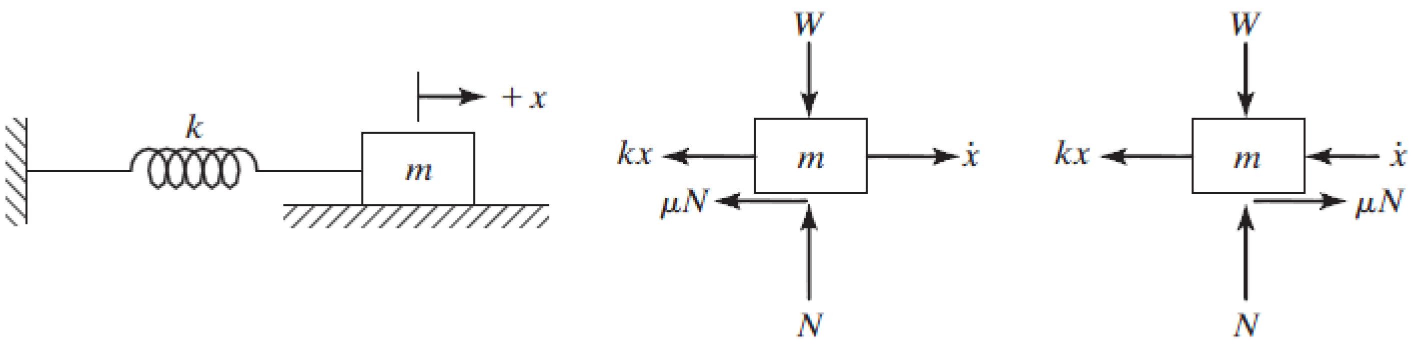
Free Vibration with Coulomb Damping

the friction force *F* is given by $F = \mu N = \mu W = \mu m g$



(a) (b) (c)

Equation of Motion

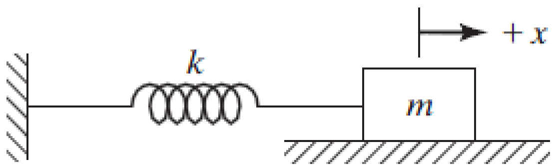
Case 1. When x is positive and dx/dt is positive or when x is negative and dx/dt is positive (i.e., for the half cycle during which the mass moves from left to right), the equation of motion can be obtained using Newton's second law (see Fig. 2.42(b)):

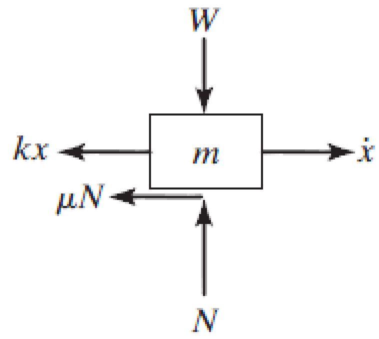
$$m\ddot{x} = -kx - \mu N \quad \text{or} \quad m\ddot{x} + kx = -\mu N \tag{2.126}$$

Case 2. When x is positive and dx/dt is negative or when x is negative and dx/dt is negative (i.e., for the half cycle during which the mass moves from right to left), the equation of motion can be derived from Fig. 2.42(c) as

$$-kx + \mu N = m\ddot{x} \quad \text{or} \quad m\ddot{x} + kx = \mu N \tag{2.128}$$

Case 1. When *x* is positive and *dx/dt* is positive





(b)

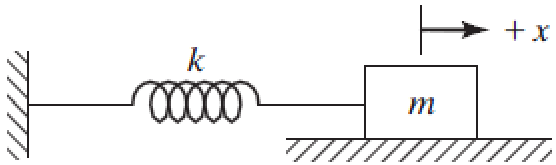
$$m\ddot{x} = -kx - \mu N \quad \text{or} \quad m\ddot{x} + kx = -\mu N$$

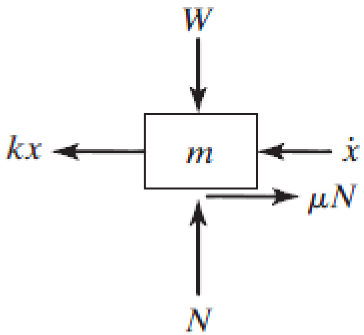
$$x(t) = A_1 \cos \omega_n t + A_2 \sin \omega_n t - \frac{\mu N}{k}$$

$$-A_1 = -x_0 + \frac{3\mu N}{k}, \qquad A_2 = 0$$

$$x(t) = \left(x_0 - \frac{3\mu N}{k} \right) \cos \omega_n t - \frac{\mu N}{k}$$

Case 2. When *x* is positive and *dx/dt* is negative or when *x* is negative and *dx/dt* is negative



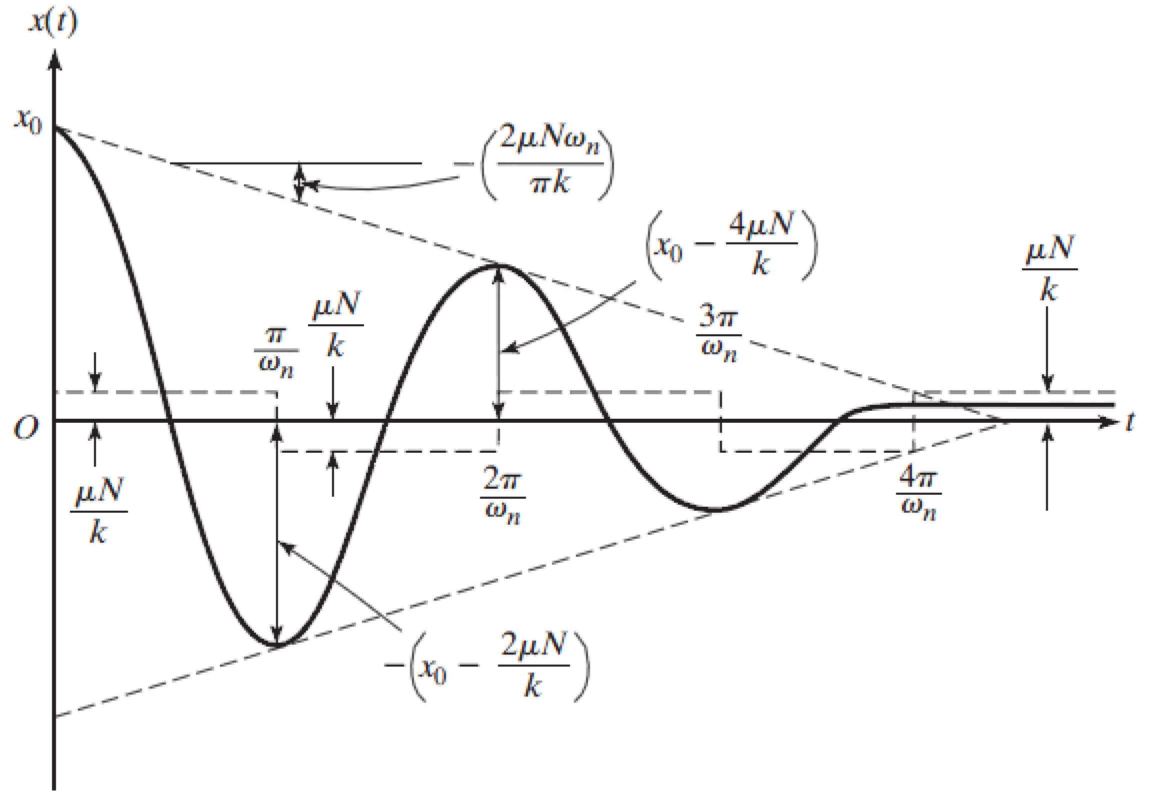


$$-kx + \mu N = m\ddot{x} \quad \text{or} \quad m\ddot{x} + kx = \mu N$$

$$x(t) = A_3 \cos \omega_n t + A_4 \sin \omega_n t + \frac{\mu N}{k}$$

$$A_3 = x_0 - \frac{\mu N}{k}, \qquad A_4 = 0$$

$$x(t) = \left(x_0 - \frac{\mu N}{k}\right) \cos \omega_n t + \frac{\mu N}{k}$$



The motion stops when $x_n \leq \mu N/k$,

$$x_0 - r\frac{2\mu N}{k} \leq \frac{\mu N}{k}$$

$$r \geq \left\{ \frac{x_0 - \frac{\mu N}{k}}{\frac{2\mu N}{k}} \right\}$$

In each successive cycle, the amplitude of motion is reduced by the amount $4\mu N/k$, so the amplitudes at the end of any two consecutive cycles are related:

$$X_m = X_{m-1} - \frac{4\mu N}{k}$$

As the amplitude is reduced by an amount $4\mu N/k$ in one cycle (i.e., in time $2\pi/\omega_n$), the slope of the enveloping straight lines (shown dotted) in Fig. 2.43 is

$$-\left(\frac{4\mu N}{k}\right)\Big/\left(\frac{2\pi}{\omega_n}\right) = -\left(\frac{2\mu N\omega_n}{\pi k}\right)$$

EXAMPLE 2.14 Coefficient of Friction from Measured Positions of Mass

A metal block, placed on a rough surface, is attached to a spring and is given an initial displacement of 10 cm from its equilibrium position. After five cycles of oscillation in 2 s, the final position of the metal block is found to be 1 cm from its equilibrium position. Find the coefficient of friction between the surface and the metal block.

Solution: Since five cycles of oscillation were observed to take place in 2 s, the period (τ_n) is $2/5 = 0.4$ s, and hence the frequency of oscillation is $\omega_n = \sqrt{\frac{k}{m}} = \frac{2\pi}{\tau_n} = \frac{2\pi}{0.4} = 15.708$ rad/s. Since the amplitude of oscillation reduces by

$$\frac{4\mu N}{k} = \frac{4\mu mg}{k}$$

the reduction in amplitude in five cycles is

$$5\left(\frac{4\mu mg}{k}\right) = 0.10 - 0.01 = 0.09 \text{ m}$$

$$\mu = \frac{0.09k}{20mg} = \frac{0.09\omega_n^2}{20g} = \frac{0.09(15.708)^2}{20(9.81)} = 0.1132$$