Mechanical Vibrations

Lecture 11

C H A P T E R 3

Harmonically Excited Vibration

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Learning Objectives

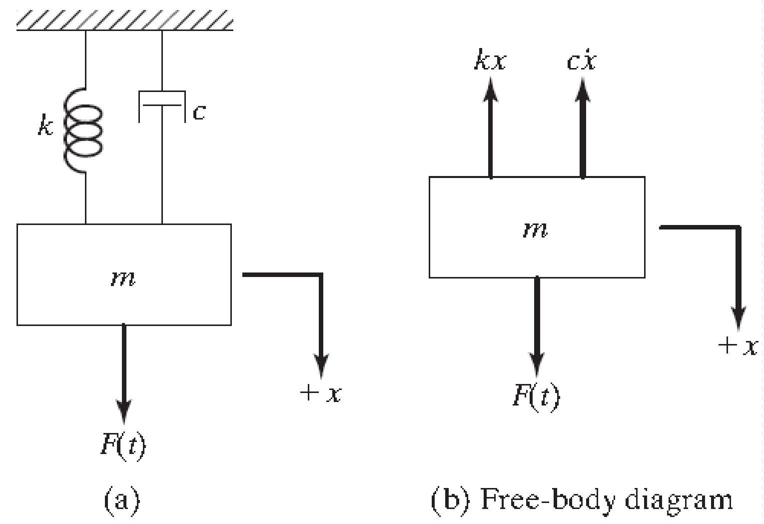
After completing this chapter, you should be able to do the following:

- Find the responses of undamped and viscously damped single-degree-of-freedom systems subjected to different types of harmonic force, including base excitation and rotating unbalance
- Distinguish between transient, steady-state, and total solutions
- Understand the variations of magnification factor and phase angles with the frequency of excitation and the phenomena of resonance and beats.
- Find the response of systems involving Coulomb, hysteresis, and other types of damping
- Derive transfer functions of systems governed by linear differential equations with constant coefficients.

In this chapter, we shall consider the dynamic response of a single-degree-of-freedom system under harmonic excitations of the form $F(t) = F_0 e^{i(\omega t+\phi)}$ or $F(t) = F_0 \cos(\omega t + \phi)$ or $F(t) = F_0 \sin(\omega t + \phi)$, where F_0 is the amplitude, ω is the frequency, and ϕ is the phase angle of the harmonic excitation. The value of ϕ depends on the value of $F(t)$ at $t = 0$ and is usually taken to be zero.

Equation of Motion

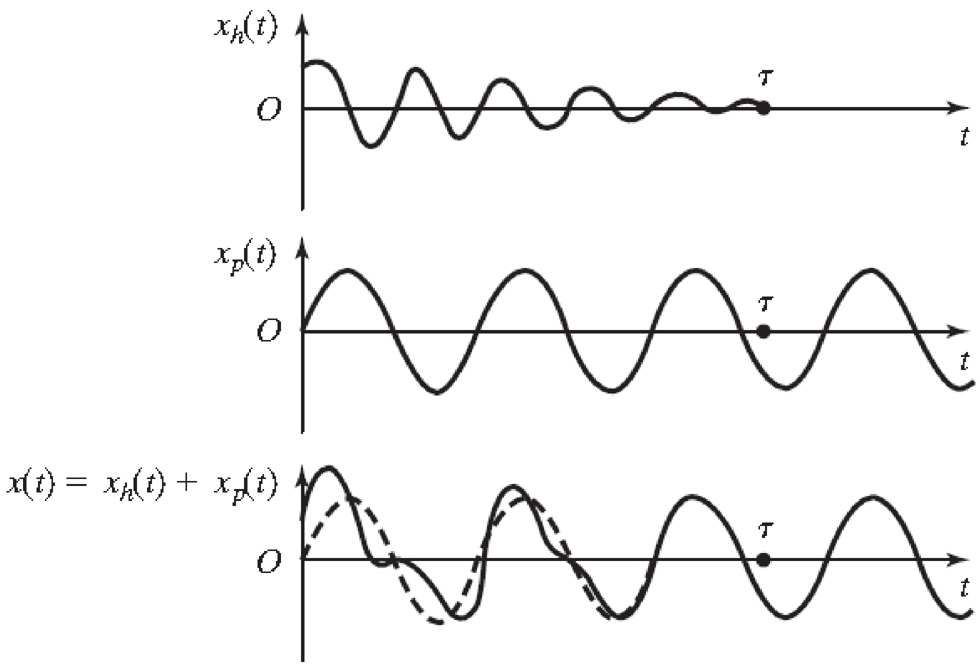
$$m\ddot{x} + c\dot{x} + kx = F(t)$$



the solution of the homogeneous equation

$$m\ddot{x} + c\dot{x} + kx = 0$$

The variations of homogeneous, particular, and general solutions with time for a typical case are shown in Fig. 3.2



3.3 Response of an Undamped System Under Harmonic Force

If a force $F(t) = F_0 \cos \omega t$

$$m\ddot{x} + kx = F_0 \cos \omega t$$

The homogeneous solution of this equation is given by $x_h(t) = C_1 \cos \omega_n t + C_2 \sin \omega_n t$

$$x_p(t) = X \cos \omega t$$

$$X = \frac{F_0}{k - m\omega^2} = \frac{\delta_{st}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

where $\delta_{st} = F_0 / k$

Thus the total solution of Eq. (3.3) becomes

$$x(t) = C_1 \cos \omega_n t + C_2 \sin \omega_n t + \frac{F_0}{k - m\omega^2} \cos \omega t$$

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Using the initial conditions $x(t = 0) = x_0$ and $\dot{x}(t = 0) = \dot{x}_0$, we find that

$$C_1 = x_0 - \frac{F_0}{k - m\omega^2}, \qquad C_2 = \frac{\dot{x}_0}{\omega_n}$$

$$x(t) = \left(x_0 - \frac{F_0}{k - m\omega^2} \right) \cos \omega_n t + \left(\frac{\dot{x}_0}{\omega_n} \right) \sin \omega_n t + \left(\frac{F_0}{k - m\omega^2} \right) \cos \omega t$$

$$X = \frac{F_0}{k - m\omega^2} = \frac{\delta_{st}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

The maximum amplitude X in Eq. (3.6) can be expressed as

$$\frac{X}{\delta_{st}} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$$r = \omega/\omega_n$$

Case 1. When $0 < \omega/\omega_n < 1$

Case 2. When $\omega/\omega_n > 1$

Case 3. When $\omega/\omega_n = 1$,