Mechanical Vibrations

Lecture 14

C H A P T E R 3

Harmonically Damped Excited Vibration and Vibration Isolation

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3.5 Response of a Damped System Under $F(t) = F_0 e^{i\omega t}$

$$m\ddot{x} + c\dot{x} + kx = F_0 e^{i\omega t}$$

$$x_p(t) = X e^{i\omega t}$$

$$X = \frac{F_0}{(k - m\omega^2) + ic\omega}$$

$$X = F_0 \left[\frac{k - m\omega^2}{(k - m\omega^2)^2 + c^2\omega^2} - i \frac{c\omega}{(k - m\omega^2)^2 + c^2\omega^2} \right]$$

Using the relation $x + iy = Ae^{i\phi}$, where $A = \sqrt{x^2 + y^2}$ and $\tan \phi = y/x$, can be expressed as

$$X = \frac{F_0}{[(k - m\omega^2)^2 + c^2\omega^2]^{1/2}} e^{-i\phi}$$

$$\phi = \tan^{-1}\left(\frac{c\omega}{k - m\omega^2} \right)$$

$$x_p(t) = \frac{F_0}{[(k - m\omega^2)^2 + (c\omega)^2]^{1/2}} e^{i(\omega t - \phi)}$$

$$\frac{kX}{F_0} = \frac{1}{1 - r^2 + i2\zeta r}$$

$$\left|\frac{kX}{F_0}\right| = \frac{1}{[(1 - r^2)^2 + (2\zeta r)^2]^{1/2}}$$

$$H(i\omega) = |H(i\omega)|e^{-i\phi}$$

$$\phi = \tan^{-1}\left(\frac{2\zeta r}{1 - r^2}\right)$$

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If $F(t) = F_0 \cos \omega t$, the corresponding steady-state solution is given by the real part of Eq. (3.53):

$$x_p(t) = \frac{F_0}{\left[(k - m\omega^2)^2 + (c\omega)^2\right]^{1/2}} \cos(\omega t - \phi)$$

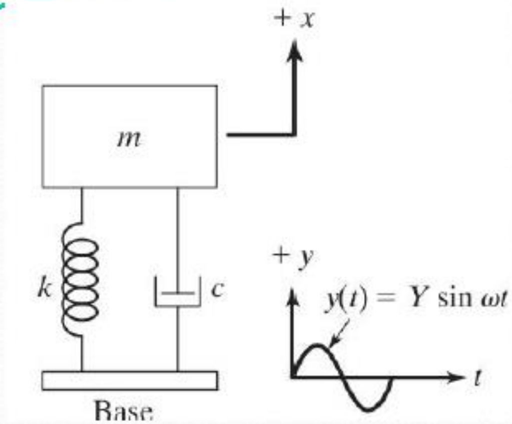
$$= \mathrm{Re}\left[\frac{F_0}{k} H(i\omega) e^{i\omega t}\right] = \mathrm{Re}\left[\frac{F_0}{k} |H(i\omega)| e^{i(\omega t - \phi)}\right] \tag{3.59}$$

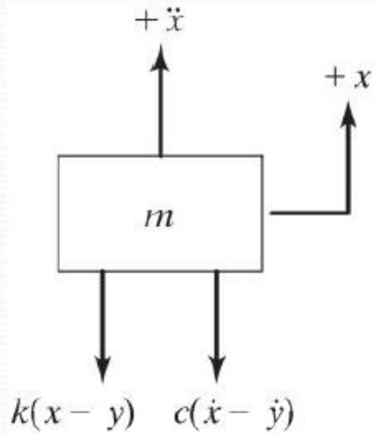
which can be seen to be the same as Eq. (3.25). Similarly, if $F(t) = F_0 \sin \omega t$, the corresponding steady-state solution is given by the imaginary part of Eq. (3.53):

$$x_p(t) = \frac{F_0}{\left[(k - m\omega^2)^2 + (c\omega)^2\right]^{1/2}} \sin(\omega t - \phi)$$

$$= \mathrm{Im}\left[\frac{F_0}{k} |H(i\omega)| e^{i(\omega t - \phi)}\right] \tag{3.60}$$

3.6 Response of a Damped System Under the Harmonic Motion of the Base





$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$

If $y(t) = Y \sin \omega t$, Eq. (3.64) becomes

$$m\ddot{x} + c\dot{x} + kx = ky + c\dot{y} = kY \sin \omega t + c\omega Y \cos \omega t$$

$$= A \sin(\omega t - \alpha) \tag{3.65}$$

where $A = Y\sqrt{k^2 + (c\omega)^2}$ and $\alpha = \tan^{-1}\left[-\frac{c\omega}{k}\right]$. This shows that giving excitation to

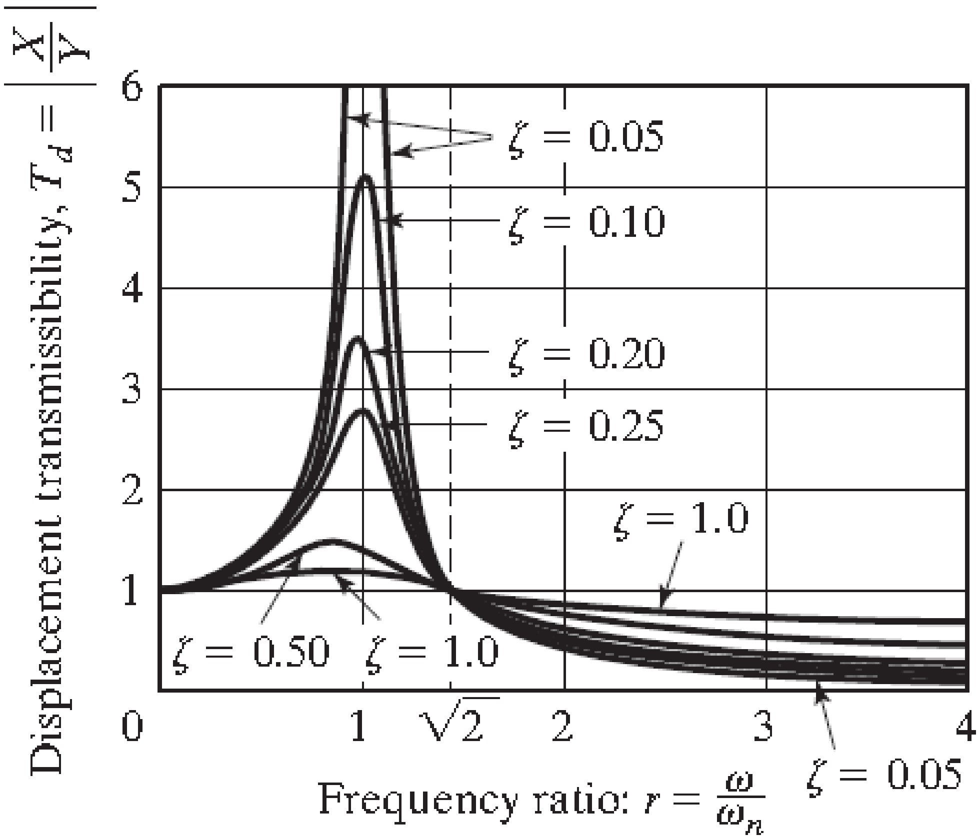
$$x_p(t) = \frac{Y\sqrt{k^2 + (c\omega)^2}}{\left[(k - m\omega^2)^2 + (c\omega)^2\right]^{1/2}} \sin(\omega t - \phi_1 - \alpha)$$

$$\phi_1 = \tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right)$$

$$x_p(t) = X \sin(\omega t - \phi)$$

$$\frac{X}{Y} = \left[\frac{k^2 + (c\omega)^2}{(k - m\omega^2)^2 + (c\omega)^2}\right]^{1/2} = \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}\right]^{1/2}$$

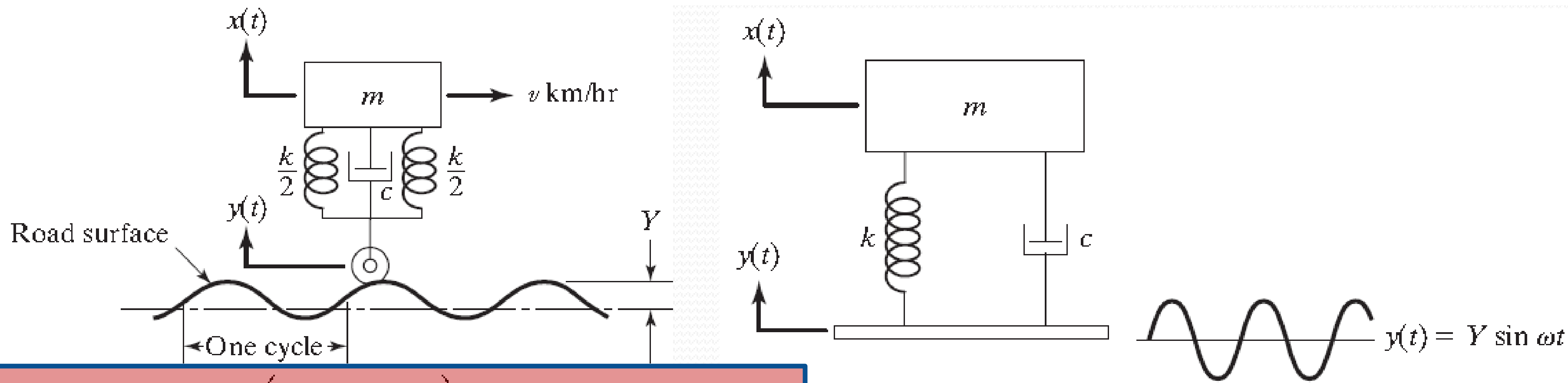
$$\phi = \tan^{-1}\left[\frac{mc\omega^3}{k(k - m\omega^2) + (\omega c)^2}\right] = \tan^{-1}\left[\frac{2\zeta r^3}{1 + (4\zeta^2 - 1)r^2}\right]$$



1. The value of T_d is unity at $r = 0$ and close to unity for small values of r.
2. For an undamped system $(\zeta = 0)$, $T_d \to \infty$ at resonance $(r = 1)$.
3. The value of T_d is less than unity $(T_d < 1)$ for values of $r > \sqrt{2}$ (for any amount of damping ζ).
4. The value of T_d is unity for all values of ζ at $r = \sqrt{2}$.
5. For $r < \sqrt{2}$, smaller damping ratios lead to larger values of T_d. On the other hand, for $r > \sqrt{2}$, smaller values of damping ratio lead to smaller values of T_d.

EXAMPLE 3.4 Vehicle Moving on a Rough Road

Figure 3.18 shows a simple model of a motor vehicle that can vibrate in the vertical direction while traveling over a rough road. The vehicle has a mass of 1200 kg. The suspension system has a spring constant of 400 kN/m and a damping ratio of $\zeta = 0.5$. If the vehicle speed is 20 km/hr, determine the displacement amplitude of the vehicle. The road surface varies sinusoidally with an amplitude of $Y = 0.05$ m and a wavelength of 6 m.



$$\omega = 2\pi f = 2\pi\left(\frac{v \times 1000}{3600}\right)\frac{1}{6} = 0.290889v \text{ rad/s}$$

For $v = 20$ km/hr, $\omega = 5.81778$ rad/s. The natural frequency of the vehicle is given by

$$\omega_n = \sqrt{\frac{k}{m}} = \left(\frac{400 \times 10^3}{1200}\right)^{1/2} = 18.2574 \text{ rad/s}$$

$$r = \frac{\omega}{\omega_n} = \frac{5.81778}{18.2574} = 0.318653$$

$$\frac{X}{Y} = \left\{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}\right\}^{1/2} = \left\{\frac{1 + (2 \times 0.5 \times 0.318653)^2}{(1 - 0.318653)^2 + (2 \times 0.5 \times 0.318653)^2}\right\}^{1/2}$$

$$= 1.100964$$

$$X = 1.100964Y = 1.100964(0.05) = 0.055048 \text{ m}$$

EXAMPLE 3.5 Machine on Resilient Foundation

A heavy machine, weighing 3000 N, is supported on a resilient foundation. The static deflection of the foundation due to the weight of the machine is found to be 7.5 cm. It is observed that the machine vibrates with an amplitude of 1 cm when the base of the foundation is subjected to harmonic oscillation at the undamped natural frequency of the system with an amplitude of 0.25 cm. Find

a. the damping constant of the foundation,
b. the dynamic force amplitude on the base, and

Solution:

a. The stiffness of the foundation can be found from its static deflection: k = weight of machine$/\delta_{st} = 3000/0.075 = 40{,}000$ N/m.

At resonance ($\omega = \omega_n$ or $r = 1$), Eq. (3.68) gives

$$\frac{X}{Y} = \frac{0.010}{0.0025} = 4 = \left[\frac{1 + (2\zeta)^2}{(2\zeta)^2}\right]^{1/2} \tag{E.1}$$

The solution of Eq. (E.1) gives $\zeta = 0.1291$. The damping constant is given by

$$\begin{aligned} c = \zeta \cdot c_c = \zeta 2\sqrt{km} &= 0.1291 \times 2 \times \sqrt{40{,}000 \times (3000/9.81)} \\ &= 903.0512 \text{ N-s/m} \end{aligned} \tag{E.2}$$

b. The dynamic force amplitude on the base at $r = 1$ can be found from Eq. (3.74):

$$F_T = Yk\left[\frac{1 + 4\zeta^2}{4\zeta^2}\right]^{1/2} = kX = 40{,}000 \times 0.01 = 400 \text{ N}$$