Mechanical Vibrations

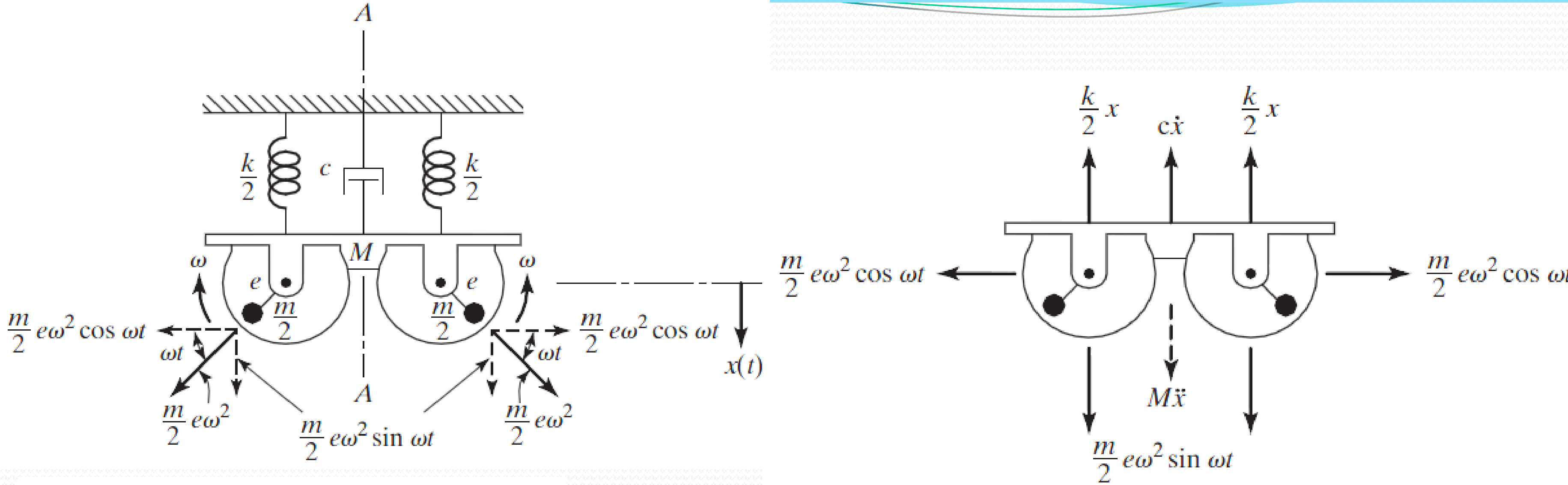
Lecture 15

CHAPTER 3

Harmonically Damped Excited Vibration
Rotating Unbalance

Dr. Juma Yousuf Alaydi
Mechanical Engineering
Islamic University

Response of a Damped System Under Rotating Unbalance



$$M\ddot{x} + c\dot{x} + kx = me\omega^2 \sin \omega t$$

$$x_p(t) = X \sin(\omega t - \phi) = \text{Im}\left[\frac{me}{M}\left(\frac{\omega}{\omega_n}\right)^2 |H(i\omega)| e^{i(\omega t - \phi)}\right]$$

$$X = \frac{me\omega^2}{\left[(k - M\omega^2)^2 + (c\omega)^2\right]^{1/2}} = \frac{me}{M}\left(\frac{\omega}{\omega_n}\right)^2 |H(i\omega)| \qquad \phi = \tan^{-1}\left(\frac{c\omega}{k - M\omega^2}\right)$$

$$\frac{MX}{me} = \frac{r^2}{\left[(1 - r^2)^2 + (2\zeta r)^2\right]^{1/2}} = r^2 |H(i\omega)| \qquad \phi = \tan^{-1}\left(\frac{2\zeta r}{1 - r^2}\right)$$

The force transmitted to the foundation due to rotating unbalanced force (F) can be found as $F(t) = kx(t) + c\dot{x}(t)$. The magnitude (or maximum value) of F can be derived as (see Problem 3.73):

$$|F| = me\omega^2 \left[\frac{1 + 4\zeta^2 r^2}{(1 - r^2)^2 + 4\zeta^2 r^2} \right]^{\frac{1}{2}} \quad (3.84)$$

EXAMPLE 3.6 Deflection of an Electric Motor due to Rotating Unbalance

An electric motor of mass M, mounted on an elastic foundation, is found to vibrate with a deflection of 0.15 m at resonance (Fig. 3.20). It is known that the unbalanced mass of the motor is 8% of the mass of the rotor due to manufacturing tolerances used, and the damping ratio of the foundation is $\zeta = 0.025$. Determine the following:

a. the eccentricity or radial location of the unbalanced mass (e),
b. the peak deflection of the motor when the frequency ratio varies from resonance, and
c. the additional mass to be added uniformly to the motor if the deflection of the motor at resonance is to be reduced to 0.1 m.