Mechanical Vibrations

Lecture 17

C H A P T E R 4
Vibration Under General Forcing Conditions

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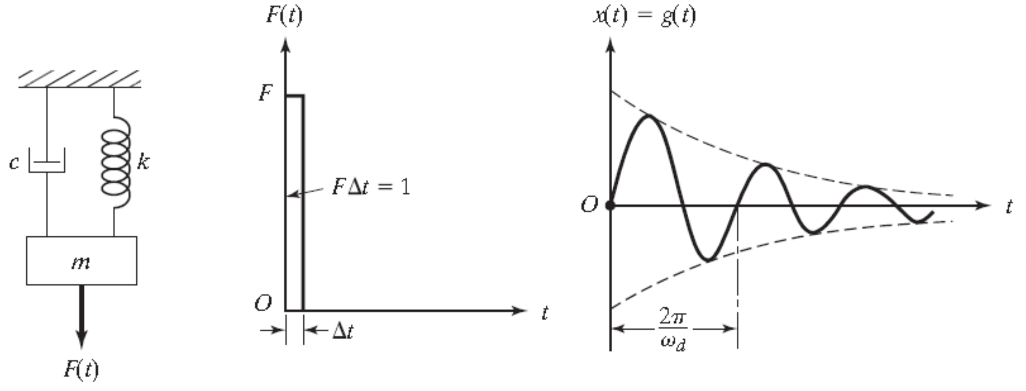
Response to an Impulse

$$\text{Impulse} = F\Delta t = m\dot{x}_2 - m\dot{x}_1$$



(a) (b) (c)

$$f = \lim_{\Delta t \to 0} \int_t^{t+\Delta t} F\,dt = F\,dt = 1$$

The unit impulse, $f = 1$, acting at $t = 0$,

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$x(t) = e^{-\zeta\omega_n t}\left\{x_0 \cos\omega_d t + \frac{\dot{x}_0 + \zeta\omega_n x_0}{\omega_d}\sin\omega_d t\right\}$$

$$\text{Impulse} = f = 1 = m\dot{x}(t = 0) - m\dot{x}(t = 0^-) = m\dot{x}_0$$

$$x(t = 0) = x_0 = 0$$

$$\dot{x}(t = 0) = \dot{x}_0 = \frac{1}{m}$$

$$x(t) = g(t) = \frac{e^{-\zeta\omega_n t}}{m\omega_d}\sin\omega_d t$$

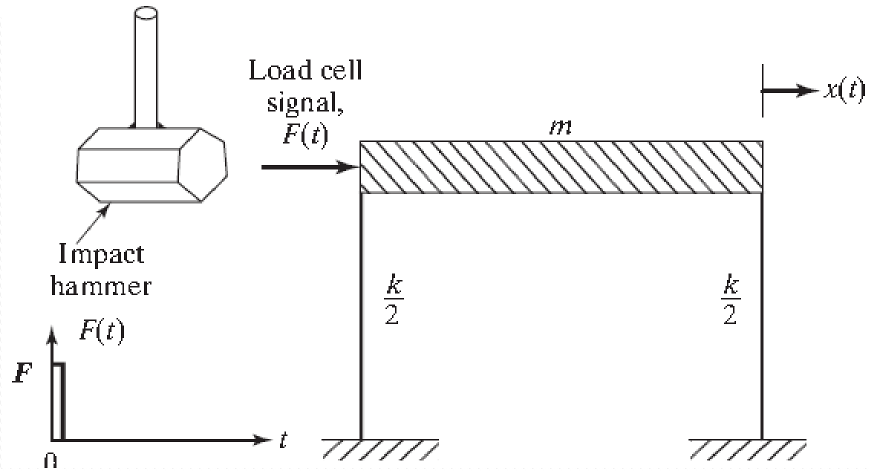
If the magnitude of the impulse is F instead of unity, the initial velocity $\dot{x}_0$ is F/m and the response of the system becomes

$$x(t) = \frac{Fe^{-\zeta\omega_n t}}{m\omega_d}\sin\omega_d t = Fg(t) \qquad (4.26)$$

EXAMPLE 4.7

Response of a Structure Under Impact

In the vibration testing of a structure, an impact hammer with a load cell to measure the impact force is used to cause excitation, as shown in Fig. 4.8(a). Assuming $m = 5$ kg, $k = 2000$ N/m, $c = 10$ N-s/m, and $F = 20$ N-s, find the response of the



Solution: From the known data, we can compute

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{2000}{5}} = 20 \text{ rad/s}, \qquad \zeta = \frac{c}{c_c} = \frac{c}{2\sqrt{km}} = \frac{10}{2\sqrt{2000(5)}} = 0.05$$

$$\omega_d = \sqrt{1-\zeta^2}\,\omega_n = 19.975 \text{ rad/s}$$

Assuming that the impact is given at $t = 0$, we find (from Eq. (4.26)) the response of the system as

$$x_1(t) = F\frac{e^{-\zeta\omega_n t}}{m\omega_d}\sin\omega_d t$$

$$= \frac{20}{(5)(19.975)}e^{-0.05(20)t}\sin 19.975t = 0.20025e^{-t}\sin 19.975t \text{ m} \tag{E.1}$$

EXAMPLE 4.8

Response of a Structure Under Double Impact

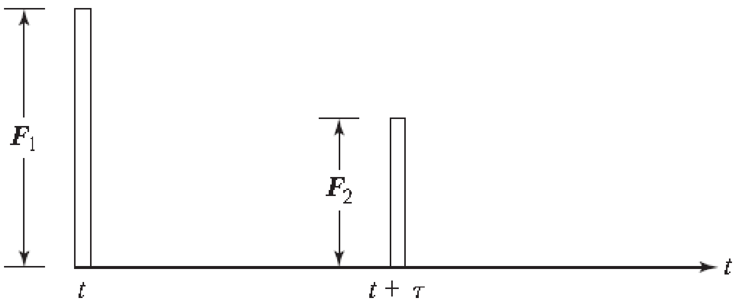
In many cases, providing only one impact to the structure using an impact hammer is difficult. Sometimes a second impact takes place after the first, as shown in Fig. 4.8(b), and the applied force, $F(t)$, can be expressed as

$$F(t) = F_1\,\delta(t) + F_2\,\delta(t - \tau)$$

where $\delta(t)$ is the Dirac delta function and τ indicates the time between the two impacts of magnitudes F_1 and F_2. For a structure with $m = 5$ kg, $k = 2000$ N/m, $c = 10$ N-s/m and $F(t) = 20\,\delta(t) + 10\,\delta(t - 0.2)$ N, find the response of the structure.

Solution: From the known data, we find $\omega_n = 20$ rad/s (see the solution for Example 4.7), $\zeta = 0.05$, and $\omega_d = 19.975$ rad/s. The response due to the impulse $F_1\,\delta(t)$ is given by Eq. (E.1) of Example 4.7, while the response due to the impulse $F_2\,\delta(t - 0.2)$ can be determined from Eqs. (4.27) and (4.26) as

$$x_2(t) = F_2 \frac{e^{-\zeta\omega_n(t-\tau)}}{m\omega_d} \sin \omega_d (t| - \tau) \qquad \text{(E.1)}$$



1. (E.1) becomes

$$x_2(t) = \frac{10}{(5)(19.975)} e^{-0.05(20)(t-0.2)} \sin 19.975(t - 0.2)$$

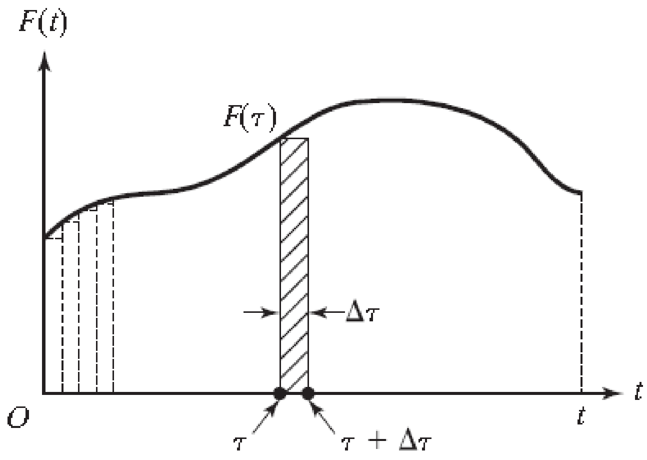
$$= 0.100125e^{-(t-0.2)} \sin 19.975(t - 0.2); \qquad t > 0.2 \qquad \text{(E.2)}$$

Using the superposition of the two responses $x_1(t)$ and $x_2(t)$, the response due to two impacts, in meters, can be expressed as

$$x(t) = \begin{Bmatrix} 0.20025e^{-t} \sin 19.975t;\ 0 \le t \le 0.2 \\ 0.20025e^{-t} \sin 19.975t + 0.100125e^{-(t-0.2)} \sin 19.975(t - 0.2);\ t > 0.2 \end{Bmatrix} \qquad \text{(E.3)}$$

Response to a General Forcing Condition

since the impulse is $t - \tau$, so the response of the system at t due to this impulse alone is given by Eq. (4.27) with $\boldsymbol{F} = F(\tau)\,\Delta\tau$:



$$\Delta x(t) = F(\tau)\,\Delta\tau\, g(t - \tau)$$

$$x(t) \simeq \sum F(\tau) g(t - \tau)\,\Delta\tau$$

Letting $\Delta\tau \rightarrow 0$ and replacing the summation by integration, we obtain

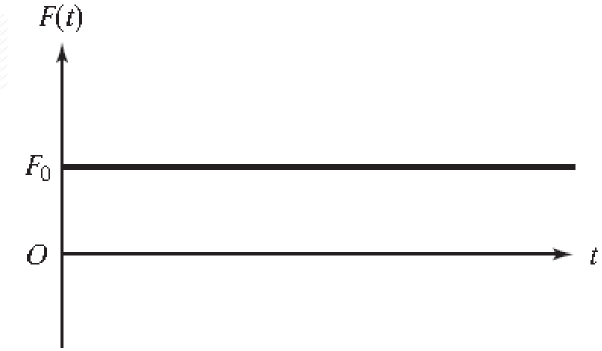
$$x(t) = \int_0^t F(\tau) g(t - \tau)\, d\tau$$

By substituting Eq. (4.25) into Eq. (4.30), we obtain

$$x(t) = \frac{1}{m\omega_d} \int_0^t F(\tau)\, e^{-\zeta\omega_n(t-\tau)} \sin \omega_d(t - \tau)\, d\tau$$

EXAMPLE 4.9 Step Force on a Compacting Machine

A compacting machine, modeled as a single-degree-of-freedom system, is shown in Fig. 4.10(a). The force acting on the mass m (m includes the masses of the piston, the platform, and the material being compacted) due to a sudden application of the pressure can be idealized as a step force, as shown in Fig. 4.10(b). Determine the response of the system.

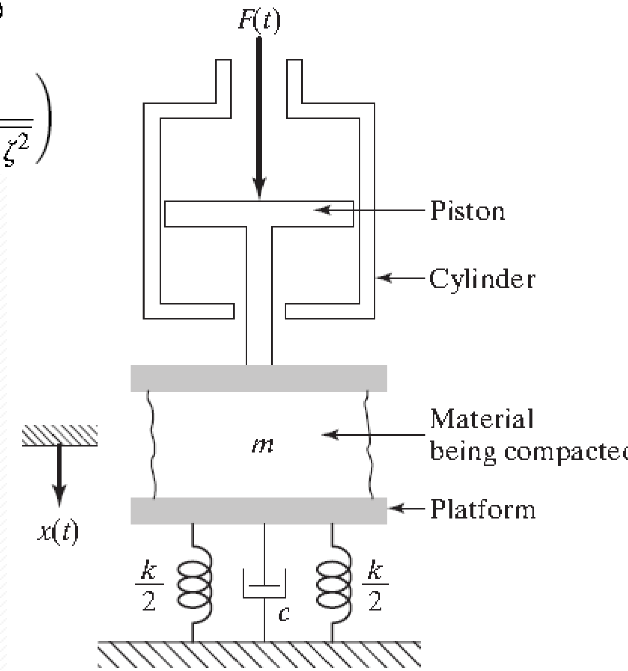


noting that $F(t) = F_0$, we can write Eq. (4.31) as

$$x(t) = \frac{F_0}{m\omega_d}\int_0^t e^{-\zeta\omega_n(t-\tau)}\sin\omega_d(t-\tau)\,d\tau$$

$$= \frac{F_0}{m\omega_d}\left[e^{-\zeta\omega_n(t-\tau)}\left\{\frac{\zeta\omega_n\sin\omega_d(t-\tau)+\omega_d\cos\omega_d(t-\tau)}{(\zeta\omega_n)^2+(\omega_d)^2}\right\}\right]_{\tau=0}^{t}$$

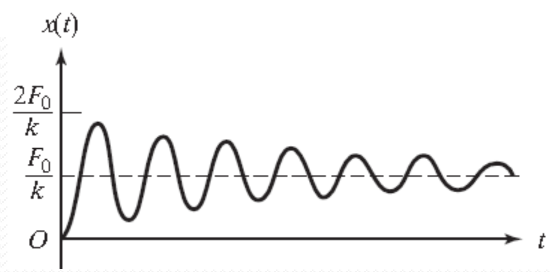
$$= \frac{F_0}{k}\left[1-\frac{1}{\sqrt{1-\zeta^2}}\cdot e^{-\zeta\omega_n t}\cos(\omega_d t-\phi)\right] \qquad \phi = \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)$$



This response is shown in Fig. 4.10(c). If the system is undamped ($\zeta = 0$ and $\omega_d = \omega_n$), Eq. (E.1) reduces to

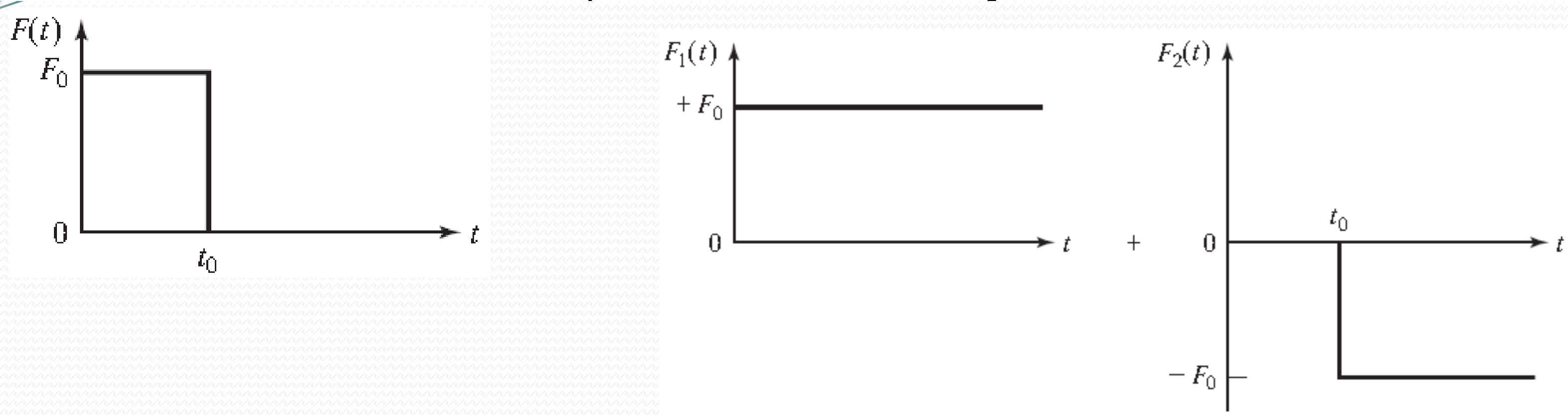
$$x(t) = \frac{F_0}{k}[1-\cos\omega_n t] \tag{E.3}$$

Equation (E.3) is shown graphically in Fig. 4.10(d). It can be seen that if the load is instantaneously applied to an undamped system, a maximum displacement of twice the static displacement will be attained—that is, $x_{max} = 2F_0/k$.



EXAMPLE 4.11 Rectangular Pulse Load

If the compacting machine shown in Fig. 4.10(a) is subjected to a constant force only during the time $0 \leq t \leq t_0$ (Fig. 4.12a), determine the response of the machine.



Solution: The given forcing function, $F(t)$, can be considered as the sum of a step function $F_1(t)$ of magnitude $+F_0$ beginning at $t = 0$ and a second step function $F_2(t)$ of magnitude $-F_0$ starting at time $t = t_0$, as shown in Fig. 4.12(b).

Thus the response of the system can be obtained by subtracting Eq. (E.1) of Example 4.10 from Eq. (E.1) of Example 4.9. This gives

$$x(t) = \frac{F_0 e^{-\zeta\omega_n t}}{k\sqrt{1-\zeta^2}}\left[-\cos(\omega_d t - \phi) + e^{\zeta\omega_n t_0}\cos\{\omega_d(t - t_0) - \phi\}\right] \qquad \phi = \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)$$

EXAMPLE 4.13

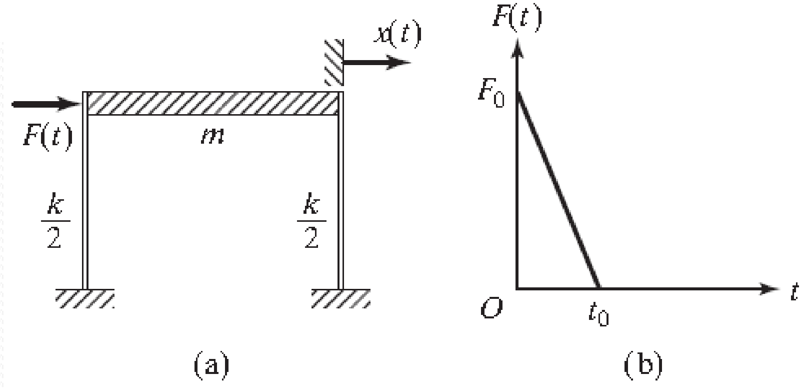
Blast Load on a Building Frame

A building frame is modeled as an undamped single-degree-of-freedom system (Fig. 4.14(a)). Find the response of the frame if it is subjected to a blast loading represented by the triangular pulse shown in Fig. 4.14(b).

$$F(\tau) = F_0\left(1 - \frac{\tau}{t_0}\right) \quad \text{for } 0 \le \tau \le t_0$$

$$F(\tau) = 0 \quad \text{for } \tau > t_0$$

$$x(t) = \frac{1}{m\omega_n}\int_0^t F(\tau)\sin\omega_n(t-\tau)\,d\tau$$



$$x(t) = \frac{F_0}{k}\left[1 - \frac{t}{t_0} - \cos\omega_n t + \frac{1}{\omega_n t_0}\sin\omega_n t\right]$$

Response during $t > t_0$: Here also we use Eq. (E.1) for $F(\tau)$, but the upper limit of integration in Eq. (E.3) will be t_0, since $F(\tau) = 0$ for $\tau > t_0$. Thus the response can be found from Eq. (E.7) by setting $t = t_0$ within the square brackets. This results in

$$x(t) = \frac{F_0}{k\omega_n t_0}\left[(1 - \cos\omega_n t_0)\sin\omega_n t - (\omega_n t_0 - \sin\omega_n t_0)\cos\omega_n t\right] \tag{E.9}$$