Mechanical Vibrations

Lecture 2

1.7 Spring Elements

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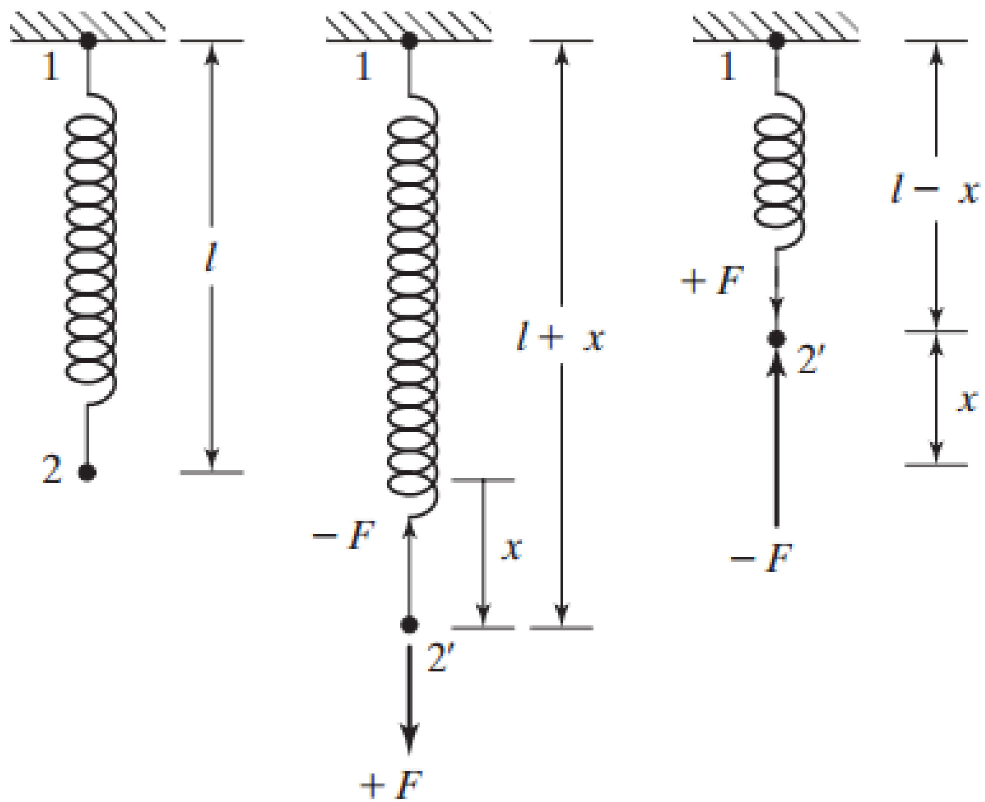
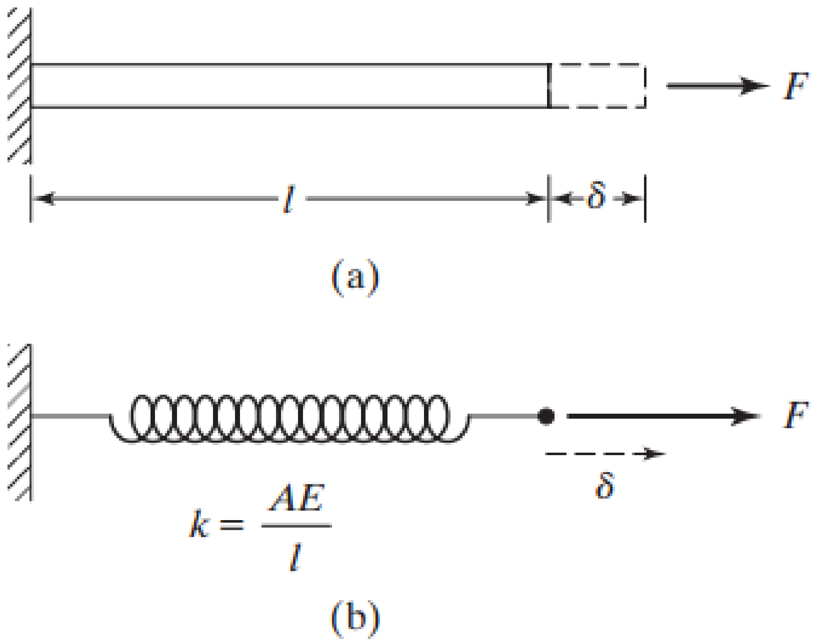


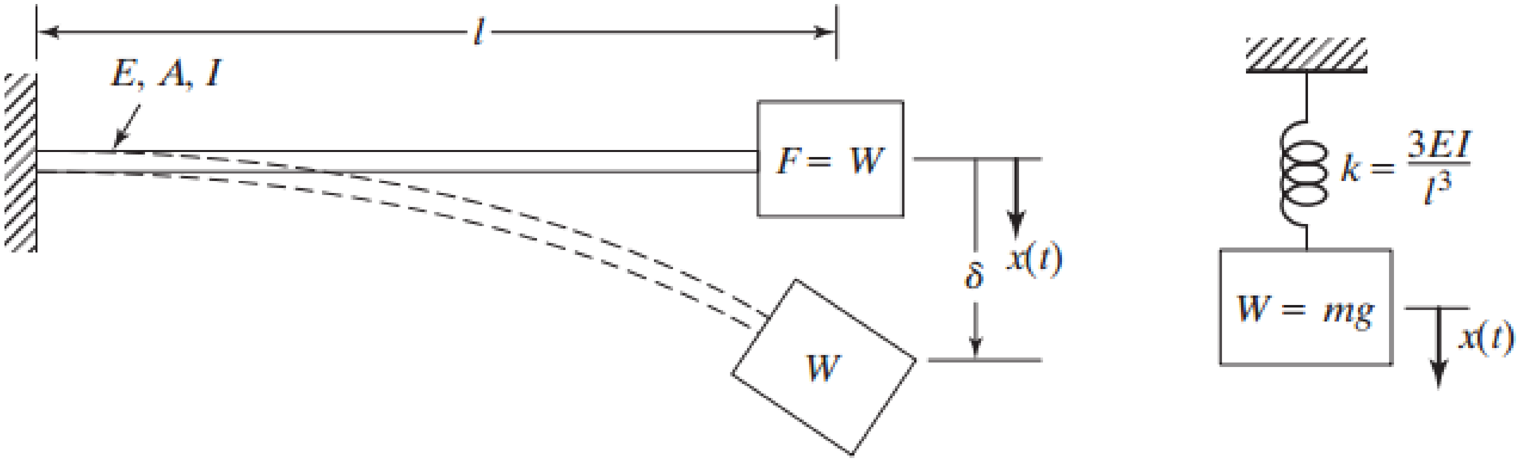
(a) (b) (c)

Spring Constant of a Rod

Find the equivalent spring constant of a uniform rod of length l, cross-sectional area A, and Young's modulus E subjected to an axial tensile (or compressive) force F as shown in Fig. 1.24(a).



$$k = \frac{W}{\delta} = \frac{3EI}{l^3} \quad \text{(E.2)}$$

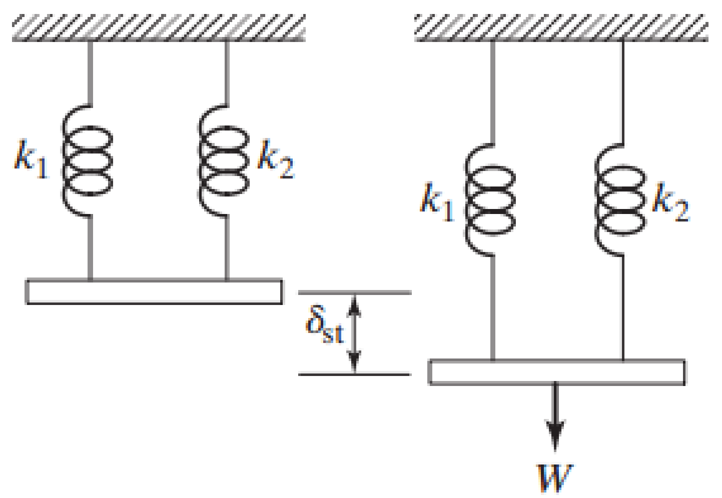


(a) Cantilever with end force

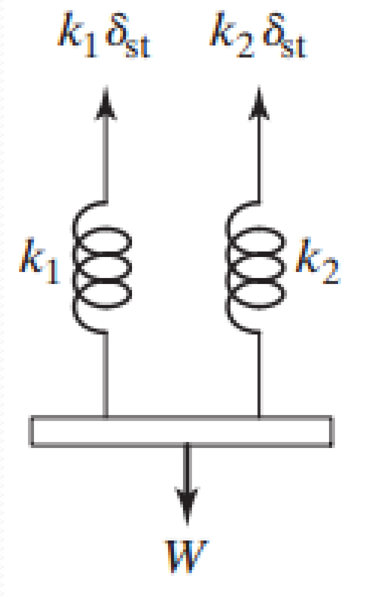
(b) Equivalent spring

Combination of Springs

Case 1: Springs in Parallel

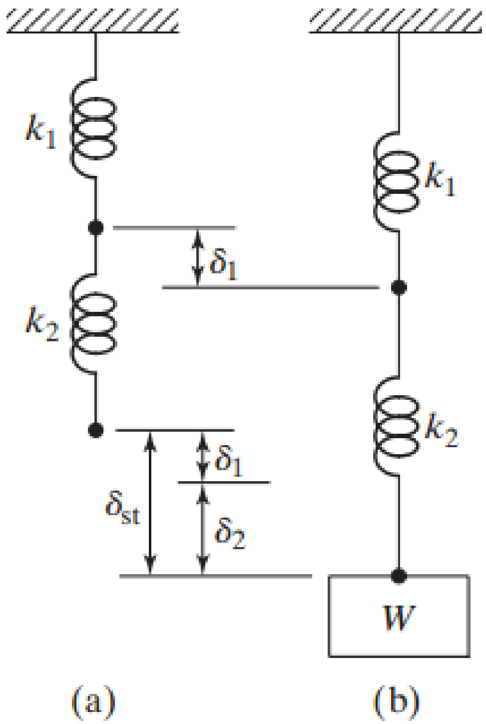


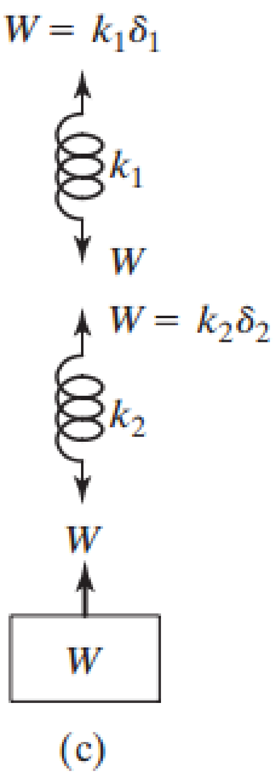
(a) (b)



$$W = k_1\delta_{st} + k_2\delta_{st}$$

Case 2: Springs in Series.



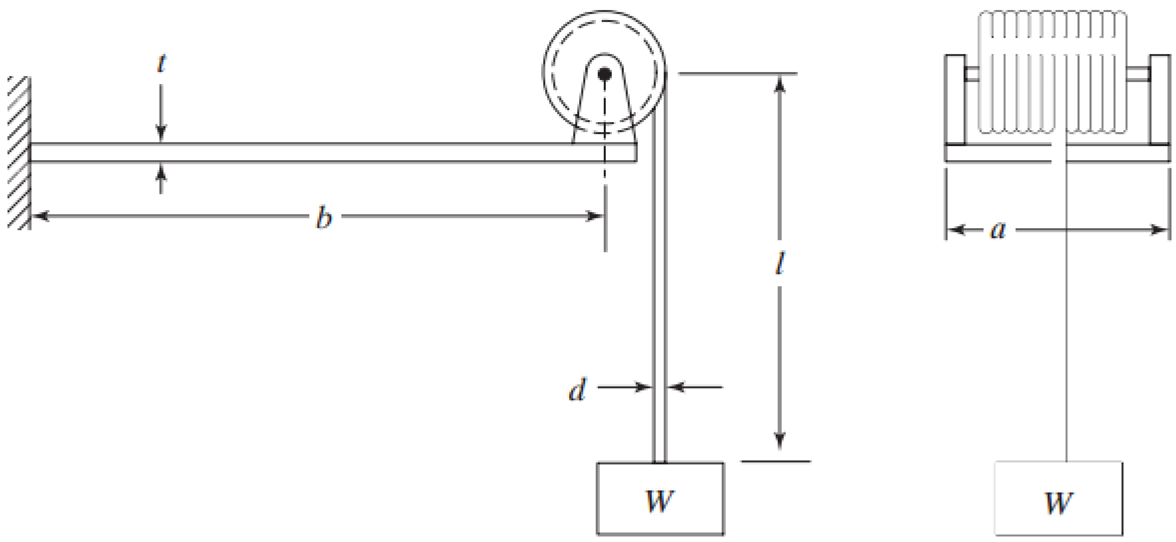


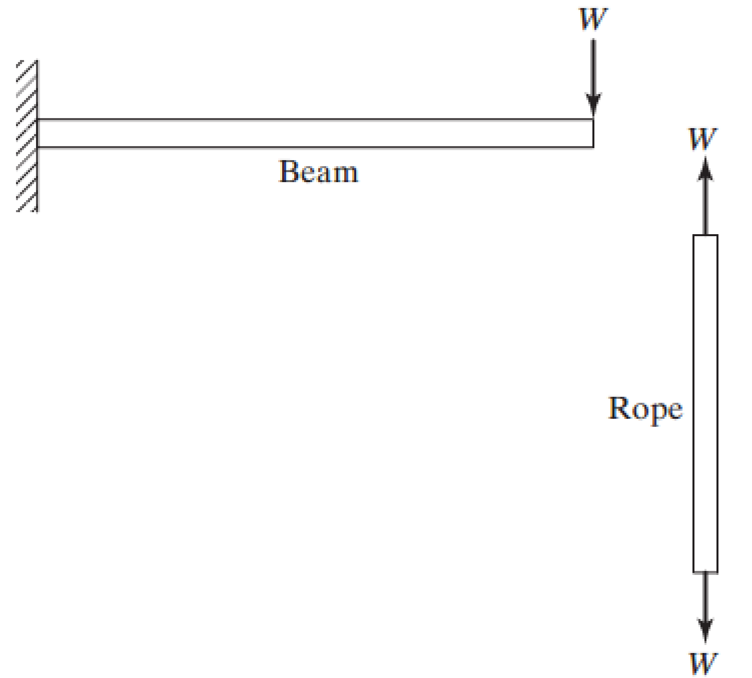
$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

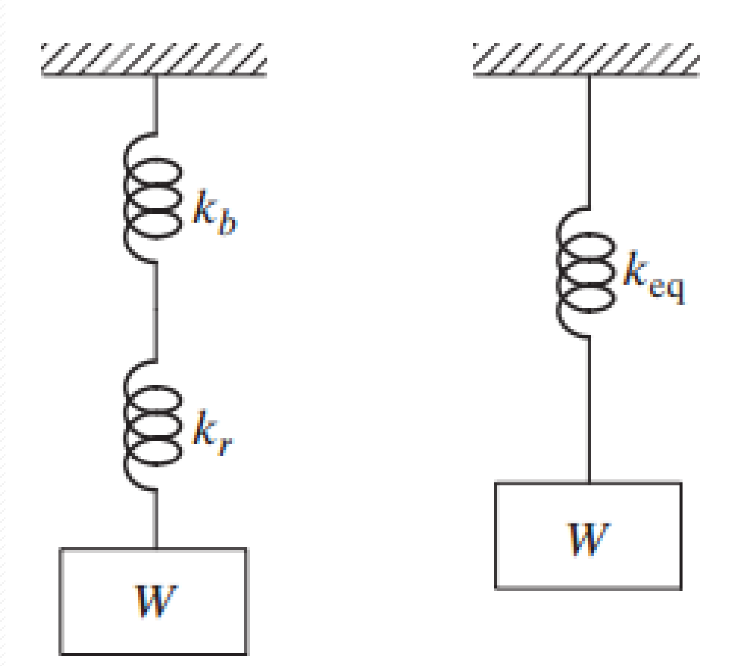
EXAMPLE 1.7

Equivalent *k* of Hoisting Drum

A hoisting drum, carrying a steel wire rope, is mounted at the end of a cantilever beam as shown in Fig. 1.31 (a). Determine the equivalent spring constant of the system when the suspended length of the wire rope is l. Assume that the net cross-sectional diameter of the wire rope is d and the Young's modulus of the beam and the wire rope is E.



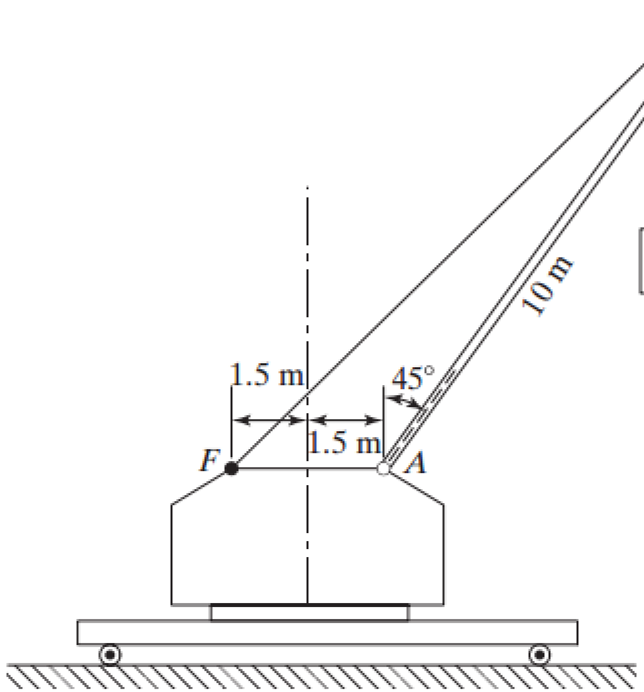

W
Beam
W
Rope
W

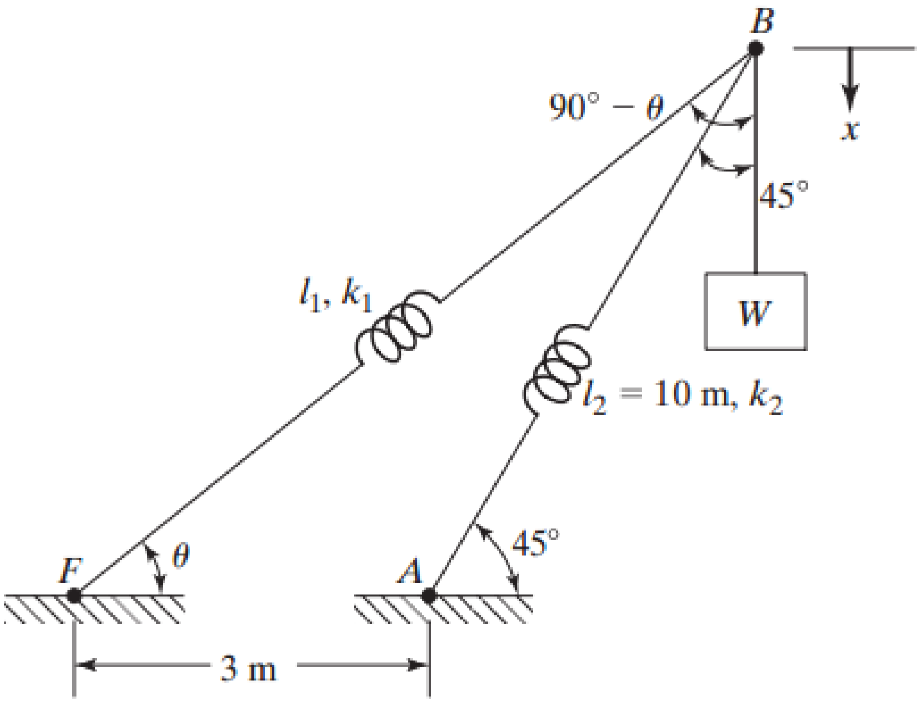

k_b
k_r
W
k_{eq}
W

EXAMPLE 1.8

Equivalent k of a Crane

The boom AB of the crane shown in Fig. 1.32(a) is a uniform steel bar of length 10 m and area of cross section 2,500 mm^2. A weight W is suspended while the crane is stationary. The cable $CDEBF$ is made of steel and has a cross-sectional area of 100 mm^2. Neglecting the effect of the cable $CDEB$, find the equivalent spring constant of the system in the vertical direction.





A vertical displacement x of point B will cause the spring (boom) to deform by an amount and the spring (cable) to deform by an amount The length of the cable FB, l_1, is given by Fig. 1.32(b):

$$l_1^2 = 3^2 + 10^2 - 2(3)(10)\cos 135^\circ = 151.426, \quad l_1 = 12.3055 \text{ m}$$

The angle θ satisfies the relation

$$l_1^2 + 3^2 - 2(l_1)(3)\cos\theta = 10^2, \quad \cos\theta = 0.8184, \quad \theta = 35.0736^\circ$$

The total potential energy (U) stored in the springs k_1 and k_2 can be expressed, using Eq. (1.2) as

$$U = \frac{1}{2}k_1[x\cos(90^\circ - \theta)]^2 + \frac{1}{2}k_2[x\cos(90^\circ - 45^\circ)]^2 \qquad \text{(E.1)}$$

where

$$k_1 = \frac{A_1E_1}{l_1} = \frac{(100\times10^{-6})(207\times10^9)}{12.3055} = 1.6822\times10^6 \text{ N/m}$$

and

$$k_2 = \frac{A_2E_2}{l_2} = \frac{(2500\times10^{-6})(207\times10^9)}{10} = 5.1750\times10^7 \text{ N/m}$$

Since the equivalent spring in the vertical direction undergoes a deformation x, the potential energy of the equivalent spring (U_{eq}) is given by

$$U_{eq} = \tfrac{1}{2}k_{eq}x^2 \qquad \text{(E.2)}$$

By setting $U = U_{eq}$, we obtain the equivalent spring constant of the system as

$$k_{eq} = k_1\sin^2\theta + k_2\sin^2 45^\circ = k_1\sin^2 35.0736^\circ + k_2\sin^2 45^\circ = 26.4304\times10^6 \text{ N/m}$$