Soil Properties

Physical Properties
- Gradation and Structure
- Soil-Water Relationships
- Atterberg’s Limits
- Soil Compaction

Mechanical Properties
- Compressibility
- Shear Strength
- Bearing Capacity
- Permeability
Permeability is the measure of the soil’s ability to permit water to flow through its pores or voids.

Figure 6.1 Pressure, elevation, and total heads for flow of water through soil.
Applications

- Rate of settlement under load
- Dams
- Stability of slopes
- Filters
Permeability and seepage

- Soils are assemblages of solid particles with interconnected voids through which water can flow.
- The study of the flow of water through porous soil media is important in soil mechanics.
Permeability and seepage

• For example:
  • Pumping of water for underground constructions
  • Stability analysis of earth dams
  • Earth retaining structures subjected to seepage forces.
What is permeability?

A measure of how easily a fluid (e.g., water) can pass through a porous medium (e.g., soils)

Loose soil
- easy to flow
- high permeability

Dense soil
- difficult to flow
- low permeability
Hydraulic gradient \( i \)

Total head loss per unit length between A and B

\[
i = \frac{TH_A - TH_B}{l_{AB}}
\]

length AB, along the stream line
Hydraulic Gradient

\[ i = \frac{\Delta h}{L} \]
Bernoulli’s Equation

The energy of a fluid particle is made of:

1. Kinetic energy
   - due to velocity

2. Strain energy
   - due to pressure

3. Potential energy
   - due to elevation \((z)\) with respect to a datum
Bernoulli’s Equation

If the equation is applied to the flow of water through porous soil medium, the term containing the velocity head can be neglected since the seepage velocity is small.

Total head = Pressure head + Elevation head
Bernoulli’s equation

\[ H = \frac{u}{\gamma} + \frac{v^2}{2g} + z \]

- **Total Head**
- **Dynamic Head**
- **Elevation Head**

**H**: total head

**P**: water pressure

**γ**: unit weight of water

**v**: velocity of water

**g**: gravity acceleration

**Z**: elevation head
Bernoulli’s equation

The seepage flow velocity in soil is very small.

Therefore, the dynamic head (velocity head) can be neglected.

So that the total head at any points is:

\[ H = \frac{u}{\gamma} + Z \]
\[ \Delta h = h_A - h_B = \left( \frac{u_A}{\gamma_w} + Z_A \right) - \left( \frac{u_B}{\gamma_w} + Z_B \right) \]

The head loss, \( \Delta h \), can be expressed in a nondimensional form as

\[ i = \frac{\Delta h}{L} \]
1. Laminar flow zone (Zone I)
2. Transition zone (Zone II)
3. Turbulent flow zone (Zone III)
Darcy’s Law

\[ v \propto i \]

\[ v = -ki \]

Negative sign refers to the hydraulic gradient that is negative. (i.e. total head decreases in the direction of flow)
Darcy’s Law

Darcy (1856) found an equation for the discharge velocity of water through saturated soils:

\[ v = k \ i \]

*Permeability*

- or hydraulic conductivity
- unit of velocity (cm/s)
For coarse grain soils, \( k = f (e \text{ or } D_{10}) \)
Darcy's Velocity

\[ v = -ki \]

Flow Rate

\[ q = -kiA \]
## Hydraulic Conductivity

### Typical Values for Hydraulic Conductivity

<table>
<thead>
<tr>
<th>Soil</th>
<th>cm/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean gravel</td>
<td>100-1.0</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>1.0-0.01</td>
</tr>
<tr>
<td>Fine sand</td>
<td>0.01-0.001</td>
</tr>
<tr>
<td>Silty clay</td>
<td>0.001-0.00001</td>
</tr>
<tr>
<td>Clay</td>
<td>&lt;0.000001</td>
</tr>
</tbody>
</table>
Seepage Velocity (True Velocity)

\[ q = vA = A_v v_s \]

\[ q = v(A_v + A_s) = A_v v_s \]

\[ v_s = \frac{v(A_v + A_s)}{A_v} = \frac{v(A_v + A_s)L}{A_v L} = \frac{v(V_v + V_s)}{V_v} \]

\[ v_s = v \left[ 1 + \left( \frac{V_v}{V_s} \right) \right] = v \left( \frac{1 + e}{e} \right) = \frac{v}{n} \]

Seepage Velocity \[ = v_s = \frac{V}{n} \]
Hydraulic Conductivity

Hydraulic conductivity of soils depends on several factors:

- Fluid viscosity
- Pore size distribution
- Grain size distribution
- Void ratio
- Degree of soil saturation
Hydraulic Conductivity

\[ k = \frac{\gamma_w}{\eta} \bar{K} \]

- \( \gamma_w \) = Unit weight of water
- \( \eta \) = Viscosity of water
- \( K \) = Absolute permeability (\( L^2 \))
Hydraulic Conductivity with Temp.

\[ k_{20^\circ C} = \left( \frac{\eta_{T^\circ C}}{\eta_{20^\circ C}} \right) k_{T^\circ C} \]

Table 6.2 Variation of $\eta_{T^\circ C}/\eta_{20^\circ C}$

<table>
<thead>
<tr>
<th>Temperature, $T$ ($^\circ C$)</th>
<th>$\eta_{T^\circ C}/\eta_{20^\circ C}$</th>
<th>Temperature, $T$ ($^\circ C$)</th>
<th>$\eta_{T^\circ C}/\eta_{20^\circ C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1.135</td>
<td>23</td>
<td>0.931</td>
</tr>
<tr>
<td>16</td>
<td>1.106</td>
<td>24</td>
<td>0.910</td>
</tr>
<tr>
<td>17</td>
<td>1.077</td>
<td>25</td>
<td>0.889</td>
</tr>
<tr>
<td>18</td>
<td>1.051</td>
<td>26</td>
<td>0.869</td>
</tr>
<tr>
<td>19</td>
<td>1.025</td>
<td>27</td>
<td>0.850</td>
</tr>
<tr>
<td>20</td>
<td>1.000</td>
<td>28</td>
<td>0.832</td>
</tr>
<tr>
<td>21</td>
<td>0.976</td>
<td>29</td>
<td>0.814</td>
</tr>
<tr>
<td>22</td>
<td>0.953</td>
<td>30</td>
<td>0.797</td>
</tr>
</tbody>
</table>
Determination of Coefficient of Permeability

- There are 2 standard types of laboratory tests for determining the coefficient of permeability of soils:
  - Constant head test
  - Falling head test
Determination of Coefficient of Permeability

- There are 2 standard types of laboratory tests for determining the coefficient of permeability of soils:
  - Constant head test
  - Falling head test
Determination of Hydraulic Conductivity in the Lab

**Constant Head Test**

**Falling Head Test**
Constant Head Permeameter

• Most suitable for coarse grained soils, that have high ‘k’:

![Diagram of Constant Head Permeameter]

A L

L

h_L

measuring cylinder
Constant Head Test

\[ V = q t \]

\[ V = v A t \]

\[ V = -k i A t \]

\[ V = -k \frac{h}{L} A t \]

\[ k = -V \frac{L}{A t h} \]
**Constant Head Permeameter**

- The total volume of water collected is:

\[ Q = Av t = A(ki)t \]

Where, 
- Q: volume of water collected
- A: area of cross section of the soil sample
- t: duration of collection of water

\[ i = \frac{\Delta H}{l} = \frac{h}{l} \rightarrow Q = A(k\frac{h}{l})t \rightarrow k = \frac{QL}{Aht} \]
Falling Head Permeameter

• Suitable for fine-grained soils with low ‘k’:
Falling Head Test

\[ q = k \frac{h}{L} A = -a \frac{dh}{dt} \]

\[ dt = \frac{aL}{Ak} \left( -\frac{dh}{h} \right) \]

\[ t = \frac{aL}{Ak} \log_e \frac{h_1}{h_2} \]

\[ k = 2.303 \frac{aL}{At} \log_{10} \frac{h_1}{h_2} \]
Falling Head Permeameter

\[ q = k i A = k \frac{h}{L} A = -a \frac{dh}{dt} \]

Where; \( q \): rate of flow  
\( a \): cross sectional area of the standpipe  
\( A \): cross sectional area of the soil sample

\[ k = \frac{aL}{At} \ln \frac{h_1}{h_2} = 2.3 \frac{aL}{At} \log \frac{h_1}{h_2} \]
Find the flow rate in m³/sec/m length (at right angles to the cross section shown) through the permeable soil layer shown in Figure 6.7 given $H = 8\text{m}$, $H_1 = 3\text{m}$, $h = 4\text{m}$, $L = 50\text{m}$, $\alpha = 8^\circ$, and $k = 0.08 \text{ cm/sec}$. 

Diagram:

- Impervious layer
- Arrow indicating direction of flow
- Parameters $H$, $H_1$, $h$, $L$, $\alpha$, and $k$
Solution

Hydraulic gradient \( (i) \) = \( \frac{h}{L \cos \alpha} \)

From Eqs. (6.17) and (6.18),

\[
q = k i A = k \left( \frac{h \cos \alpha}{L} \right) \left( H_1 \cos \alpha \times 1 \right)
\]

\[
= (0.08 \times 10^{-2} \text{ m/sec}) \left( \frac{4 \cos 8^\circ}{50} \right) (3 \cos 8^\circ \times 1)
\]

\[
= 0.19 \times 10^{-3} \text{ m}^3/\text{sec/m}
\]
For a variable-head permeability test, the following are given: length of specimen = 15 in., area of specimen = 3 in.\(^2\), and \(k = 0.0688\) in./min. What should be the area of the standpipe for the head to drop from 25 to 12 in. in 8 min.?

**Solution**

From Eq. (6.22),

\[
k = 2.303 \frac{aL}{At} \log_{10} \frac{h_1}{h_2}
\]

\[
0.0688 = 2.303 \left( \frac{a \times 15}{3 \times 8} \right) \log_{10} \left( \frac{25}{12} \right)
\]

\[
a = 0.15\text{ in.}\(^2\)\]
Example 7.2

For a falling-head permeability test, the following values are given:

- Length of specimen = 200 mm.
- Area of soil specimen = 1000 mm².
- Area of standpipe = 40 mm².
- Head difference at time \( t = 0 \) = 500 mm.
- Head difference at time \( t = 180 \) sec = 300 mm.

Determine the hydraulic conductivity of the soil in cm/sec.

Solution

From Eq. (7.22),

\[
k = 2.303 \frac{aL}{At} \log_{10} \left( \frac{h_1}{h_2} \right)
\]

We are given \( a = 40 \) mm², \( L = 200 \) mm, \( A = 1000 \) mm², \( t = 180 \) sec, \( h_1 = 500 \) mm, and \( h_2 = 300 \) mm,

\[
k = 2.303 \frac{(40)(200)}{(1000)(180)} \log_{10} \left( \frac{500}{300} \right)
\]

\[
= 2.27 \times 10^{-2} \text{ cm/sec}
\]
Example 7.4

A permeable soil layer is underlain by an impervious layer, as shown in Figure 7.9a. With $k = 5.3 \times 10^{-5}$ m/sec for the permeable layer, calculate the rate of seepage through it in m$^3$/hr/m width if $H = 3$ m and $\alpha = 8^\circ$. 

![Diagram of permeable and impervious layers](image)
Solution

From Figure 7.9b,

\[ i = \frac{\text{head loss}}{\text{length}} = \frac{S \tan \alpha}{\left( \frac{S}{\cos \alpha} \right)} = \sin \alpha \]

\[ q = kiA = (k)(\sin \alpha)(3 \cos \alpha) \]  

(1)

\[ k = 5.3 \times 10^{-5} \text{ m/sec} \]

\[ q = (5.3 \times 10^{-5})(\sin 8^\circ)(3 \cos 8^\circ)(3600) = 0.0789 \text{ m}^3/\text{hr/m} \]

\[ \uparrow \]

To change to

\[ \text{m/hr} \]
Find the flow rate in m³/sec/m length (at right angles to the cross section shown) through the permeable soil layer shown in Figure 7.10 given \( H = 8 \) m, \( H_1 = 3 \) m, \( h = 4 \) m, \( S = 50 \) m, \( \alpha = 8^\circ \), and \( k = 0.08 \) cm/sec.

![Flow through permeable layer](image)

**Figure 7.10** Flow through permeable layer

**Solution**

Hydraulic gradient \((i)\) = \( \frac{h}{S} \frac{S}{\cos \alpha} \)
Figure 6.9 Results of permeability tests on which Eq. (6.32) is based: (a) results for $C_u = 1-3$; (b) results for $C_u > 3$ (After Kenney, Lau, and Ofoegbu, 1984)
Figure 6.10: Hydraulic conductivity of granular soils (After U.S. Department of Navy, 1971).
Figure 6.12 Coefficient of permeability for sodium illite (Based on Olsen, 1961)
Equivalent Hydraulic Conductivity

Case I: Horizontal Flow

\[ q = v \cdot 1 \cdot H \]
\[ = v_1 \cdot 1 \cdot H_1 + v_2 \cdot 1 \cdot H_2 + v_3 \cdot 1 \cdot H_3 + \ldots + v_n \cdot 1 \cdot H_n \]
Equivalent Hydraulic Conductivity

Case I: Horizontal Flow

\[ q = v \cdot 1 \cdot H \]

\[ = v_1 \cdot 1 \cdot H_1 + v_2 \cdot 1 \cdot H_2 + v_3 \cdot 1 \cdot H_3 + \cdots + v_n \cdot 1 \cdot H_n \]

\[ v = k_{H(eq)}i_{eq}; \quad v_1 = k_{H_1}i_1; \quad v_2 = k_{H_2}i_2; \quad v_3 = k_{H_3}i_3; \quad \cdots \quad v_n = k_{H_n}i_n \]

\[ k_{H(eq)} = \frac{1}{H} (k_{H_1}H_1 + k_{H_2}H_2 + k_{H_3}H_3 + \cdots + k_{H_n}H_n) \]
Equivalent Hydraulic Conductivity

Case II: Vertical Flow

\[ v = v_1 = v_2 = v_3 = \cdots = v_n \]

\[ h = h_1 + h_2 + h_3 + \cdots + h_n \]

\[ k_{V(eq)} \left( \frac{h}{H} \right) = k_{V_1}i_1 = k_{V_2}i_2 = k_{V_3}i_3 = \cdots = k_{V_n}i_n \]
Equivalent Hydraulic Conductivity

Case II: Vertical Flow

\[ v = v_1 = v_2 = v_3 = \ldots = v_n \]

\[ h = h_1 + h_2 + h_3 + \ldots + h_n \]

\[ k_{V(eq)} \left( \frac{h}{H} \right) = k_{V_1} i_1 = k_{V_2} i_2 = k_{V_3} i_3 = \ldots = k_{V_n} i_n \]

\[ h = H_1 i_1 + H_2 i_2 + H_3 i_3 + \ldots + H_n i_n \]

\[ k_{V(eq)} = \frac{H}{ \left( \frac{H_1}{k_{V_1}} \right) + \left( \frac{H_2}{k_{V_2}} \right) + \left( \frac{H_3}{k_{V_3}} \right) + \ldots + \left( \frac{H_n}{k_{V_n}} \right) } \]
Example 7.12

A layered soil is shown in Figure 7.24. Given:

- $H_1 = 1.5$ m $k_1 = 10^{-4}$ cm/sec
- $H_2 = 3$ m $k_2 = 3.2 \times 10^{-2}$ cm/sec
- $H_3 = 2$ m $k_3 = 4.1 \times 10^{-5}$ cm/sec

Estimate the ratio of equivalent hydraulic conductivity,

$$\frac{k_{H(eq)}}{k_{V(eq)}}$$

$$k_{H(eq)} = \frac{1}{H} (k_{H_1}H_1 + k_{H_2}H_2 + k_{H_3}H_3)$$

$$= \frac{1}{(1.5 + 3 + 2)} \left[ (10^{-4}) (1.5) + (3.2 \times 10^{-2}) (3) + (4.1 \times 10^{-5}) (2) \right]$$

$$= 148.05 \times 10^{-4} \text{ cm/sec}$$

$$k_{V(eq)} = \frac{H}{\left( \frac{H_1}{k_{V_1}} \right) + \left( \frac{H_2}{k_{V_2}} \right) + \left( \frac{H_3}{k_{V_3}} \right)}$$

$$= \frac{1.5}{10^{-4}} + \frac{3}{3.2 \times 10^{-2}} + \frac{2}{4.1 \times 10^{-5}}$$

$$= 1.018 \times 10^{-4} \text{ cm/sec}$$

$$\frac{k_{H(eq)}}{k_{V(eq)}} = \frac{148.05 \times 10^{-4}}{1.018 \times 10^{-4}} = 145.4$$
Example 7.13

Figure 7.25 shows three layers of soil in a tube that is 100 mm $\times$ 100 mm in cross section. Water is supplied to maintain a constant-head difference of 300 mm across the sample. The hydraulic conductivities of the soils in the direction of flow through them are as follows:

<table>
<thead>
<tr>
<th>Soil</th>
<th>$k$ (cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>B</td>
<td>$3 \times 10^{-3}$</td>
</tr>
<tr>
<td>C</td>
<td>$4.9 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Find the rate of water supply in cm$^3$/hr.

Solution

From Eq. (7.45),

$$k_{V(eq)} = \frac{H}{\left(\frac{H_1}{k_1}\right) + \left(\frac{H_2}{k_2}\right) + \left(\frac{H_3}{k_3}\right)} = \frac{450}{\left(\frac{150}{10^{-2}}\right) + \left(\frac{150}{3 \times 10^{-3}}\right) + \left(\frac{150}{4.9 \times 10^{-4}}\right)}$$

$$= 0.001213 \text{ cm/sec}$$

$$q = k_{V(eq)}iA = (0.001213)\left(\frac{300}{450}\right)\left(\frac{100}{10} \times \frac{100}{10}\right)$$

$$= 0.0809 \text{ cm}^3/\text{sec} = 291.24 \text{ cm}^3/\text{hr}$$
Figure 6.25 Pumping test from a well in an unconfined permeable layer underlain by an impermeable stratum.
Figure 6.26  Pumping test from a well penetrating the full depth in a confined aquifer
\[ q = k \left( \frac{dh}{dr} \right) 2\pi rh \]

\[ \int_{r_2}^{r_1} \frac{dr}{r} = \left( \frac{2\pi k}{q} \right) \int_{h_2}^{h_1} h \, dh \]

\[ k = \frac{2.303q \log_{10} \left( \frac{r_1}{r_2} \right)}{\pi (h_1^2 - h_2^2)} \]
Pumped well in confined aquifer

- Elevation
- Aquifer heads
- Pumped well
- Observation well
- Radial flow
- Impermeable stratum
• Pumped well in **confined** aquifer

![Diagram of a confined aquifer with pumped well and observation well, showing elevation, aquifer heads, radial flow, and impermeable stratum.](image-url)
This gives the hydraulic conductivity in the direction of flow as

\[ k = \frac{q \log_{10} \left( \frac{r_1}{r_2} \right)}{2.727H(h_1 - h_2)} \]  

(7.50)

\[ q = k \left( \frac{dh}{dr} \right) 2\pi r H \]  

or

\[ \int_{r_2}^{r_1} \frac{dr}{r} = \int_{h_2}^{h_1} \frac{2\pi k H}{q} dh \]  

(7.49)
This gives the hydraulic conductivity in the direction of flow as

\[
k = \frac{q \log_{10} \left( \frac{r_1}{r_2} \right)}{2.727H(h_1 - h_2)}
\]
Permeability & Seepage
Seepage Terminology

- Concrete dam
- Impervious strata

$$TH = h_L$$

Datum

$$TH = 0$$

Soil

Impervious strata
**Equipotential line** is simply a contour of constant total head.
Stresses due to Flow

Static Situation (No flow)

\[ \sigma_v = \gamma_w h_w + \gamma_{sat} z \]
\[ u = \gamma_w (h_w + z) \]
\[ \sigma_v' = \gamma' z \]
Stresses due to Flow

Downward Flow

At \( X \),

\[
\sigma_v = \gamma_w h_w + \gamma_{sat} z \\
\]

... as for static case

\[
\begin{align*}
u &= \gamma_w h_w + \gamma_w (L-h_L)(z/L) \\
&= \gamma_w h_w + \gamma_w (z-i z) \\
&= \gamma_w (h_w+z) - \gamma_w i z
\end{align*}
\]

Reduction due to flow

\[
\sigma'_v = \gamma' z + \gamma_w i z
\]

Increase due to flow

\[
\begin{align*}
u &= \gamma_w (h_w+L-h_L)
\end{align*}
\]

\( u = \gamma_w h_w \)

\( \text{flow} \)
Stresses due to Flow

Upward Flow

At X,

\[ \sigma_v = \gamma_w h_w + \gamma_{sat} z \]

... as for static case

\[ u = \gamma_w h_w + \gamma_w (L+h_L)(z/L) \]

\[ = \gamma_w h_w + \gamma_w (z+iz) \]

\[ = \gamma_w (h_w+z) + \gamma_w iz \]

Increase due to flow

\[ \sigma'_v = \gamma' z - \gamma_w iz \]

Reduction due to flow
Flow nets

- Flow nets are useful in the study of seepage through porous media.

- A flow net consists of 2 sets of lines which for an isotropic (having equal properties in all directions) material are mutually perpendicular.
Flow nets

• Lines drawn in the direction of flow are called flow lines ($\Psi$ — lines)

• Those perpendicular to the flow lines are called equipotential lines ($\Phi$ — lines)
Stream line is simply the path of a water molecule.

From upstream to downstream, total head steadily decreases along the stream line.
**Equipotential line** is simply a contour of constant total head.
Flow net

A network of selected stream lines and equipotential lines.

curvilinear square

90°

concrete dam

soil

impervious strata
Sketching of Flow Nets

• For practical purposes, flow nets can be obtained by sketching. The boundary conditions must be satisfied.

• An impermeable surface represents a flow line

• The interface between water and a porous medium is an equipotential line.
• **The procedure is:**

  – Draw geometry of the problem to a CHOSEN SCALE

  – Prepare a first sketch of flow net trying to form equal sided figures and satisfying the boundary conditions

  – Prepare an improved new sketch of flow net

  – Correct the second sketch locally

  !!!EXAMPLE!!!
Calculation of Seepage Loss through or beneath dams

• Let :

Nd : Total number of equal drops in head

Nf : Total number of flow channels

Δl : side of a typical “square” of the flownet

k : coefficient of permeability

h : total drop in head between first and last lines
Quantity of Seepage (Q)

\[ Q = k h_L \frac{N_f}{N_d} \]

- \( N_f \) \# of flow channels
- \( N_d \) \# of equipotential drops
- \( h_L \) head loss from upstream to downstream

...per unit length normal to the plane

head loss from upstream to downstream
Heads at a Point X

Total head = $h_L$ - # of drops from upstream $\times \Delta h$

Elevation head = $-z$

Pressure head = Total head – Elevation head

$\Delta h = \frac{h_L}{N_d}$

$TH = h_L$

Elevation head = $-z$

Pressure head = Total head – Elevation head

$TH = 0$

Concrete dam

Impervious strata

Datum
Piping in **Granular Soils**

At the downstream, near the dam, the exit hydraulic gradient

\[ i_{exit} = \frac{\Delta h}{\Delta l} \]

\( \Delta h \) = total head drop

\( \Delta l \) = exit hydraulic gradient
Piping in **Granular Soils**

If $i_{exit}$ exceeds the critical hydraulic gradient ($i_c$), firstly the soil grains at exit get washed away.

This phenomenon progresses towards the upstream, forming a free passage of water ("pipe").
Piping in **Granular** Soils

Piping is a very serious problem. It leads to downstream flooding which can result in loss of lives.

Therefore, provide adequate safety factor against piping.

\[ F_{piping} = \frac{i_c}{i_{exit}} \]

typically 5-6
Baldwin Hills Dam after it failed by piping in 1963. The failure occurred when a concentrated leak developed along a crack in the embankment, eroding the embankment fill and forming this crevasse. An alarm was raised about four hours before the failure and thousands of people were evacuated from the area below the dam. The flood that resulted when the dam failed and the reservoir was released caused several millions of dollars in damage.
Piping Failures

Fontenelle Dam, USA (1965)
Quick condition

- When flow is in the upward direction an unstable “quick condition” or boiling occurs when the upward seepage force per unit volume reaches the submerged unit weight of the soil.

- This condition usually occurs in sands.

- Determination of submerged unit weight of soil is needed by using the mass-volume relationships. (Lecture 1)
Quick Condition in **Granular Soils**

During upward flow, at X:

\[
\sigma'_v = \gamma^' z - \gamma_w i z
\]

\[
= \gamma_w z \left\{ \frac{\gamma^'}{\gamma_w} - i \right\}
\]

Critical hydraulic gradient \((i_c)\)

If \(i > i_c\), the effective stresses is negative.

i.e., no inter-granular contact & thus failure.

- Quick condition
• Trench supported by sheet piles

- 6m
- 6m
- 5m
- Uniform sand
- Impermeable clay
• Trench supported by sheet piles
• Trench supported by sheet piles

\[ \Delta h = 6 \text{m} \]
\[ N_h = 10 \]
\[ N_f = 2.5 + 2.5 \]

Uniform sand

Impermeable clay
• Excavation supported by a sheet pile

Steel sheet

Water pumped away

Uniform sand

Shale
• Excavation supported by a sheet pile

- Steel sheet
- Water pumped away
- Uniform sand
- Shale
• Reduced sheet penetration; possible liquefaction $\sigma'_v = 0$
• Reduced sheet penetration; possible liquefaction $\sigma'_v = 0$
• Concrete dam or weir
- Concrete dam with cut-off; reduces uplift pressure
• Concrete dam with cut-off; reduces uplift pressure
• Pumped well in **confined** aquifer
• Pumped well in **confined** aquifer
• Clay dam, no air entry
• Clay dam, no air entry
• Clay dam, no air entry
- Clay dam, no air entry, reduced drain; seepage out of downstream face
Clay dam, with air entry

- reservoir
- clay
- Shale
- drain
• Clay dam, with air entry
- Clay dam, no capillary, reduced drain; seepage out of downstream face
- Clay dam, no capillary, reduced drain; seepage out of downstream face
Flow of water in earth dams

- The drain in a rolled clay dam will be made of gravel, which has an effectively infinite hydraulic conductivity compared to that of the clay, so far a finite quantity of flow in the drain and a finite area of drain the hydraulic gradient is effectively zero, i.e. the drain is an equipotential
Flow of water in earth dams

- The phreatic surface connects points at which the pressure head is zero. Above the phreatic surface the soil is in suction, so we can see how much capillarity is needed for the material to be saturated. If there is insufficient capillarity, we might discard the solution and try again. Alternatively: assume there is zero capillarity, the top water boundary is now atmospheric so along it and the flow net has to be adjusted within an unknown top boundary as the phreatic surface is a flow line if there is no capillarity.
Flow of water in earth dams

• If $\overline{h} = y$ then $\delta h = \delta y = \text{cons}$ in the flow net, so once we have the phreatic surface we can put on the starting points of the equipotentials on the phreatic surface directly.
Unsteady flow effects

• Consolidation of matrix
Change in pressure head within the soil due to changes in the boundary water levels may cause soil to deform, especially in compressible clays. The soil may undergo consolidation, a process in which the voids ratio changes over time at a rate determined by the pressure variation and the hydraulic conductivity, which may in turn depend on the voids ratio.
Breakdown of rigid matrix

- Liquefaction (tensile failure)

The total stress $\sigma$ normal to a plane in the soil can be separated into two components, the pore pressure $p$ and the effective inter-granular stress $\sigma'$:

$$\sigma = \sigma' + p$$

By convention in soils compressive stresses are +ve.

Tensile failure occurs when the effective stress is less than the fracture strength $\sigma'_{fracture}$, and by definition for soil $\sigma'_{fracture}=0$. When the effective stress falls to zero the soil particles are no longer in contact with each other and the soil acts like a heavy liquid. This phenomenon is called liquefaction, and is responsible to quick sands.
Large upward hydraulic gradients:

Uniform soil of unit weight $\gamma$

Upward flow of water
Uniform soil of unit weight $\gamma$

Upward flow of water

Critical potential Head $h_{\text{crit}} = h_{\text{crit}} - z$

Water table and datum

Plug of Base area $A$

Critical head Pressure $h_{\text{crit}}$

Gap opening as plug rises
At the base of the rising plug, if there is no side friction:

\[
\sigma_v = \gamma \cdot z, \quad p = h_{crit} \cdot \gamma_w
\]

So if \( \sigma_v' = 0 \) then \( \sigma_v = p \) and:

\[
\gamma \cdot z = h_{crit} \cdot \gamma_w = (\bar{h}_{crit} + z) \cdot \gamma_w
\]

\[
i_{crit} = \frac{\bar{h}_{crit}}{z} = \frac{\gamma - \gamma_w}{\gamma_w}, \quad icrit=0.8\sim1.0
\]
where $i_{crit}$ is the critical hydraulic gradient for the quick sand Condition. As $\gamma \approx 18 \sim 20$ kN/m$^3$ for many soils (especially sands and silts) and $\gamma_w \approx 10$ kN/m$^3$:

$$i_{crit} = \frac{\bar{h}_{crit}}{z} = \frac{20 - 10}{10} = 1.0$$
Frictional (shear failure)

- Sliding failure of a gravity concrete dam due to insufficient friction along the base:
Uniform sand

Reservoir

Tail water

\[ U = \int p \cdot ds \]
Stresses due to Flow

Static Situation (No flow)

At X,

\[ \sigma_v = \gamma_w h_w + \gamma_{sat} z \]

\[ u = \gamma_w (h_w + z) \]

\[ \sigma_v' = \gamma' z \]
Stresses due to Flow

Downward Flow

At X,

\[ \sigma_v = \gamma_w h_w + \gamma_{sat} z \]

... as for static case

\[ u = \gamma_w h_w + \gamma_w (L - h_L)(z/L) \]

\[ = \gamma_w h_w + \gamma_w (z - iz) \]

\[ = \gamma_w (h_w + z) - \gamma_w iz \]

Reduction due to flow

\[ \sigma_v' = \gamma' z + \gamma_w iz \]

Increase due to flow
Stresses due to Flow

Upward Flow

At X,

\[ \sigma_v = \gamma_w h_w + \gamma_{\text{sat}} z \]

... as for static case

\[ u = \gamma_w h_w + \gamma_w (L+h_L)(z/L) \]
\[ = \gamma_w h_w + \gamma_w (z+iz) \]
\[ = \gamma_w (h_w+z) + \gamma_w iz \]

\( u = \gamma_w h_w \)  
\( u = \gamma_w (h_w+L+h_L) \)

Increase due to flow

Reduction due to flow

\[ \sigma_v' = \gamma' z - \gamma_w iz \]