Waves vs. Particles

Waves are very different from particles.

<table>
<thead>
<tr>
<th>Particles have zero size.</th>
<th>Waves have a characteristic size – their wavelength.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple particles must exist at different locations.</td>
<td>Multiple waves can combine at one point in the same medium – they can be present at the same location.</td>
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Quantization

When waves are combined in systems with boundary conditions, only certain allowed frequencies can exist.

- We say the frequencies are \textit{quantized}.
- Quantization is at the heart of quantum mechanics.

The analysis of waves under boundary conditions explains many quantum phenomena.

Quantization can be used to understand the behavior of the wide array of musical instruments that are based on strings and air columns.

Waves can also combine when they have different frequencies.
Superposition Principle

Waves can be combined in the same location in space.

To analyze these wave combinations, use the superposition principle:

*If two or more traveling waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave functions of the individual waves.*

Waves that obey the superposition principle are linear waves.

- For mechanical waves, linear waves have amplitudes much smaller than their wavelengths.
Superposition and Interference

Two traveling waves can pass through each other without being destroyed or altered.

- A consequence of the superposition principle.

The combination of separate waves in the same region of space to produce a resultant wave is called **interference**.

- The term interference has a very specific usage in physics.
- It means waves pass through each other.
Superposition Example

Two pulses are traveling in opposite directions (a).

- The wave function of the pulse moving to the right is $y_1$ and for the one moving to the left is $y_2$.

The pulses have the same speed but different shapes.

The displacement of the elements is positive for both.

When the waves start to overlap (b), the resultant wave function is $y_1 + y_2$.
Superposition Example, cont

When crest meets crest (c) the resultant wave has a larger amplitude than either of the original waves.

The two pulses separate (d).

- They continue moving in their original directions.
- The shapes of the pulses remain unchanged.

This type of superposition is called \textbf{constructive interference}. 

\begin{itemize}
  \item When the crests of the two pulses align, the amplitude is the sum of the individual amplitudes.
  \item When the pulses no longer overlap, they have not been permanently affected by the interference.
\end{itemize}
Destructive Interference Example

Two pulses traveling in opposite directions.

Their displacements are inverted with respect to each other.

When these pulses overlap, the resultant pulse is $y_1 + y_2$.

This type of superposition is called **destructive interference**.

![Diagram showing destructive interference example](image-url)
Types of Interference, Summary

**Constructive interference** occurs when the displacements caused by the two pulses are in the same direction.

- The amplitude of the resultant pulse is greater than either individual pulse.

**Destructive interference** occurs when the displacements caused by the two pulses are in opposite directions.

- The amplitude of the resultant pulse is less than either individual pulse.
Analysis Model

The superposition principle is the centerpiece of the analysis model called waves in interference.

Applies in many situations

- They exhibit interesting phenomena with practical applications.

\[
\begin{align*}
\sin a + \sin b &= 2 \cos \left( \frac{a - b}{2} \right) \sin \left( \frac{a + b}{2} \right) \\
\cos a + \cos b &= 2 \cos \left( \frac{a - b}{2} \right) \cos \left( \frac{a + b}{2} \right)
\end{align*}
\]
Superposition of Sinusoidal Waves

Assume two waves are traveling in the same direction in a linear medium, with the same frequency, wavelength and amplitude.

The waves differ only in phase:

- \( y_1 = A \sin (kx - \omega t) \)
- \( y_2 = A \sin (kx - \omega t + \phi) \)
- \( y = y_1 + y_2 = 2A \cos (\phi/2) \sin (kx - \omega t + \phi/2) \)

The resultant wave function, \( y \), is also sinusoidal.

The resultant wave has the same frequency and wavelength as the original waves.

The amplitude of the resultant wave is \( 2A \cos (\phi / 2) \).

The phase of the resultant wave is \( \phi / 2 \).
Sinusoidal Waves with Constructive Interference

When $\phi = 0$, then $\cos(\phi/2) = 1$

The amplitude of the resultant wave is $2A$.
- The crests of the two waves are at the same location in space.
- The waves are everywhere in phase.
- The waves interfere constructively.

In general, constructive interference occurs when $\cos(\Phi/2) = \pm 1$.
- That is, when $\Phi = 0, 2\pi, 4\pi, \ldots$ rad
  - When $\Phi$ is an even multiple of $\pi$

The individual waves are in phase and therefore indistinguishable.
Constructive interference: the amplitudes add.
Sinusoidal Waves with Destructive Interference

When $\phi = \pi$, then $\cos (\phi/2) = 0$
- Also any odd multiple of $\pi$

The amplitude of the resultant wave is 0.
- See the straight red-brown line in the figure.

The waves interfere destructively.

The individual waves are $180^\circ$ out of phase.
Destructive interference: the waves cancel.
Sinusoidal Waves, General Interference

When $\phi$ is other than 0 or an even multiple of $\pi$, the amplitude of the resultant is between 0 and $2A$.

The wave functions still add

The interference is neither constructive nor destructive.
Sinusoidal Waves, Summary of Interference

Constructive interference occurs when $\phi = n\pi$ where $n$ is an even integer (including 0).
  - Amplitude of the resultant is $2A$

Destructive interference occurs when $\phi = n\pi$ where $n$ is an odd integer.
  - Amplitude is 0

General interference occurs when $0 < \phi < n\pi$.
  - Amplitude is $0 < A_{\text{resultant}} < 2A$
Sinusoidal Waves, Interference with Difference Amplitudes

Constructive interference occurs when $\phi = n \pi$ where $n$ is an even integer (including 0).

- Amplitude of the resultant is the sum of the amplitudes of the waves

Destructive interference occurs when $\phi = n \pi$ where $n$ is an odd integer.

- Amplitude is less, but the amplitudes do not completely cancel
Radiation from a monopole source

A monopole is a source which radiates sound equally well in all directions. The simplest example of a monopole source would be a sphere whose radius alternately expands and contracts sinusoidally. The monopole source creates a sound wave by alternately introducing and removing fluid into the surrounding area. A **boxed loudspeaker** at low frequencies acts as a monopole. The directivity pattern for a monopole source is shown in the figure at right.

The animated GIF at left shows the pressure field produced by a monopole source. Individual points on the grid simply move back and forth about some equilibrium position while the spherical wave expands outwards.
Radiation from a dipole source

A dipole source consists of two monopole sources of equal strength but opposite phase and separated by a small distance compared with the wavelength of sound. While one source expands the other source contracts. The result is that the fluid (air) near the two sources sloshes back and forth to produce the sound. A sphere which oscillates back and forth acts like a dipole source, as does an unboxed loudspeaker. A dipole source does not radiate sound in all directions equally. The directivity pattern shown at right looks like a figure-8; there are two regions where sound is radiated very well, and two regions where sound cancels. The wavefronts expanding to the right and left are 180° out of phase with each other.
Radiation from a lateral quadrupole source
The superposition principle is the centerpiece of the analysis model called \textit{waves in interference}.

Applies in many situations

- They exhibit interesting phenomena with practical applications.
Interference in Sound Waves

Sound from S can reach R by two different paths.

The distance along any path from speaker to receiver is called the **path length**, $r$.

The lower path length, $r_1$, is fixed. The upper path length, $r_2$, can be varied.

Whenever $\Delta r = |r_2 - r_1| = n \lambda$, constructive interference occurs.

- $n = 0, 1, \ldots$

A maximum in sound intensity is detected at the receiver.
Interference in Sound Waves, 2

Whenever $\Delta r = |r_2 - r_1| = (n\lambda)/2 (n \text{ is odd})$, destructive interference occurs.

No sound is detected at the receiver.

A phase difference may arise between two waves generated by the same source when they travel along paths of unequal lengths.
Example 18.1  Two Speakers Driven by the Same Source

A pair of speakers placed 3.00 m apart are driven by the same oscillator (Fig. 18.6). A listener is originally at point $O$, which is located 8.00 m from the center of the line connecting the two speakers. The listener then walks to point $P$, which is a perpendicular distance 0.350 m from $O$, before reaching the first minimum in sound intensity. What is the frequency of the oscillator?

Solution  To find the frequency, we must know the wavelength of the sound coming from the speakers. With this information, combined with our knowledge of the speed of sound, we can calculate the frequency. The wavelength can be determined from the interference information given. The first minimum occurs when the two waves reaching the listener at point $P$ are 180° out of phase—in other words, when their path difference $\Delta r$ equals $\lambda/2$. To calculate the path difference, we must first find the path lengths $r_1$ and $r_2$. 

Figure 18.6 (Example 18.1) Two speakers emit sound waves to a listener at \( P \).
Figure 18.6 shows the physical arrangement of the speakers, along with two shaded right triangles that can be drawn on the basis of the lengths described in the problem. From these triangles, we find that the path lengths are

\[ r_1 = \sqrt{(8.00 \text{ m})^2 + (1.15 \text{ m})^2} = 8.08 \text{ m} \]

and

\[ r_2 = \sqrt{(8.00 \text{ m})^2 + (1.85 \text{ m})^2} = 8.21 \text{ m} \]

Hence, the path difference is \( r_2 - r_1 = 0.13 \text{ m} \). Because we require that this path difference be equal to \( \lambda/2 \) for the first minimum, we find that \( \lambda = 0.26 \text{ m} \).

To obtain the oscillator frequency, we use Equation 16.12, \( \nu = \lambda f \), where \( \nu \) is the speed of sound in air, 343 m/s:

\[
f = \frac{\nu}{\lambda} = \frac{343 \text{ m/s}}{0.26 \text{ m}} = 1.3 \text{ kHz}
\]
Standing Waves

Assume two waves with the same amplitude, frequency and wavelength, traveling in opposite directions in a medium.

The waves combine in accordance with the waves in interference model.

\[ y_1 = A \sin (kx - \omega t) \] and \[ y_2 = A \sin (kx + \omega t) \]

They interfere according to the superposition principle.

Section 18.2
Standing Waves, cont

The resultant wave will be \( y = (2A \sin kx) \cos \omega t \).

This is the wave function of a standing wave.

- There is no \( kx - \omega t \) term, and therefore it is not a traveling wave.

In observing a standing wave, there is no sense of motion in the direction of propagation of either of the original waves.
Standing Wave Example

The amplitude of the vertical oscillation of any element of the string depends on the horizontal position of the element. Each element vibrates within the confines of the envelope function $2A \sin kx$.

Note the stationary outline that results from the superposition of two identical waves traveling in opposite directions.

The amplitude of the simple harmonic motion of a given element is $2A \sin kx$.

- This depends on the location $x$ of the element in the medium.

Each individual element vibrates at $\omega$.

Section 18.2
Note on Amplitudes

There are three types of amplitudes used in describing waves.

- The amplitude of the individual waves, \( A \)
- The amplitude of the simple harmonic motion of the elements in the medium, \( 2A \sin kx \)
  - A given element in the standing wave vibrates within the constraints of the envelope function \( 2A \sin kx \).
- The amplitude of the standing wave, \( 2A \)
Standing Waves, Definitions

A **node** occurs at a point of zero amplitude.
- These correspond to positions of $x$ where
  \[ x = \frac{n\lambda}{2} \quad n = 0, 1, 2, 3, \ldots \]

An **antinode** occurs at a point of maximum displacement, $2A$.
- These correspond to positions of $x$ where
  \[ x = \frac{n\lambda}{4} \quad n = 1, 3, 5, \ldots \]
Features of Nodes and Antinodes

The distance between adjacent antinodes is $\lambda/2$.
The distance between adjacent nodes is $\lambda/2$.
The distance between a node and an adjacent antinode is $\lambda/4$. 
Nodes and Antinodes, cont

The diagrams above show standing-wave patterns produced at various times by two waves of equal amplitude traveling in opposite directions.

In a standing wave, the elements of the medium alternate between the extremes shown in (a) and (c).
Example 18.2  Formation of a Standing Wave

Two waves traveling in opposite directions produce a standing wave. The individual wave functions are

\[ y_1 = (4.0 \text{ cm}) \sin(3.0x - 2.0t) \]
\[ y_2 = (4.0 \text{ cm}) \sin(3.0x + 2.0t) \]

where \( x \) and \( y \) are measured in centimeters.

(A) Find the amplitude of the simple harmonic motion of the element of the medium located at \( x = 2.3 \text{ cm} \).

Solution  The standing wave is described by Equation 18.3; in this problem, we have \( A = 4.0 \text{ cm} \), \( k = 3.0 \text{ rad/cm} \), and \( \omega = 2.0 \text{ rad/s} \). Thus,

\[ y = (2A \sin kx) \cos \omega t = [(8.0 \text{ cm}) \sin 3.0x] \cos 2.0t \]

Thus, we obtain the amplitude of the simple harmonic motion of the element at the position \( x = 2.3 \text{ cm} \) by evaluating the coefficient of the cosine function at this position:

\[ y_{\text{max}} = (8.0 \text{ cm}) \sin 3.0x \bigg|_{x=2.3} \]
\[ = (8.0 \text{ cm}) \sin (6.9 \text{ rad}) = 4.6 \text{ cm} \]

(B) Find the positions of the nodes and antinodes if one end of the string is at \( x = 0 \).
Solution  With \( k = \frac{2\pi}{\lambda} = 3.0 \text{ rad/cm} \), we see that the wavelength is \( \lambda = \frac{(2\pi)}{3.0} \text{ cm} \). Therefore, from Equation 18.4 we find that the nodes are located at

\[
x = n \frac{\lambda}{2} = n \left( \frac{\pi}{3} \right) \text{ cm} \quad n = 0, 1, 2, 3, \ldots
\]

and from Equation 18.5 we find that the antinodes are located at

\[
x = n \frac{\lambda}{4} = n \left( \frac{\pi}{6} \right) \text{ cm} \quad n = 1, 3, 5, \ldots
\]
18.3 Standing Waves in a String

Consider a string fixed at both ends.

The string has length $L$.

Waves can travel both ways on the string.

Standing waves are set up by a continuous superposition of waves incident on and reflected from the ends.

There is a boundary condition on the waves.

The ends of the strings must necessarily be nodes.

- They are fixed and therefore must have zero displacement.
The boundary condition results in the string having a set of natural patterns of oscillation, called **normal modes**.

- Each mode has a characteristic frequency.
  - This situation in which only certain frequencies of oscillations are allowed is called **quantization**.

- The normal modes of oscillation for the string can be described by imposing the requirements that the ends be nodes and that the nodes and antinodes are separated by $\lambda/4$.

We identify an analysis model called **waves under boundary conditions**.
This is the first normal mode that is consistent with the boundary conditions.

There are nodes at both ends.

There is one antinode in the middle.

This is the longest wavelength mode:

- \( \frac{1}{2} \lambda_1 = L \) so \( \lambda_1 = 2L \)

The section of the standing wave between nodes is called a loop.

In the first normal mode, the string vibrates in one loop.
Consecutive normal modes add a loop at each step.

- The section of the standing wave from one node to the next is called a **loop**.

The second mode (c) corresponds to to $\lambda = L$.

The third mode (d) corresponds to $\lambda = \frac{2L}{3}$.

Section 18.3
Standing Waves on a String, Summary

The wavelengths of the normal modes for a string of length $L$ fixed at both ends are $\lambda_n = \frac{2L}{n}$ where $n = 1, 2, 3, \ldots$

- $n$ is the $n^{th}$ normal mode of oscillation
- These are the possible modes for the string:

The natural frequencies are

$$f_n = n \frac{v}{2L} = n \frac{T}{2L} \sqrt{\frac{T}{\mu}}$$

- Also called quantized frequencies
Waves on a String, Harmonic Series

The **fundamental frequency** corresponds to \( n = 1 \).
- It is the lowest frequency, \( f_1 \)

The frequencies of the remaining natural modes are integer multiples of the fundamental frequency.
- \( f_n = nf_1 \)

Frequencies of normal modes that exhibit this relationship form a **harmonic series**.

The normal modes are called **harmonics**.
Musical Note of a String

The musical note is defined by its fundamental frequency.
The frequency of the string can be changed by changing either its length or its tension.
Example 18.3  Give Me a C Note!

Middle C on a piano has a fundamental frequency of 262 Hz, and the first A above middle C has a fundamental frequency of 440 Hz.

(A) Calculate the frequencies of the next two harmonics of the C string.

Solution Knowing that the frequencies of higher harmonics are integer multiples of the fundamental frequency $f_1 = 262$ Hz, we find that

\[ f_2 = 2f_1 = 524 \text{ Hz} \]
\[ f_3 = 3f_1 = 786 \text{ Hz} \]

(B) If the A and C strings have the same linear mass density $\mu$ and length $L$, determine the ratio of tensions in the two strings.

Solution Using Equation 18.9 for the two strings vibrating at their fundamental frequencies gives

\[ f_{1A} = \frac{1}{2L} \sqrt{\frac{T_A}{\mu}} \text{ and } f_{1C} = \frac{1}{2L} \sqrt{\frac{T_C}{\mu}} \]
Setting up the ratio of these frequencies, we find that

\[
\frac{f_{1A}}{f_{1C}} = \sqrt{\frac{T_A}{T_C}}
\]

\[
\frac{T_A}{T_C} = \left( \frac{f_{1A}}{f_{1C}} \right)^2 = \left( \frac{440}{262} \right)^2 = 2.82
\]
Resonance

A system is capable of oscillating in one or more normal modes.

Assume we drive a string with a vibrating blade.

If a periodic force is applied to such a system, the amplitude of the resulting motion of the string is greatest when the frequency of the applied force is equal to one of the natural frequencies of the system.

This phenomena is called **resonance**.
Because an oscillating system exhibits a large amplitude when driven at any of its natural frequencies, these frequencies are referred to as resonance frequencies.

If the system is driven at a frequency that is not one of the natural frequencies, the oscillations are of low amplitude and exhibit no stable pattern.

Resonance occurs when a system is able to store and easily transfer energy between two or more different storage modes. However, there are some losses from cycle to cycle, called damping. When damping is small, the resonant frequency is approximately equal to the natural frequency of the system, which is a frequency of unforced vibrations. Some systems have multiple, distinct, resonant frequencies.

Resonance phenomena occur with all types of vibrations or waves: there is mechanical resonance, acoustic resonance, electromagnetic resonance, nuclear magnetic resonance (NMR), electron spin resonance (ESR) and resonance of quantum wave functions. Resonant systems can be used to generate vibrations of a specific frequency (e.g., musical instruments),
Resonance

- When we apply a periodically varying force to a system that can oscillate, the system is forced to oscillate with a frequency equal to the frequency of the applied force (driving frequency): forced oscillation. When the applied frequency is close to a characteristic frequency of the system, a phenomenon called resonance occurs.

- Resonance also occurs when a periodically varying force is applied to a system with normal modes.
Problem
Resonance

The sound waves generated by the fork are reinforced when the length of the air column corresponds to one of the resonant frequencies of the tube. Suppose the smallest value of $L$ for which a peak occurs in the sound intensity is 9.00 cm.

$L_{\text{smallest}} = 9.00 \text{ cm}$

(a) Find the frequency of the tuning fork.

\[ f_1 = \frac{v}{4L_1} = \frac{345}{4(9.00 \times 10^{-2})} \text{ Hz} = 985 \text{ Hz} \]

(b) Find the wavelength and the next two water levels giving resonance.

\[ \lambda = 4L_1 = 4(9.00 \times 10^{-2}) \text{ m} = 0.360 \text{ m} \]

\[ L_2 = L_1 + \frac{\lambda}{2} = 0.270 \text{ m}, \quad L_3 = L_2 + \frac{\lambda}{2} = 0.450 \text{ m}. \]
Example 18.6  Wind in a Culvert

A section of drainage culvert 1.23 m in length makes a howling noise when the wind blows.

(A) Determine the frequencies of the first three harmonics of the culvert if it is cylindrical in shape and open at both ends. Take $v = 343 \text{ m/s}$ as the speed of sound in air.

Solution  The frequency of the first harmonic of a pipe open at both ends is

$$f_1 = \frac{v}{2L} = \frac{343 \text{ m/s}}{2(1.23 \text{ m})} = 139 \text{ Hz}$$

Because both ends are open, all harmonics are present; thus,

$$f_2 = 2f_1 = 278 \text{ Hz} \quad \text{and} \quad f_3 = 3f_1 = 417 \text{ Hz}$$

(B) What are the three lowest natural frequencies of the culvert if it is blocked at one end?

Solution  The fundamental frequency of a pipe closed at one end is
\[ f_1 = \frac{v}{4L} = \frac{343 \text{ m/s}}{4(1.23 \text{ m})} = 69.7 \text{ Hz} \]

In this case, only odd harmonics are present; hence, the next two harmonics have frequencies \( f_3 = 3f_1 = 209 \text{ Hz} \) and \( f_5 = 5f_1 = 349 \text{ Hz} \).

**(C)** For the culvert open at both ends, how many of the harmonics present fall within the normal human hearing range (20 to 20,000 Hz)?

**Solution** Because all harmonics are present for a pipe open at both ends, we can express the frequency of the highest harmonic heard as \( f_n = nf_1 \) where \( n \) is the number of harmonics that we can hear. For \( f_n = 20,000 \text{ Hz} \), we find that the number of harmonics present in the audible range is

\[ n = \frac{20,000 \text{ Hz}}{139 \text{ Hz}} = 143 \]
Beats and interference

Wave interference is the phenomenon that occurs when two waves meet while traveling along the same medium. The interference of waves causes the medium to take on a shape that results from the net effect of the two individual waves upon the particles of the medium.

If two upward displaced pulses having the same shape meet up with one another while traveling in opposite directions along a medium, the medium will take on the shape of an upward displaced pulse with twice the amplitude of the two interfering pulses. This type of interference is known as constructive interference.

If an upward displaced pulse and a downward displaced pulse having the same shape meet up with one another while traveling in opposite directions along a medium, the two pulses will cancel each other's effect upon the displacement of the medium and the medium will assume the equilibrium position. This type of interference is known as destructive interference.
The diagrams below show two waves - one is blue and the other is red - interfering in such a way to produce a resultant shape in a medium; the resultant is shown in green. In two cases (on the left and in the middle), constructive interference occurs and in the third case (on the far right, destructive interference occurs.

- **Constructive Interference:**
  - Upward displaced pulse meets upward displaced pulse
  - Downward displaced pulse meets downward displaced pulse

- **Destructive Interference:**
  - Upward displaced pulse meets downward displaced pulse
An animation below shows two sound waves interfering constructively in order to produce very large oscillations in pressure at a variety of anti-nodal locations. Note that compressions are labeled with a C and rarefactions are labeled with an R.
**Beats**

Two sound waves with **different but close** frequencies give rise to **BEATS**

Consider

\[ s_1(x,t) = s_m \cos \omega_1 t \]
\[ s_2(x,t) = s_m \cos \omega_2 t \]

\[ s = s_1 + s_2 = \left[ 2s_m \cos \omega' t \right] \cos \omega t \]

\[ \omega' = \frac{1}{2} (\omega_1 - \omega_2) \]
\[ \omega = \frac{1}{2} (\omega_1 + \omega_2) \]

\[ s = \left[ 2s_m \cos \omega' t \right] \cos \omega t \]

Very small

On top of the almost same frequency, the amplitude takes maximum twice in a cycle: \( \cos \omega' t = 1 \) and \(-1\): Beats

Beat frequency \( f_{\text{beat}} \): \[ f_{\text{beat}} = f_1 - f_2 \]
Sum and Difference Frequencies

When you superimpose two sine waves of different frequencies, you get components at the sum and difference of the two frequencies. This can be shown by using a sum rule from trigonometry. For equal amplitude sine waves

\[ \cos(2\pi f_1 t) + \cos(2\pi f_2 t) = 2 \cos \left( \frac{2\pi}{2} \frac{f_1 + f_2}{t} \right) \cos \left( \frac{2\pi}{2} \frac{f_1 - f_2}{t} \right) \]

The first term gives the phenomenon of beats with a beat frequency equal to the difference between the frequencies mixed. The beat frequency is given by

\[ f_{\text{beat}} = |f_1 - f_2| \]

since the first term above drives the output to zero (or a minimum for unequal amplitudes) at this beat frequency. Both the sum and difference frequencies are exploited in radio communication.

For instance, if one tuning fork vibrates at 438 Hz and a second one vibrates at 442 Hz, the resultant sound wave of the combination has a frequency of 440 Hz (the musical note A) and a beat frequency of 4 Hz. A listener would hear a 440-Hz sound wave go through an intensity maximum four times every second.
1. Two waves in one string are described by the wave functions

\[ y_1 = 3.0 \cos(4.0x - 1.6t) \]

and

\[ y_2 = 4.0 \sin(5.0x - 2.0t) \]

where \( y \) and \( x \) are in centimeters and \( t \) is in seconds. Find the superposition of the waves \( y_1 + y_2 \) at the points (a) \( x = 1.00, \ t = 1.00 \), (b) \( x = 1.00, \ t = 0.500 \), and (c) \( x = 0.500, \ t = 0 \). (Remember that the arguments of the trigonometric functions are in radians.)

4. Two waves are traveling in the same direction along a stretched string. The waves are 90.0° out of phase. Each wave has an amplitude of 4.00 cm. Find the amplitude of the resultant wave.
6. Two identical sinusoidal waves with wavelengths of 3.00 m travel in the same direction at a speed of 2.00 m/s. The second wave originates from the same point as the first, but at a later time. Determine the minimum possible time interval between the starting moments of the two waves if the amplitude of the resultant wave is the same as that of each of the two initial waves.

8. Two loudspeakers are placed on a wall 2.00 m apart. A listener stands 3.00 m from the wall directly in front of one of the speakers. A single oscillator is driving the speakers at a frequency of 300 Hz. (a) What is the phase difference between the two waves when they reach the observer? (b) What If? What is the frequency closest to 300 Hz to which the oscillator may be adjusted such that the observer hears minimal sound?
13. Two sinusoidal waves traveling in opposite directions interfere to produce a standing wave with the wave function

\[ y = (1.50 \text{ m}) \sin(0.400x) \cos(200t) \]

where \( x \) is in meters and \( t \) is in seconds. Determine the wavelength, frequency, and speed of the interfering waves.

15. Two speakers are driven in phase by a common oscillator at 800 Hz and face each other at a distance of 1.25 m. Locate the points along a line joining the two speakers where relative minima of sound pressure amplitude would be expected. (Use \( v = 343 \text{ m/s} \).)

19. Find the fundamental frequency and the next three frequencies that could cause standing-wave patterns on a string that is 30.0 m long, has a mass per length of \( 9.00 \times 10^{-3} \text{ kg/m} \), and is stretched to a tension of 20.0 N.

20. A string with a mass of 8.00 g and a length of 5.00 m has one end attached to a wall; the other end is draped over a pulley and attached to a hanging object with a mass of 4.00 kg. If the string is plucked, what is the fundamental frequency of vibration?
26. A 60.000-cm guitar string under a tension of 50.000 N has a mass per unit length of 0.100 00 g/cm. What is the highest resonant frequency that can be heard by a person capable of hearing frequencies up to 20 000 Hz?

27. A cello A-string vibrates in its first normal mode with a frequency of 220 Hz. The vibrating segment is 70.0 cm long and has a mass of 1.20 g. (a) Find the tension in the string. (b) Determine the frequency of vibration when the string vibrates in three segments.

32. The chains suspending a child’s swing are 2.00 m long. At what frequency should a big brother push to make the child swing with largest amplitude?

36. The overall length of a piccolo is 32.0 cm. The resonating air column vibrates as in a pipe open at both ends. (a) Find the frequency of the lowest note that a piccolo can play, assuming that the speed of sound in air is 340 m/s. (b) Opening holes in the side effectively shortens the length of the resonant column. If the highest note a piccolo can sound is 4 000 Hz, find the distance between adjacent antinodes for this mode of vibration.

37. Calculate the length of a pipe that has a fundamental frequency of 240 Hz if the pipe is (a) closed at one end and (b) open at both ends.