The Islamic University of Gaza
Department of Mathematics
First Exam.27-10-2016

Abstract algebra1 MATH3415
First Semester
Time:1 Hour

Student Name:-	Q1	Q2	Q3	Q4	Q5	Total
Student Number:-	5	5	10	10	10	40
Instructor name:Prof.Dr. M. Al-Ashker						

Solve the following questions:-

Q1. Determine which of the following statements are true and which are false

(1) The permutation $\sigma = (134)(28567)$ is odd permutation.

(2) The order of the group $U(8)$ is 4.

(3) Every group of order 4 is cyclic.

(4) The only generators of $2Z$ under addition are 2 and -2.

(5) The group S_3 contains cyclic subgroups.

Q2. Circle the correct answer of the following:-

(a) Let G be a cyclic group of order 45, then the number of elements of order 9 in G is $\{4, \ 8, \ 2, \ 6 \}$.

(b) The order of 22 in $(Z_{48}, \oplus)$ is equal:$\{48, \ 24, \ 12, \ 6 \}$.

(c) The number of subgroups in $(Z_{20}, \oplus)$ is: $\{2, \ 3, \ 6, \ 4 \}$.

(d) The group $Aut(Z_{15})$ is isomorphic to :
$\{U(15), \ S_{15}, \ A_{15}, \ D_{15} \}$.

(e) The order of the permutation $\sigma = (123)(45678)(2345)$:
is $\{2, \ 8, \ 4, \ 10 \}$.

Q3. (a) Let $G = \langle a \rangle$ be a cyclic group of order 16. Find all generators of G.

(b) Find all cyclic subgroups of G in part (a).

Q4. (a) Prove that if H and K are subgroups of a group G, then $H \cap K$ is also a subgroup of G.

(b) Let G be a group, show that G is abelian if and only if $(ab)^2 = a^2b^2,\ \forall\, a,\ b \in G$.

Q5. (a) Suppose that g and h induced the same inner automorphism of a group G. Prove that $h^{-1}g \in Z(G)$.

(b) Let G be a group and let $a,\ b \in G$, assume that $a \neq e$ and $|b| = 2$, and $bab^{-1} = a^2$. Find $|a|$.