The Islamic University of Gaza **Abstract algebra1 First Exam.**

Department of Mathematics **First Semester**

10-10-2012 **Time:1 Hour**

	Q1	Q2	Q3	Q4	Total
Student Name:-	Q1	Q2	Q3	Q4	Total
Student Number:-	10	12	12	6	40
Instructor name:					

Solve the following questions:-

Q1. Determine which of the following statements are true and which are false

(a) Every subgroup of a cyclic group is abelian.

(b) $(\mathbb{Z}_{12}, \oplus)$ has a subgroup of order 8, since 4 divides both 8 and 12.

(c) Every abelian group is cyclic.

(d) Every set of numbers that is a group under addition is a group under multiplication.

(e) There is no group of order 11.

(f) The only generators of $(Z_{12}, \oplus)$ are 1, 11.

(g) The only generators of $(Z, +)$ are 1 and (-1).

(h) $(Z_5, \oplus)$ is a cyclic group.

(i) The set $GL(2, R)$ under usual addition of matrices is infinite abelian group.

(j) It is possible for abelian group to contain a non-abelian subgroup.

Q2. Circle the correct answer of the following:-

(a) The inverse of 11 in $U(15)$ is equal: $\{4, \quad , 7 \quad , 13 \quad , 11 \quad \}$.

(b) The order of 77 in $(Z_{210}, \oplus)$ is equal:$\{77, \quad , 7 \quad , 30 \quad , 210 \quad \}$.

(c) The number of subgroups in $(Z_6, \oplus)$ is: $\{6, \quad , 5 \quad , 3 \quad , 2 \quad \}$.

(d) The order of $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \in SL(2, Z_{10})$ is equal :$\{4, \quad , 5 \quad , 10 \quad , \infty \quad \}$.

(e) The identity element in the group $(Z, \star)$, where $a \star b = a + b - 3$ is: $\{0, \quad , 1 \quad , 3 \quad , -3 \quad \}$.

(f) Let G be a cyclic group of order 30, then the number of elements of order 6 in G is: $\{2, \quad , 3 \quad , 5 \quad , 6 \quad \}$.

Q3. (a) Prove that in any group G, there is only one identity element.

(b) For each a in a group G, prove the centralizer of a, $C(a)$ is a subgroup of G.

Q4. Let G be a group in which $a^2 = 2$ for all $a \in G$, prove that G is abelian.