The Islamic University of Gaza **Abstract algebra1 MATH3415**

Department of Mathematics **First Semester**

Final Exam.2-2-2016 **Time:2 Hours**

Student Name:- Class No.	Q1	Q2	Q3	Q4	Q5	Q6	Total
Student Number:-	10	10	10	10	10	10	60
Instructor name:Prof.Dr. M. Al-Ashker							

Solve the following questions:-

Q1. Determine which of the following statements are true and which are false:

(1) $H \lhd G$ if and only if $ghg^{-1} \in G \ \forall \ g \in G, \ h \in H$.

(2) Let $G = \mathbb{Z}_{91}$ and $H =< 50 >$ in G, then $G = H$.

(3) There exists a non-zero homomorphism from the group $\mathbb{Z}_{33}$ to the group $\mathbb{Z}_{10}$

(4) The group $\mathbb{Z}_8 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_5 \oplus \mathbb{Z}_{25}$ is isomorphic to $\mathbb{Z}_{50} \oplus \mathbb{Z}_{40}$.

(5) If $H = 5\mathbb{Z}$ and $K = 7\mathbb{Z}$, where $\mathbb{Z}$ is the group of integers under addition, then $\mathbb{Z} = H \times K$.

(6) If all proper subgroups of a group G are Abelian, then G is Abelian.

(7) The group $\mathbb{Z} \oplus \mathbb{Z}/ < (2, 2) >$ is isomorphic to $\mathbb{Z}$.

(8) $|Aut(\mathbb{Z}_{10})| = 4$.

(9) $|D_5| = |Inn(D_5)|$.

(10) Let $\phi_g : G \longrightarrow G$ given by $\phi_g(x) = gxg^{-1}$, $\forall x \in G$, then ϕ_g is an automorphism of G for a fixed $g \in G$.

Q2. Circle the correct answer of the following:-

(a) If Φ is a homomorphism from $\mathbb{Z}_{30}$ to $\mathbb{Z}_{30}$ such that $ker\Phi = \{0, 10, 20\}$.
If $\Phi(23) = 6$, then $\Phi^{-1}(6)$ is the set :
1) $\{23, \ 3, \ 13\}$
2) $\{6, \ 16, \ 26\}$
3) $\{23, \ 6, \ 13\}$
4) $\{6, \ 16, \ 23\}$.

(b) If $\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \in S_3$. The the order of $(\alpha, 5) \in S_3 \oplus U(12)$ is equal:
1)10
2)15
3)6
4)20

(c) The order of $(2, 4, 7)$ in $\mathbb{Z}_8 \oplus \mathbb{Z}_5 \oplus \mathbb{Z}_{10}$ is equal:
1)35
2)20
3)210
4) 140.

(d) Let G be a cyclic group of order 30, then the number of elements of order 6 in G is:
1)2
2)3
3)5
4)6.

(e) Let G be acyclic group with generator a. Suppose that $|a| = 26$. Then the number of distinct left cosets of $H =< a^4 >$ in G is :
1)3
2)6
3)8
4)2.

Q3. Prove the following theorems:

(a)Prove that for each a in group G, then the centralizer of a, $C(a)$ is a subgroup of G.

(b)Prove that If N is a normal subgroup of G, then the natural homomorphism $\Phi : G \longrightarrow G/N$, given by $\Phi(g) = gN$ is a homomorphism from G onto G/N.

Q4. (a)Let G be a group of order pq where p and q are not necessarily distinct primes. Prove that the center $Z(G)$ equals either $\{e\}$ or G.

(b) Let $\phi : G \longrightarrow G'$ be a homomorphism. Prove that $ker\phi$ is a subgroup of G.

Q5. Let $\sigma = (12)(58)(346)(52)(41)(37)(67)$

(a) Write σ as a product of disjoint cycles.

(b) Find $|\sigma|$.

(c) Is $\sigma \in A_8$?.

(d) What is σ^{-1}?.

(e) What is the maximum possible order of an element in S_8?.

(f) What is the maximum possible order of an element in A_8?.

Q6. (a)Give all non-isomorphic groups of order 4.

(b)Give all non-isomorphic groups of order 8.

(c)Give all non-isomorphic groups of order 10.