The Islamic University of Gaza **Abstract algebra1 MATH3415**

Department of Mathematics **First Semester**

Final Exam.31-12-2016 **Time:2 Hours**

Student Name:-	Q1	Q2	Q3	Q4	Q5	Q6	Total
Student Number:-	10	10	10	10	10	10	60
Instructor name:Prof.Dr. M. Al-Ashker							

Solve the following questions:-

Q1. Determine which of the following statements are true and which are false:

(1) The empty set ϕ is a subgroup of any group G. (   ).

(2) The union of two subgroups is a subgroup. (   ).

(3) $2\mathbb{Z} \cup 6\mathbb{Z} = 2\mathbb{Z}$. (   ).

(4) Any group of order 5 is cyclic. (   ).

(5) Every group G contaims at least two subgroups. (   ).

(6) If G is a cyclic group, then G has at least two distinct generators. (   ).

(7) $\mathbb{Z}_3 \oplus \mathbb{Z}_3 \simeq \mathbb{Z}_9$. (   ).

(8) $(2\mathbb{Z}, +) \simeq (3\mathbb{Z}, +)$. (   ).

(9) Let $G = (\mathbb{R}/\{0\}, \cdot)$ and $H = (\mathbb{R}^+, \cdot)$ be groups. Let $\phi : G \longrightarrow H$ is given by $\phi(x) = |x|$, the absolute value of x. Then ϕ is an isomorphism. (   ).

(10) The factor group S_n/A_n has three elements. (   ).

Q2. Circle the correct answer of the following:-

(a) The order of the element $14+ < 8 >$ in $\mathbb{Z}_{24}/ < 8 >$ is:

1) 4

2) 2

3) 3

4) 6

(b) The order of $4U_5(105)$ in the factor group $U(105)/U_5(105)$ is:

1)4

2)3

3)2

4)6

(c) Let $\phi : \mathbb{Z} \oplus \mathbb{Z} \longrightarrow \mathbb{Z}$ defined by $\phi(a, b) = a - b$ be a homomorphism, then $ker\phi$ is:

1)$\{(a, 0) : a \in \mathbb{Z}\}$

2)$\{(0, a) : a \in \mathbb{Z}\}$

3)$\{(a, a) : a \in \mathbb{Z}\}$

4)Non of above.

(d) Let $G = \mathbb{Z}_{48},\ H =< 32 >$ be a subgroup of G, then $|G : H|$ is equal:

1)8

2)4

3)2

4)16.

(e) Let $G =< x >$ be a cyclic group of order 144. Then the order of x^{54} is :

1)8

2)27

3)72

4)2.

Q3. Prove the following :

(a)Let a be an element of a finite group G. Prove that a and a^{-1} have the same order.

(b)Let $\phi : G \longrightarrow H$ be a homomorphism.

(i) Show that $\phi(e_G) = e_H$.

(ii)Show that if H is abelian and ϕ is one-to- one, then G is abelian.

Q4. (a)Let $G = \mathbb{Z}_{23}/\{0\}$ be a cyclic group under multiplication and that 5 is a generator of G.

(i) Find $|G|$

(ii) Find the inverse of 5 and its order.

(b)Suppose that G is agroup of permutations on some set. If $|G| = 60$ and $orb_G(5) = \{1, 5\}$. Prove that $Stab_G(5)$ is normal in G.

Q5. (a) Let a and b belong to a group G. If $|a|$ and $|b|$ are relatively prime, show that $< a > \cap < b >= \{e\}$.

(b) let $\alpha = (1, 3, 5, 7, 9, 8, 6)(2, 4, 10)$ be an element in S_{10}. What is the smalest positive integer n such that $\alpha^n = \alpha^{-5}$?.

Q6. (a) How many homomorphisms are there from $\mathbb{Z}_{20}$ to $\mathbb{Z}_8$. How many are there onto $\mathbb{Z}_8$. Find all the homomorphisms if any exist.

(b) Prove the theorem: let G and H be finite cyclic groups. Then $G \oplus H$ is cyclic if and only if $|G|$ and $|H|$ are relatively prime.