Islamic University of Gaza

Faculty of Science **Abstract Algebra 1** **Department of Math**

Date:- 7-1-2013 **Final Exam** **Time:-2 Hour**

Student name:	Q1	Q2	Q3	Q4	Q5	Total
Student number:	10	10	10	10	10	50
Instructor name:						

Solve each of the following questions:-

(Q1) Determine which of the following statements true or false:-

1. ( ) **The group** $Aut(Z_{38})$ **is cyclic.**
2. ( ) **The group** $Z_8 \oplus Z_{18}$ **is isomorphic to** Z_{36}.
3. ( ) **Any group of order** 4 **must be isomorphic to** Z_4.
4. ( ) **If** $G/Z(G)$ **is cyclic, then** $Inn(G)$ **is trivial group.**
5. ( ) **The alternating subgroup** A_n **in** S_n **is normal subgroup of** S_n **because** $|S_n : A_n| = 2$.

(Q2) Circle the correct answer for each of the following statements:-

1. **The order of** 21 **in** Z_{60} **is**

   (a) 10 **(b)**20 **(c)**15 **(d)**30.

2. **The order of** $(6,5)$ **in** $Z_{10} \oplus Z_{15}$ **is**

   (a) 30 **(b)**5 **(c)**15 **(d)**150.

3. **The order of** $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 5 & 6 & 7 & 2 & 1 \end{pmatrix}$ **in** S_7 **is**

   (a)12 **(b)**44 **(c)**3 **(d)**7.

4. **The order of the element** $14+ < 8 >$ **in the factor group** $Z_{24}/ < 8 >$ **is equal**

   a) 3 **(b)**6 **(c)**8 **(d)**4.

5. **The inverse of** $(6,5)$ **in** $Z_{10} \oplus Z_{15}$ **is**

   a) $(4,10)$ **(b)**$(6,5)$ **(c)**$(5,6)$ **(d)**$(10,4)$.

(Q3) Prove each of the following:

1. **Every subgroup of cyclic group is cyclic.**

2. **Let** $\phi : G \to \overline{G}$ **be a group homomorphism, prove that the mapping** $\Psi : G/\ker\phi \to \phi(G)$ **given by** $\Psi(g\ker\phi) = \phi(g)$ **is an isomorphism.**

(Q4) (a) Determine all homomorphism from Z_{12} to Z_{30}

(b) Determine the number of elements of order 5 in $Z_{25} \oplus Z_5$.

(Q5) (a) Let ϕ be a homomorphism from a group G to a group $\overline{G}$. If $\overline{K}$ is normal subgroup of $\overline{G}$, then $\phi^{-1}(\overline{K}) = \{k \in G : \phi(k) \in \overline{K}\}$ is normal subgroup of G.

(b) Let G be a finite group and let H be a normal subgroup such that $|G : H| = 20$ and $|H| = 7$. Suppose $x \in G$ and $x^7 = e$, Show that $x \in H$.