Chapter 5
Simple Mechanisms
Assignment #1

- All questions at the end of chapter
- 1st Exam: Monday 5/10/2015
Kinematic Link or Element

- **kinematic link (link) or element:** Each part of a machine, which moves relative to some other part.
  - A link need not to be a rigid body
  - It must be a resistant body

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Kinematic Link or Element

- **A resistant body:** capable of transmitting required forces with negligible deformation.

- **A link has two characteristics:**
  1. It should have relative motion
  2. It must be a resistant body

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Types of Links

1. **Rigid link**: does not undergo any deformation while transmitting motion

2. **Flexible link**: partly deformed in a manner not to affect the transmission of motion
   - **Example**: belts, ropes, chains and wires

3. **Fluid link**: formed by having a fluid in a container and motion is transmitted through the fluid pressure
   - **Example**: hydraulic presses, jacks and brakes
Structure and a Machine

- **Structure:** assembly of a number of resistant bodies having **no relative motion** between them and meant for **carrying loads** having straining action.
  - **Examples:** A railway bridge, a roof truss, machine frames
# Structure and a Machine

<table>
<thead>
<tr>
<th>Machine</th>
<th>Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parts move relative to one another</td>
<td>Parts do not move relative to one another</td>
</tr>
<tr>
<td>A machine transforms available energy into some useful work</td>
<td>No energy is transformed into useful work</td>
</tr>
<tr>
<td>Links of a machine may transmit both power &amp; motion</td>
<td>Parts of a structure transmit forces only</td>
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Kinematic Pair

- **A pair**: two links of a machine, in contact with each other.
- **kinematic pair**: the relative motion between the links is completely or successfully constrained.
Types of Constrained Motions

1. Completely constrained motion
2. Incompletely constrained motion
3. Successfully constrained motion
Types of Constrained Motions

Completely constrained motion

- Motion between a pair is limited to a definite direction irrespective of direction of force applied
- **Example:** piston and cylinder form a pair

![Diagram of square bar in square hole](image1)

**Fig. 5.2.** Square bar in a square hole.

![Diagram of shaft with collars in circular hole](image2)

**Fig. 5.3.** Shaft with collars in a circular hole.

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Types of Constrained Motions

*Incompletely constrained motion*

Motion between a pair can take place in more than one direction.

**Fig. 5.4.** Shaft in a circular hole.
Types of Constrained Motions

Successfully constrained motion

Motion between elements, forming a pair is such that the constrained motion is not completed by itself, but by some other means.
Types of Constrained Motions

Successfully constrained motion

**Fig. 5.5**: shaft may rotate in a bearing or it may move upwards.

- a case of incompletely constrained motion.
- If load is placed on shaft to prevent axial upward movement of shaft, then motion of pair is said to be successfully constrained motion.
Classification of Kinematic Pairs

1. According to type of relative motion between elements
   a. Sliding pair
   b. Turning pair
   c. Rolling pair
   d. Screw pair
   e. Spherical pair

2. According to type of contact between elements
   a. Lower pair
   b. Higher pair

3. According to type of closure
   a. Self closed pair
   b. Force - closed pair
Classification of Kinematic Pairs
According to type of relative motion between elements

a. Sliding pair
   - one pair can only slide relative to the other
   - completely constrained motion
   - Examples:
     - Piston & cylinder
     - Tail stock on lathe bed
Classification of Kinematic Pairs
According to type of relative motion between elements

b. Turning pair

- One pair can only turn or revolve about a fixed axis of another link
- Completely constrained motion

Examples:
- Crankshaft in a journal bearing in an engine
- Lathe spindle supported in head stock
Classification of Kinematic Pairs

According to type of relative motion between elements

c. Rolling pair

- One pair rolls over another fixed link
- **Example:** Ball and roller bearings

d. Screw pair

- One element can turn about the other by screw threads
- **Example:** Lead screw of a lathe with nut
Classification of Kinematic Pairs
According to type of relative motion between elements

e. **Spherical pair**
   - One element (with spherical shape) turns or swivels about the other fixed element
   
   **Examples:**
   - Ball and socket joint
   - Attachment of a car mirror
   - Pen stand
Classification of Kinematic Pairs

According to type of contact between the elements

a. **Lower pair**

- Two elements of pair have surface contact when relative motion takes place
- Surface of one element slides over surface of the other
- **Examples:** Sliding pairs, turning pairs and screw pairs
Classification of Kinematic Pairs

According to type of contact between the elements

b. **Higher pair**

- Two elements of a pair have a line or point contact when relative motion takes place
- Motion between the two elements is partly turning and partly sliding

**Examples:**
- A pair of friction discs
- Toothed gearing
- Belt and rope drives
- Ball and roller bearings, and cam & follower
Classification of Kinematic Pairs
According to type of closure

a. **Self closed pair**
   - Two elements of a pair are connected together mechanically in such a way that only required kind of relative motion occurs.
   - Lower pairs are self closed pair
Classification of Kinematic Pairs

According to type of closure

b. Force - closed pair

- Two elements of a pair are not connected mechanically but are kept in contact by action of external forces

- **Example:** cam & follower, kept in contact by the forces exerted by spring & gravity
Kinematic Chain (KC)

A kinematic chain:

- A combination of kinematic pairs, joined in such a way that each link forms a part of two pairs
- Relative motion between links or elements is completely or successfully constrained
Kinematic Chain (KC)

Example:

✓ Crankshaft of engine forms a kinematic pair with the bearings which are fixed in a pair
✓ ConRod with crank forms a 2\(^{\text{nd}}\) pair
✓ Piston with ConRod forms a 3\(^{\text{rd}}\) pair
✓ Piston with cylinder forms a 4\(^{\text{th}}\) pair
✓ Total combination of these links is a KC
Kinematic Chain (KC)

If each link is assumed to form two pairs with two adjacent links, the relation between # of pairs \(p\) forming a KC & # of links \(l\) is:

\[ l = 2p - 4 \]
Kinematic Chain (KC)

- Relation between # of links ($l$) & # of joints ($j$) which constitute a KC is given by:

\[ j = \frac{3}{2} l - 2 \]
Kinematic Chain (KC)

- Above Equations are applicable only to KC, in which lower pairs are used
- These equations may be applied to KC, in which higher pairs are used:
  - each higher pair = two lower pairs with an additional element or link
Kinematic Chain (KC) Examples

- Arrangement of three links with pin joints
- It is not a KC: no relative motion is possible

Number of links, $l = 3$
Number of pairs, $p = 3$
number of joints, $j = 3$
From equation (i),

\[
3 = 2p - 4 = 2
\]

L.H.S. > R.H.S.

Now from equation (ii),

\[
j = \frac{3}{2} l - 2 \quad \text{or} \quad 3 = \frac{3}{2} \times 3 - 2 = 2.5
\]

L.H.S. > R.H.S.

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Kinematic Chain

Examples — constrained kinematic chain of one DOF

\[ l = 4, \ p = 4, \ \text{and} \ j = 4 \]
\[ l = 2p - 4 \]
\[ 4 = 2 \times 4 - 4 = 4 \]
L.H.S. = R.H.S.

\[ j = \frac{3}{2} l - 2 \]
\[ 4 = \frac{3}{2} \times 4 - 2 = 4 \]
L.H.S. = R.H.S.
Kinematic Chain

Examples

- It is not a KC
- Unconstrained chain

Fig. 5.8. Arrangement of five links.

\[
\begin{align*}
  l &= 5, \ p = 5, \text{ and } j = 5 \\
  \text{From equation (i),} \\
  l &= 2p - 4 \quad \text{or} \quad 5 = 2 \times 5 - 4 = 6 \\
  \text{L.H.S. < R.H.S.} \\
  \text{From equation (ii),} \\
  j &= \frac{3}{2} l - 2 \quad \text{or} \quad 5 = \frac{3}{2} \times 5 - 2 = 5.5 \\
  \text{L.H.S. < R.H.S.}
\end{align*}
\]
Kinematic Chain

Examples – Compound Kinematic Chain

A chain having more than four links

From equation (i),
\[ l = 2p - 4 \]
L.H.S. = R.H.S.
From equation (ii),
\[ j = \frac{3}{2}l - 2 \]
L.H.S. = R.H.S.

or \[ 6 = 2 \times 5 - 4 = 6 \]

or \[ 7 = \frac{3}{2} \times 6 - 2 = 7 \]
Types of Joints in a Chain

1. **Binary joint**
   - Two links joined at same connection
   - 4 links & 4 binary joins
   - Use **A.W. Klein equation** to determine whether chain is:
     - locked chain (structure)
     - kinematic chain
     - unconstrained chain

\[ j + \frac{h}{2} = \frac{3}{2} l - 2 \]

\( j = \# \) of binary joints
\( h = \# \) of higher pairs, \( l = \# \) of links

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Types of Joints in a Chain

2. Ternary joint

- Three links joined at same connection

Example:
- 6 links
- 3 binary joints at A, B & D
- 2 ternary joints at C & E
- One ternary joint ≈ two binary joints
- Equivalent binary joints = 3 + 2×2 = 7

\[ j = \frac{3}{2} l - 2 \]
\[ 7 = \frac{3}{2} \times 6 - 2 = 7 \]
Types of Joints in a Chain

3. Quaternary joint

- Four links joined at same connection
- Equivalent to three binary joints
- When \( l \) number of links are joined at same connection, joint is equivalent to \((l - 1)\) binary joints
Types of Joints in a Chain

3. Quaternary joint

- Fig. 5.12 (a):
  - one binary joint at D
  - 4 ternary joints at A, B, E and F
  - 2 quaternary joints at C & G
  - # of binary joints in chain are

\[ 1 + 4 \times 2 + 2 \times 3 = 15 \]

\[ j = \frac{3}{2} l - 2, \quad \text{or} \quad 15 = \frac{3}{2} \times 11 - 2 = 14.5, \ i.e., \ L.H.S. > R.H.S. \]

- Chain is not a KC
- locked chain: structure
Types of Joints in a Chain

3. Quaternary joint

- Total number of binary joints = \(1 + 2 \times 6 = 13\)
- \(LHS = RHS\), chain is a KC or constrained chain

\[
j = \frac{3}{2}l - 2
\]

\[
13 = \frac{3}{2} \times 10 - 2 = 13,\ \text{i.e. L.H.S.} = \text{R.H.S.}
\]
Mechanism

- **Mechanism:** one of the links of a KC is fixed
  - Used for transmitting or transforming motion e.g. engine indicators, typewriter etc.
- **Simple mechanism:** 4 links
- **Compound mechanism:** > 4 links
- **Machine:** When a mechanism is required to transmit power or to do some particular type of work
Number of DOF for Plane Mechanisms

**DOF:** number of input parameters which must be independently controlled in order to bring the mechanism into a useful engineering purpose.
Number of DOF for Plane Mechanisms

Fig. 5.13 (a)
- One variable is needed to define relative positions of all links
- \# of DOF = 1

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Number of DOF for Plane Mechanisms

Fig. 5.13 (b)

- Two variables $\theta_1$ & $\theta_2$ are needed to completely define relative positions of all links
- # of DOF = 2
Number of DOF for Plane Mechanisms

- Two links AB & CD in a plane motion
- Each link has 3 DOF before it is connected to any other link
- When CD is connected to AB by a turning pair at A, position of link CD is now determined by a single variable & thus has one DOF

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Number of DOF for Plane Mechanisms

When a link is connected to a fixed link by a turning pair (lower pair), two DOF are destroyed

(a) $n = 9$  
(b) $n = 7$  
(c) $n = 5$  
(d) $n = 3$  
(e) $n = 1$
Number of DOF for Plane Mechanisms

- Consider a plane mechanism with \( l \) number of links
- In a mechanism, one of the links is to be fixed, therefore:
  - number of movable links = \((l - 1)\)
  - total number of DOF = \(3 \ (l - 1)\) before they are connected to any other link
Number of DOF for Plane Mechanisms

- # of DOF of a mechanism with \( l \) number of links connected by \( j \) number of binary joints or lower pairs (single DOF pairs) and \( h \) number of higher pairs (two DOF pairs) is given by:

\[
n = 3(l - 1) - 2j - h
\]

- Kutzbach criterion for movability of a mechanism having plane motion
Application of Kutzbach Criterion to Plane Mechanisms

A structure and no relative motion between the links is possible

\[ n = 3 (l - 1) - 2j - h \]

\[ n = 3 (3 - 1) - 2 \times 3 = 0 \]

Redundant constraints in the chain and it forms a statically indeterminate structure

\[ n = 3 (6 - 1) - 2 \times 8 = -1 \]
Application of Kutzbach Criterion to Plane Mechanisms

\[ n = 3 \ (l - 1) - 2 \ j - h \]

\[ n = 3 \ (5 - 1) - 2 \times 6 = 0 \]

\[ n = 3 \ (l - 1) - 2 \ j \]

\[ n = 3 \ (4 - 1) - 2 \times 4 = 1 \]

\[ n = 3 \ (5 - 1) - 2 \times 5 = 2 \]
Application of Kutzbach Criterion to Plane Mechanisms

**Fig. 5.17 (a):** 3 links, 2 binary joints and one higher pair: $l = 3$, $j = 2$ & $h = 1$
Application of Kutzbach Criterion to Plane Mechanisms

\[ n = 3 \left( (l - 1) - 2j - h \right) \]
\[ n = 3 (4 - 1) - 2 \times 3 - 1 = 2 \]

Fig. 5.17 (b): 4 links, 3 binary joints and one higher pair: \( l = 4, j = 3 \) & \( h = 1 \)
Grubler’s Criterion (GC) for Plane Mechanisms

- GC applies to mechanisms with only single DOF joints
- Substituting $n = 1$ and $h = 0$ in Kutzbach equation,

$$1 = 3 (l - 1) - 2j \text{ or } 3l - 2j - 4 = 0$$
Grubler’s Criterion (GC) for Plane Mechanisms

- A plane mechanism with a movability of 1 & only single DOF joints **can not have odd number of links**.

- **Simplest possible mechanisms:**
  - four bar mechanism
  - slider-crank mechanism
Types of Kinematic Chains

Three types of kinematic chains with four lower pairs are considered:

1. Four bar chain
2. Single slider crank chain
3. Double slider crank chain
Four Bar Chain

- Four links, turning pairs at A, B, C & D.
- Grashof’s law for a four bar mechanism: For a continuous relative motion between links 2 & 4,

\[ \sum \text{shortest & longest link lengths} \leq \sum \text{remaining two link lengths} \]
Four Bar Chain

- **Crank or driver**: One of the links, will make a complete revolution relative to the other three links, if it satisfies **Grashof’s law**.
- **Follower**: Link BC (2)
- **ConRod**: Link CD (3)
- **Frame of mechanism**: Fixed link AB (1)
Single Slider Crank Chain

- Modification of the basic four bar chain
- One sliding pair and three turning pairs
- Converts rotary motion into reciprocating motion and vice versa
Inversion of Mechanism

- **Inversion of the mechanism:** method of obtaining different mechanisms by fixing different links in a kinematic chain

- **Relative motions between various links is not changed**, but **their absolute motions may be changed drastically**
Inversions of Single Slider Crank Chain

2. Oscillating cylinder engine

- Link 3 is fixed
- When crank (2) rotates, piston attached to piston rod (1) reciprocates & cylinder (4) oscillates about a pin pivoted to the fixed link at A

Fig. 5.24. Oscillating cylinder engine.
Inversions of Single Slider Crank Chain

4. Crank and slotted lever quick return motion mechanism

- Used in shaping machines, slotting machines and in rotary internal combustion engines.
Inversions of Single Slider Crank Chain

4. Crank and slotted lever quick return motion mechanism

- Forward stroke occurs when crank rotates from position $CB_1$ to $CB_2$ (angle $\beta$) cw.
- Return stroke occurs when crank rotates from position $CB_2$ to $CB_1$ (angle $\alpha$) cw.
- Crank has uniform angular speed,

\[
\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} \quad \text{or} \quad \frac{360^\circ - \alpha}{\alpha}
\]
Inversions of Single Slider Crank Chain

5. Whitworth quick return motion mechanism

Used in shaping & slotting machines

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Fig. 5.27. Whitworth quick return motion mechanism.
Inversions of Single Slider Crank Chain

5. *Whitworth quick return motion mechanism*

- Time of cutting stroke $= \frac{\alpha}{\beta}$
- Time of return stroke $= \frac{\alpha}{360^\circ - \alpha}$
- Crank link CA rotates at uniform angular velocity

\[
\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360^\circ - \alpha} \quad \text{or} \quad \frac{360^\circ - \beta}{\beta}
\]
Example 5.1

A crank and slotted lever mechanism used in a shaper has a center distance of 300 mm between the center of oscillation of the slotted lever and the center of rotation of the crank. The radius of the crank is 120 mm. Find the ratio of the time of cutting to the time of return stroke.
Example 5.2

In a crank and slotted lever quick return motion mechanism, distance between the fixed centers is 240 mm and the length of driving crank is 120 mm. Find the inclination of the slotted bar with the vertical in the extreme position and the time ratio of cutting stroke to the return stroke.

If the length of the slotted bar is 450 mm, find the length of the stroke if the line of stroke passes through the extreme positions of the free end of the lever.
Example 5.2
Example 5.3

The driving crank BC is 30 mm long & time ratio of the working stroke to the return stroke is to be 1.7. If the length of the working stroke of R is 120 mm, determine the dimensions of AC & AP.
Example 5.3
Example 5.4

In a Whitworth quick return motion mechanism, distance between the fixed centers is 50 mm & length of driving crank is 75 mm. The length of slotted lever is 150 mm and length of ConRod is 135 mm. Find the ratio of time of cutting stroke to time of return stroke and also the effective stroke.
Example 5.4
Double Slider Crank Chain

Two turning pairs & two sliding pairs

Fig. 5.34. Elliptical trammels.
Inversions of Double Slider Crank Chain

1. Elliptical trammels

- When links 1 & 3 slide along their respective grooves, any point on link 2 such as $P$ traces out an ellipse on the surface of link 4.
- $AP$ & $BP$ are semi-major axis & minor axis of the ellipse.

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Inversions of Double Slider Crank Chain

1. Elliptical trammels

Coordinates of point $P$ on link $BA$ will be

\[
x = PQ = AP \cos \theta; \text{ and } y = PR = BP \sin \theta
\]

\[
\frac{x}{AP} = \cos \theta; \text{ and } \frac{y}{BP} = \sin \theta
\]

\[
\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2 \theta + \sin^2 \theta = 1
\]
Inversions of Double Slider Crank Chain

2. *Scotch yoke mechanism*

- Convert rotary motion into a reciprocating motion.
- Inversion is obtained by fixing either link 1 or link 3.
- When Link 2 (crank) rotates about B, link 4 (frame) reciprocates.
- Fixed link 1 guides the frame.

*Fig. 5.35. Scotch yoke mechanism.*

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Inversions of Double Slider Crank Chain

3. Oldham’s coupling

- Used for connecting two parallel shafts whose axes are at a small distance apart.
- Inversion is obtained by fixing link 2, Fig. 5.36 (a).
- Shafts to be connected have two flanges (link 1 & link 3) rigidly fastened at their ends.

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Inversions of Double Slider Crank Chain

3. Oldham’s coupling

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