Chapter 7

Shafts and Shaft Components
Chapter Outline

- Introduction
- Shaft Materials
- Shaft Layout
- Shaft Design for Stress
- Deflection Considerations
- Critical Speeds for Shafts
- Miscellaneous Shaft Components
- Limits and Fits
Considerations of Shaft Design

- Material Selection
- Geometric Layout
- Stress & strength: Static strength, Fatigue strength
- Deflection and rigidity
  - Bending deflection
  - Torsional deflection
  - Slope at bearings & shaft-supported elements
  - Shear deflection due to transverse loading of short shafts
- Vibration due to natural frequency
Shaft Materials

- **Deflection**: controlled by geometry & material
- **Stress**: controlled by geometry, not material
- **Strength**: controlled by material
Shaft Materials

- **Shafts**: commonly made from low carbon, CD or HR steel, such as ANSI 1020–1050 steels.
- Surface hardening usually used when shaft is being used as a bearing surface.

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Shaft Materials

- **CD steel**: typical for $d < 3$ in
  - Should be machined all over

- **HR steel**: common for larger sizes

- **Low production quantities**
  - Lathe machining

- **High production quantities**
  - Forming or casting
Shaft Layout

Issues to consider for shaft layout:

- Axial layout of components
- Supporting axial loads
- Torque transmission
- Assembly & Disassembly
Axial Layout of Components

Fig. 7-2
Axial Layout of Components

- Support load-carrying components between bearings
- **Pulleys & sprockets:** mounted outboard for ease of installation of belt or chain
- **Axial space between components:**
  - allow for lubricant flow
  - provide access space for disassembly
- Load bearing components should be placed near the bearings.
Axial Layout of Components

- **Primary means of locating components:** position them against a shoulder of the shaft.

- A shoulder provides a solid support to minimize deflection & vibration of the component.
Axial Layout of Components

- **When magnitudes of forces are reasonably low, shoulders can be constructed with:**
  - retaining rings in grooves
  - sleeves between components
  - clamp-on collars

- **Where axial loads are very small:**
  - feasible to do without shoulders
  - rely on press fits, pins, or collars with setscrews to maintain an axial location
Supporting Axial Loads

Figure 7–3

- Tapered roller bearings used in a mowing machine spindle.
Supporting Axial Loads

Figure 7–4

- A bevel-gear drive in which both pinion and gear are straddle-mounted.

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Providing for Torque Transmission

Common means of transferring torque to shaft:

- Keys
- Splines
- Setscrews
- Pins
- Press or shrink fits
- Tapered fits

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Providing for Torque Transmission

**Keys:**

- Moderate to high levels of torque
- Slip fit of component onto shaft for easy assembly
- Designed to fail if torque exceeds acceptable operating limits, protecting more expensive components.

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Providing for Torque Transmission

Splines

- Gear teeth formed on outside of shaft and on inside of hub of load-transmitting component.
- Transfer high torques
- Can be made with a reasonably loose slip fit to allow for large axial motion between shaft and component while still transmitting torque.
Providing for Torque Transmission

Pins, setscrews in hubs, tapered fits, and press fits

- Low torque transmission
- **Press and shrink fits**: used both for torque transfer and for preserving axial location.
- A **split hub** with screws to clamp hub to the shaft.

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Providing for Torque Transmission

Pins, setscrews in hubs, tapered fits, and press fits

- *Tapered fits* between shaft and the shaft-mounted device, such as a wheel
- Used on the overhanging end of a shaft
- Screw threads at shaft end then permit use of a nut to lock the wheel tightly to shaft.
- Easy disassembled
- Does not provide good axial location of the wheel on the shaft

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Assembly and Disassembly

Figure 7–5

- Bearing inner rings press-fitted to shaft while outer rings float in the housing.
- Axial clearance should be sufficient only to allow for machinery vibrations. Note the labyrinth seal on the right.
Assembly and Disassembly

Figure 7–6

- Similar to Fig. 7–5 except that outer bearing rings are preloaded.
Assembly and Disassembly

Figure 7–7

- Inner ring of LH bearing is locked to the shaft between a nut and a shaft shoulder.
- The snap ring in the outer race is used to positively locate the shaft assembly in the axial direction.
- Note the floating RH bearing and the grinding run-out grooves in the shaft.
Assembly and Disassembly

**Figure 7–8:** Similar to Fig. 7–7

- LH bearing positions the entire shaft assembly
- Inner ring is secured to shaft using a snap ring

- Note the use of a shield to prevent dirt generated from within the machine from entering the bearing
Shaft Design for Stress

- Stresses evaluated at critical locations
- Critical locations are usually
  - On the outer surface
  - Where bending moment is large
  - Where torque is present
  - Where stress concentrations exist
Shaft Stresses

- Axial loads are generally small and constant, so will be ignored
- Standard alternating & midrange stresses

\[
\sigma_a = K_f \frac{M_{ac}}{I} \quad \sigma_m = K_f \frac{M_{mc}}{I} \\
\tau_a = K_{fs} \frac{T_{ac}}{J} \quad \tau_m = K_{fs} \frac{T_{mc}}{J}
\]  

(7–1)  
(7–2)

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Shaft Stresses

- Customized for round shafts

\[
\sigma_a = K_f \frac{32M_a}{\pi d^3}, \quad \sigma_m = K_f \frac{32M_m}{\pi d^3}
\]  

(7–3)

\[
\tau_a = K_{fs} \frac{16T_a}{\pi d^3}, \quad \tau_m = K_{fs} \frac{16T_m}{\pi d^3}
\]  

(7–4)

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Shaft Stresses

Combine stresses into von Mises stresses

\[ \sigma_a' = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[ \left( \frac{32K_f M_a}{\pi d^3} \right)^2 + 3 \left( \frac{16K_f s T_a}{\pi d^3} \right)^2 \right]^{1/2} \]  

(7–5)

\[ \sigma_m' = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[ \left( \frac{32K_f M_m}{\pi d^3} \right)^2 + 3 \left( \frac{16K_f s T_m}{\pi d^3} \right)^2 \right]^{1/2} \]  

(7–6)
Shaft Stresses

Modified Goodman

\[ \frac{1}{n} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} \]

\[ \frac{1}{n} = \frac{16}{\pi d^3} \left[ \frac{1}{S_e} \left[ 4(K_f M_a)^2 + 3(K_{fs} T_a)^2 \right]^{1/2} + \frac{1}{S_{ut}} \left[ 4(K_f M_m)^2 + 3(K_{fs} T_m)^2 \right]^{1/2} \right] \]

(7–7)

\[ d = \left( \frac{16n}{\pi} \right) \left( \frac{1}{S_e} \left[ 4(K_f M_a)^2 + 3(K_{fs} T_a)^2 \right]^{1/2} \right. \]

\[ + \left. \frac{1}{S_{ut}} \left[ 4(K_f M_m)^2 + 3(K_{fs} T_m)^2 \right]^{1/2} \right) \left) \right]^{1/3} \]

(7–8)
Shaft Stresses

**DE-Gerber**

\[
\frac{1}{n} = \frac{8A}{\pi d^3 S_e} \left\{ 1 + \left[ 1 + \left( \frac{2BS_e}{AS_{ut}} \right)^2 \right]^{1/2} \right\}
\]

\[
d = \left( \frac{8nA}{\pi S_e} \left\{ 1 + \left[ 1 + \left( \frac{2BS_e}{AS_{ut}} \right)^2 \right]^{1/2} \right\} \right)^{1/3}
\]

\[
A = \sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2}
\]

\[
B = \sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2}
\]
Shaft Stresses

**DE-ASME Elliptic**

\[
\frac{1}{n} = \frac{16}{\pi d^3} \left[ 4 \left( \frac{K_f M_a}{S_e} \right)^2 + 3 \left( \frac{K_{fs} T_a}{S_e} \right)^2 + 4 \left( \frac{K_f M_m}{S_y} \right)^2 + 3 \left( \frac{K_{fs} T_m}{S_y} \right)^2 \right]^{1/2}
\]

(7–11)

\[
d = \left\{ \frac{16n}{\pi} \left[ 4 \left( \frac{K_f M_a}{S_e} \right)^2 + 3 \left( \frac{K_{fs} T_a}{S_e} \right)^2 + 4 \left( \frac{K_f M_m}{S_y} \right)^2 + 3 \left( \frac{K_{fs} T_m}{S_y} \right)^2 \right]^{1/2} \right\}^{1/3}
\]

(7–12)
Shaft Stresses

DE-Soderberg

\[
\frac{1}{n} = \frac{16}{\pi d^3} \left\{ \frac{1}{S_e} \left[ 4(K_f M_a)^2 + 3(K_{fs} T_a)^2 \right]^{1/2} + \frac{1}{S_{yt}} \left[ 4(K_f M_m)^2 + 3(K_{fs} T_m)^2 \right]^{1/2} \right\}
\]

(7–13)

\[
d = \left( \frac{16n}{\pi} \left\{ \frac{1}{S_e} \left[ 4(K_f M_a)^2 + 3(K_{fs} T_a)^2 \right]^{1/2}
\right. \right.

\left. + \frac{1}{S_{yt}} \left[ 4(K_f M_m)^2 + 3(K_{fs} T_m)^2 \right]^{1/2} \right\} \right)^{1/3}
\]

(7–14)
Shaft Stresses for Rotating Shaft

For rotating shaft with steady bending and torsion

✓ Bending stress is completely reversed
✓ Torsional stress is steady
✓ Previous equations simplify with $M_m$ & $T_a = 0$

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Checking for Yielding in Shafts

- Soderberg criteria inherently guards against yielding
- ASME-Elliptic criteria takes yielding into account, but is not entirely conservative
- Gerber and modified Goodman criteria require specific check for yielding
Checking for Yielding in Shafts

- Use von Mises max stress to check for yielding,

\[
\sigma'_{\text{max}} = \left[ (\sigma_m + \sigma_a)^2 + 3 (\tau_m + \tau_a)^2 \right]^{1/2}
\]

\[
= \left[ \left( \frac{32K_f (M_m + M_a)}{\pi d^3} \right)^2 + 3 \left( \frac{16K_{fs} (T_m + T_a)}{\pi d^3} \right)^2 \right]^{1/2}
\]

(7–15)

\[
n_y = \frac{S_y}{\sigma'_{\text{max}}}
\]

(7–16)
Example 7-1

At a machined shaft shoulder the small diameter $d$ is 1.100 in, the large diameter $D$ is 1.65 in, and the fillet radius is 0.11 in. The bending moment is 1260 lbf·in and the steady torsion moment is 1100 lbf·in. The heat-treated steel shaft has an ultimate strength of $S_{ut} = 105$ kpsi and a yield strength of $S_y = 82$ kpsi. The reliability goal is 0.99.

a) Determine the fatigue factor of safety of the design using each of the fatigue failure criteria described in this section.

b) Determine the yielding factor of safety.
Estimating Stress Concentrations

- Stress concentrations depend on size specifications, which are not known the first time through a design process.
- Standard shaft elements such as shoulders and keys have standard proportions, making it possible to estimate stress concentrations factors before determining actual sizes.
Estimating Stress Concentrations

- Shoulders for bearing & gear support should match catalog recommendation for the specific bearing or gear.

- Fillet radius at shoulder needs to be sized to avoid interference with fillet radius of the mating component.

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Reducing Stress Concentration at Shoulder Fillet

Fig. 7-9

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Reducing Stress Concentration at Shoulder Fillet

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Reducing Stress Concentration at Shoulder Fillet

Large radius relief groove

Fig. 7-9
Example 7-2

This example problem is part of a larger case study. A double reduction gearbox design has developed to the point that the general layout and axial dimensions of the countershaft carrying two spur gears has been proposed, as shown in Fig. 7–10. The gears and bearings are located and supported by shoulders, and held in place by retaining rings. The gears transmit torque through keys. Gears have been specified as shown, allowing the tangential and radial forces transmitted through the gears to the shaft to be determined as follows.

\[
\begin{align*}
W_{23}^t &= 540 \text{ lbf} \\
W_{23}^r &= 197 \text{ lbf} \\
W_{54}^t &= 2431 \text{ lbf} \\
W_{54}^r &= 885 \text{ lbf}
\end{align*}
\]
Example 7-2

Proceed with the next phase of the design, in which a suitable material is selected, and appropriate diameters for each section of the shaft are estimated, based on providing sufficient fatigue and static stress capacity for infinite life of the shaft, with minimum safety factors of 1.5.

\[
W_{23}^f = 540 \text{ lbf} \quad W_{54}^f = 2431 \text{ lbf} \\
W_{23}^r = 197 \text{ lbf} \quad W_{54}^r = 885 \text{ lbf}
\]

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Example 7-2

Bearing A

Gear 3
\( d_3 = 12 \)

D_1 \hspace{1cm} D_2 \hspace{1cm} D_3

Bearing B

Gear 4
\( d_4 = 2.67 \)

D_5 \hspace{1cm} D_6 \hspace{1cm} D_7

Datum

C 0.25 0.75 1.25 1.75 2.0 2.75 3.50

I 7.50 8.50 9.50 9.75 10.25 10.75 11.25 11.50

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Deflection Considerations

- Size critical locations for stress
- Fill in reasonable size estimates for other locations
- Perform deflection analysis
- Deflection of shaft (linear & angular) should be checked at gears & bearings
Deflection Considerations

- **Table 7–2**: Maximum Ranges for Slopes & Transverse Deflections

<table>
<thead>
<tr>
<th>Slopes</th>
<th>Transverse Deflections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tapered roller</td>
<td>0.0005–0.0012 rad</td>
</tr>
<tr>
<td>Cylindrical roller</td>
<td>0.0008–0.0012 rad</td>
</tr>
<tr>
<td>Deep-groove ball</td>
<td>0.001–0.003 rad</td>
</tr>
<tr>
<td>Spherical ball</td>
<td>0.026–0.052 rad</td>
</tr>
<tr>
<td>Self-align ball</td>
<td>0.026–0.052 rad</td>
</tr>
<tr>
<td>Uncrowned spur gear</td>
<td>&lt; 0.0005 rad</td>
</tr>
<tr>
<td>Spur gears with $P &lt; 10$ teeth/in</td>
<td>0.010 in</td>
</tr>
<tr>
<td>Spur gears with $11 &lt; P &lt; 19$</td>
<td>0.005 in</td>
</tr>
<tr>
<td>Spur gears with $20 &lt; P &lt; 50$</td>
<td>0.003 in</td>
</tr>
</tbody>
</table>

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Example 7-3

In Ex. 7–2, a preliminary shaft geometry was obtained on the basis of design for stress. The resulting shaft is shown in Fig. 7–10, with proposed diameters.

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Example 7-3

Check that the deflections and slopes at the gears and bearings are acceptable. If necessary, propose changes in the geometry to resolve any problems.
Adjusting Diameters for Allowable Deflections

- If any deflection is larger than allowed, since $I$ is proportional to $d^4$, a new diameter can be found from

$$d_{new} = d_{old} \left| \frac{n_d y_{old}}{y_{all}} \right|^{1/4}$$

(7–17)

- Similarly, for slopes,

$$d_{new} = d_{old} \left| \frac{n_d (dy/dx)_{old}}{(slope)_{all}} \right|^{1/4}$$

(7–18)
Adjusting Diameters for Allowable Deflections

- Determine largest $d_{new}/d_{old}$ ratio
- Multiply all diameters by this ratio
Example 7-4

For the shaft in Ex. 7–3, it was noted that the slope at the right bearing is near the limit for a cylindrical roller bearing. Determine an appropriate increase in diameters to bring this slope down to 0.0005 rad.
Angular Deflection of Shafts

For **stepped** shaft with **individual** cylinder length $l_i$ and **torque** $T_i$, the **angular deflection** can be estimated from

$$\theta = \sum \theta_i = \sum \frac{T_i l_i}{G_i J_i} \quad (7-19)$$

Experiments show that these equations slightly underestimate angular deflection.
Critical Speeds for Shafts

- A shaft with mass has a critical speed at which its deflections become unstable.
- Components attached to the shaft have lower critical speed than the shaft.

- **Lowest critical speed ≥ twice the operating speed**

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Critical Speeds for Shafts

- For a simply supported shaft of uniform diameter, 1\textsuperscript{st} critical speed is
  \[ \omega_1 = \left( \frac{\pi}{l} \right)^2 \sqrt{\frac{EI}{m}} = \left( \frac{\pi}{l} \right)^2 \sqrt{\frac{gEI}{A\gamma}} \]  
  \[ (7-22) \]

- For a group of attachments, Rayleigh’s method for lumped masses gives
  \[ \omega_1 = \sqrt{\frac{g \sum w_i y_i}{\sum w_i y_i^2}} \]  
  \[ (7-23) \]
Critical Speeds for Shafts

- Eq. (7–23) can be applied to the shaft itself by partitioning the shaft into segments.
Critical Speeds for Shafts

- **Influence coefficient**: transverse deflection at location \( i \) due to a unit load at location \( j \)
- From Table A–9–6 for a simply supported beam with a single unit load

\[
\delta_{ij} = \begin{cases} 
\frac{b_j x_i}{6EIl} \left( l^2 - b_j^2 - x_i^2 \right) & x_i \leq a_i \\
\frac{a_j (l - x_i)}{6EIl} \left( 2lx_i - a_j^2 - x_i^2 \right) & x_i > a_i 
\end{cases}
\]  

(7–24)
Critical Speeds for Shafts

Fig. 7–13

\[
\delta_{ij} = \begin{cases} 
\frac{b_j x_i}{6EI l} (l^2 - b_j^2 - x_i^2) & x_i \leq a_i \\
\frac{a_j (l - x_i)}{6EI l} (2l x_i - a_j^2 - x_i^2) & x_i > a_i 
\end{cases}
\]

(7–24)

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Critical Speeds for Shafts

- **Simply supported shaft** with **three loads**, deflections corresponding to location of each load is

\[
\begin{align*}
y_1 &= F_1 \delta_{11} + F_2 \delta_{12} + F_3 \delta_{13} \\
y_2 &= F_1 \delta_{21} + F_2 \delta_{22} + F_3 \delta_{23} \\
y_3 &= F_1 \delta_{31} + F_2 \delta_{32} + F_3 \delta_{33}
\end{align*}
\]

(7–25)
Critical Speeds for Shafts

- If forces are due only to centrifugal force due to shaft mass,

\[
y_1 = m_1 \omega^2 y_1 \delta_{11} + m_2 \omega^2 y_2 \delta_{12} + m_3 \omega^2 y_3 \delta_{13}
\]

\[
y_2 = m_1 \omega^2 y_1 \delta_{21} + m_2 \omega^2 y_2 \delta_{22} + m_3 \omega^2 y_3 \delta_{23}
\]

\[
y_3 = m_1 \omega^2 y_1 \delta_{31} + m_2 \omega^2 y_2 \delta_{32} + m_3 \omega^2 y_3 \delta_{33}
\]
Critical Speeds for Shafts

- If forces are due only to *centrifugal* force due to shaft mass,

\[
\begin{align*}
(m_1 \delta_{11} - 1/\omega^2)y_1 + (m_2 \delta_{12})y_2 + (m_3 \delta_{13})y_3 &= 0 \\
(m_1 \delta_{21})y_1 + (m_2 \delta_{22} - 1/\omega^2)y_2 + (m_3 \delta_{23})y_3 &= 0 \\
(m_1 \delta_{31})y_1 + (m_2 \delta_{32})y_2 + (m_3 \delta_{33} - 1/\omega^2)y_3 &= 0
\end{align*}
\]
Critical Speeds for Shafts

Non-trivial solutions to this set of simultaneous equations will exist when:

\[
\begin{vmatrix}
(m_1 \delta_{11} - 1/\omega^2) & m_2 \delta_{12} & m_3 \delta_{13} \\
 m_1 \delta_{21} & (m_2 \delta_{22} - 1/\omega^2) & m_3 \delta_{23} \\
 m_1 \delta_{31} & m_2 \delta_{32} & (m_3 \delta_{33} - 1/\omega^2)
\end{vmatrix} = 0 \quad (7-26)
\]

Expanding the determinant,

\[
\left(\frac{1}{\omega^2}\right)^3 - (m_1 \delta_{11} + m_2 \delta_{22} + m_3 \delta_{33}) \left(\frac{1}{\omega^2}\right)^2 + \cdots = 0 \quad (7-27)
\]
Critical Speeds for Shafts

- Eq. (7–27) can be written in terms of its three roots as

\[
\left( \frac{1}{\omega^2} - \frac{1}{\omega_1^2} \right) \left( \frac{1}{\omega^2} - \frac{1}{\omega_2^2} \right) \left( \frac{1}{\omega^2} - \frac{1}{\omega_3^2} \right) = 0
\]

\[
\left( \frac{1}{\omega^2} \right)^3 - \left( \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{1}{\omega_3^2} \right) \left( \frac{1}{\omega^2} \right)^2 + \cdots = 0 \quad (7–28)
\]
Critical Speeds for Shafts

Comparing Eqs. (7–27) and (7–28),

\[ \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{1}{\omega_3^2} = m_1 \delta_{11} + m_2 \delta_{22} + m_3 \delta_{33} \]  

(7–29)
Critical Speeds for Shafts

Define $\omega_{ii}$ as the critical speed if $m_i$ is acting alone. From Eq. (7–29),

$$\frac{1}{\omega_{ii}^2} = m_i \delta_{ii}$$

Thus, Eq. (7–29) can be rewritten as

$$\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{1}{\omega_3^2} = \frac{1}{\omega_{11}^2} + \frac{1}{\omega_{22}^2} + \frac{1}{\omega_{33}^2}$$

(7–30)
Critical Speeds for Shafts

- Note that: $\frac{1}{\omega_1^2} \gg \frac{1}{\omega_2^2}$, and $\frac{1}{\omega_3^2}$
- 1st critical speed can be approximated from Eq. (7–30) as

$$\frac{1}{\omega_1^2} = \frac{1}{\omega_{11}^2} + \frac{1}{\omega_{22}^2} + \frac{1}{\omega_{33}^2} \quad (7–31)$$

- For n-body shaft, we obtain Dunkerley’s equation,

$$\frac{1}{\omega_1^2} = \sum_{i=1}^{n} \frac{1}{\omega_{ii}^2} \quad (7–32)$$
Critical Speeds for Shafts

- For the load at station 1, placed at the center of the span, the equivalent load is found from

\[
\omega_{11}^2 = \frac{1}{m_1 \delta_{11}} = \frac{g}{w_1 \delta_{11}} = \frac{g}{w_{1c} \delta_{cc}}
\]

\[
w_{1c} = w_1 \frac{\delta_{11}}{\delta_{cc}}
\]

(7–33)
Example 7-5

Consider a simply supported steel shaft with 1 in diameter and a 31-in span between bearings, carrying two gears weighing 35 and 55 lbf.
Example 7-5

a) Find the influence coefficients.
b) Find $\% wy$ and $\% wy2$ and the first critical speed using Rayleigh’s equation, Eq. (7–23).
c) From the influence coefficients, find $\omega_{11}$ and $\omega_{22}$.
d) Using Dunkerley’s equation, Eq. (7–32), estimate the first critical speed.
e) Use superposition to estimate the first critical speed.
f) Estimate the shaft’s intrinsic critical speed.
g) Suggest a modification to Dunkerley’s equation to include the effect of the shaft’s mass on the first critical speed of the attachments.
Setscrews

- Setscrews resist axial & rotational motion
- They apply a compressive force to create friction

Fig. 7–15
Setscrews

- **Holding power**: Resistance to axial motion of collar or hub relative to shaft
- Factors of safety = 1.5 to 2 for static, 4 to 8 for dynamic loads
- Length ≈ 0.5 shaft diameter

### Table 7-4

<table>
<thead>
<tr>
<th>Size, in</th>
<th>Seating Torque, lbf · in</th>
<th>Holding Power, lbf</th>
</tr>
</thead>
<tbody>
<tr>
<td>#0</td>
<td>1.0</td>
<td>50</td>
</tr>
<tr>
<td>#1</td>
<td>1.8</td>
<td>65</td>
</tr>
<tr>
<td>#2</td>
<td>1.8</td>
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<td>120</td>
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<td>250</td>
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<td>1500</td>
</tr>
<tr>
<td>3/8</td>
<td>290</td>
<td>2000</td>
</tr>
<tr>
<td>7/16</td>
<td>430</td>
<td>2500</td>
</tr>
<tr>
<td>1/2</td>
<td>620</td>
<td>3000</td>
</tr>
<tr>
<td>9/16</td>
<td>620</td>
<td>3500</td>
</tr>
<tr>
<td>5/8</td>
<td>1325</td>
<td>4000</td>
</tr>
<tr>
<td>3/4</td>
<td>2400</td>
<td>5000</td>
</tr>
<tr>
<td>7/8</td>
<td>5200</td>
<td>6000</td>
</tr>
<tr>
<td>1</td>
<td>7200</td>
<td>7000</td>
</tr>
</tbody>
</table>

*Source: Unbrako Division, SPS Technologies, Jenkintown, Pa.*

Mohammad Suliman Abuaiba, Ph.D., PE
Keys and Pins

Used to secure rotating elements and to transmit torque

Fig. 7–16
Tapered Pins

- Taper pins are sized by diameter at large end
- Small end diameter is

\[ d = D - 0.0208L \]  

(7–35)
# Tapered Pins

## Table 7–5: Some standard sizes in inches

<table>
<thead>
<tr>
<th>Size</th>
<th>Commercial Maximum</th>
<th>Commercial Minimum</th>
<th>Precision Maximum</th>
<th>Precision Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/0</td>
<td>0.1103</td>
<td>0.1083</td>
<td>0.1100</td>
<td>0.1090</td>
</tr>
<tr>
<td>2/0</td>
<td>0.1423</td>
<td>0.1403</td>
<td>0.1420</td>
<td>0.1410</td>
</tr>
<tr>
<td>0</td>
<td>0.1573</td>
<td>0.1553</td>
<td>0.1570</td>
<td>0.1560</td>
</tr>
<tr>
<td>2</td>
<td>0.1943</td>
<td>0.1923</td>
<td>0.1940</td>
<td>0.1930</td>
</tr>
<tr>
<td>4</td>
<td>0.2513</td>
<td>0.2493</td>
<td>0.2510</td>
<td>0.2500</td>
</tr>
<tr>
<td>6</td>
<td>0.3423</td>
<td>0.3403</td>
<td>0.3420</td>
<td>0.3410</td>
</tr>
<tr>
<td>8</td>
<td>0.4933</td>
<td>0.4913</td>
<td>0.4930</td>
<td>0.4920</td>
</tr>
</tbody>
</table>

Mohammad Suliman Abuhaiba, Ph.D., PE
## Keys

- Keys come in standard square & rectangular sizes
- Shaft diameter determines key size

### Table 7–6

<table>
<thead>
<tr>
<th>Shaft Diameter Over (Incl.)</th>
<th>Key Size</th>
<th>Keyway Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{5}{16})</td>
<td>(\frac{7}{16})</td>
<td>(\frac{3}{32})</td>
</tr>
<tr>
<td>(\frac{7}{16})</td>
<td>(\frac{9}{16})</td>
<td>(\frac{1}{8})</td>
</tr>
<tr>
<td>(\frac{9}{16})</td>
<td>(\frac{7}{8})</td>
<td>(\frac{3}{16})</td>
</tr>
<tr>
<td>(\frac{7}{8})</td>
<td>(1\frac{1}{4})</td>
<td>(\frac{1}{4})</td>
</tr>
<tr>
<td>(1\frac{1}{4})</td>
<td>(1\frac{3}{8})</td>
<td>(\frac{5}{16})</td>
</tr>
<tr>
<td>(1\frac{3}{8})</td>
<td>(1\frac{3}{4})</td>
<td>(\frac{3}{8})</td>
</tr>
<tr>
<td>(1\frac{3}{4})</td>
<td>(2\frac{1}{4})</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>(2\frac{1}{4})</td>
<td>(2\frac{3}{4})</td>
<td>(\frac{5}{8})</td>
</tr>
<tr>
<td>(2\frac{3}{4})</td>
<td>(3\frac{1}{4})</td>
<td>(\frac{3}{4})</td>
</tr>
</tbody>
</table>

Mohammad Suliman Abuhaiba, Ph.D., PE
Keys

- Failure of keys
  - direct shear
  - bearing stress

- Key length is designed to provide desired factor of safety

- Factor of safety should not be excessive, so the inexpensive key is the weak link

- Key length is limited to hub length

- **Key length \( \leq 1.5 \text{ times shaft diameter} \)** to avoid problems from twisting

Mohammad Suliman Abuhaiba, Ph.D., PE
Keys

- **Stock key material**: low carbon CR steel, with dimensions slightly under nominal dimensions to easily fit end-milled keyway
- A setscrew is sometimes used with a key:
  - for axial positioning
  - to minimize rotational backlash
Gib-head Key

- Tapered: when firmly driven it prevents axial motion
- Head makes removal easy
- Projection of head may be hazardous

![Diagram of Gib-head Key]

Fig. 7–17

Mohammad Suliman Abuhaiba, Ph.D., PE
Woodruff Key

- Deeper penetration
- Useful for smaller shafts to prevent key from rolling

Fig. 7–17
### Woodruff Key – Inch Series

<table>
<thead>
<tr>
<th>Key Size</th>
<th>Height $b$</th>
<th>Offset $e$</th>
<th>Keyseat Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{16}$</td>
<td>$\frac{1}{4}$</td>
<td>0.109</td>
<td>$\frac{1}{64}$</td>
</tr>
<tr>
<td>$\frac{1}{16}$</td>
<td>$\frac{3}{8}$</td>
<td>0.172</td>
<td>$\frac{1}{64}$</td>
</tr>
<tr>
<td>$\frac{3}{32}$</td>
<td>$\frac{3}{8}$</td>
<td>0.172</td>
<td>$\frac{1}{64}$</td>
</tr>
<tr>
<td>$\frac{3}{32}$</td>
<td>$\frac{1}{2}$</td>
<td>0.203</td>
<td>$\frac{3}{64}$</td>
</tr>
<tr>
<td>$\frac{3}{32}$</td>
<td>$\frac{5}{8}$</td>
<td>0.250</td>
<td>$\frac{1}{16}$</td>
</tr>
<tr>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{2}$</td>
<td>0.203</td>
<td>$\frac{3}{64}$</td>
</tr>
<tr>
<td>$\frac{1}{8}$</td>
<td>$\frac{5}{8}$</td>
<td>0.250</td>
<td>$\frac{1}{16}$</td>
</tr>
<tr>
<td>$\frac{1}{8}$</td>
<td>$\frac{3}{4}$</td>
<td>0.313</td>
<td>$\frac{1}{16}$</td>
</tr>
<tr>
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<td>$\frac{3}{8}$</td>
<td>0.250</td>
<td>$\frac{1}{16}$</td>
</tr>
<tr>
<td>$\frac{5}{32}$</td>
<td>$\frac{5}{8}$</td>
<td>0.313</td>
<td>$\frac{1}{16}$</td>
</tr>
<tr>
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<td>$\frac{7}{8}$</td>
<td>0.375</td>
<td>$\frac{1}{16}$</td>
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<td>$1$</td>
<td>0.438</td>
<td>$\frac{1}{16}$</td>
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<tr>
<td>$\frac{1}{4}$</td>
<td>$\frac{7}{8}$</td>
<td>0.375</td>
<td>$\frac{1}{16}$</td>
</tr>
<tr>
<td>$\frac{1}{4}$</td>
<td>$1$</td>
<td>0.438</td>
<td>$\frac{1}{16}$</td>
</tr>
<tr>
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<td>$\frac{1}{4}$</td>
<td>0.547</td>
<td>$\frac{5}{64}$</td>
</tr>
<tr>
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<td>$\frac{1}{16}$</td>
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<tr>
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<td>$\frac{1}{4}$</td>
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<td>$\frac{1}{2}$</td>
<td>0.641</td>
<td>$\frac{7}{64}$</td>
</tr>
<tr>
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<td>$\frac{1}{4}$</td>
<td>0.547</td>
<td>$\frac{5}{64}$</td>
</tr>
<tr>
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<td>$\frac{1}{2}$</td>
<td>0.641</td>
<td>$\frac{7}{64}$</td>
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</tbody>
</table>
Woodruff Key

<table>
<thead>
<tr>
<th>Keyseat Width, in</th>
<th>Shaft Diameter, in From</th>
<th>To (inclusive)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/16</td>
<td>5/16</td>
<td>1/2</td>
</tr>
<tr>
<td>3/32</td>
<td>3/8</td>
<td>7/8</td>
</tr>
<tr>
<td>1/8</td>
<td>3/8</td>
<td>1 1/2</td>
</tr>
<tr>
<td>5/32</td>
<td>1/2</td>
<td>1 5/8</td>
</tr>
<tr>
<td>3/16</td>
<td>9/16</td>
<td>2</td>
</tr>
<tr>
<td>1/4</td>
<td>11/16</td>
<td>2 1/4</td>
</tr>
<tr>
<td>5/16</td>
<td>3/4</td>
<td>2 3/8</td>
</tr>
<tr>
<td>3/8</td>
<td>1</td>
<td>2 5/8</td>
</tr>
</tbody>
</table>

Table 7-8: Sizes for various shaft diameters

Mohammad Suliman Abuhaiba, Ph.D., PE
Stress Concentration Factors for Keys

- For keyseats cut by standard end-mill cutters, with \( r/d = 0.02 \), give
  - \( K_t = 2.14 \) for bending
  - \( K_t = 2.62 \) for torsion without key in place
  - \( K_t = 3.0 \) for torsion with key in place

- To prevent two stress concentrations from combining, **Keep end of key seat at least a distance of \( d/10 \) from shoulder fillet**

Mohammad Suliman Abuhaiba, Ph.D., PE
Example 7-6

A UNS G10350 steel shaft, heat-treated to a minimum yield strength of 75 kpsi, has a diameter of 1 7/16 in. The shaft rotates at 600 rpm and transmits 40 hp through a gear. Select an appropriate key for the gear.

Mohammad Suliman Abuhaiba, Ph.D., PE
Retaining Rings

Retaining rings are often used instead of a shoulder to provide axial positioning.

Fig. 7–18

Mohammad Suliman Abuhaiba, Ph.D., PE
Retaining Rings

- Retaining ring must seat well in bottom of groove to support axial loads against the sides of the groove.

- Table A–15–16 and A–15–17: Stress concentrations for flat-bottomed grooves

- Typical stress concentration factors are high, around 5 for bending & axial, and 3 for torsion

Mohammad Suliman Abuhaiba, Ph.D., PE
Nomenclature for Cylindrical Fit

Fig. 7–20

Mohammad Suliman Abuhaiba, Ph.D., PE
Nomenclature for Cylindrical Fit

- Upper case letters: HOLE
- Lower case letters: shaft
- **Basic size:** nominal diameter, same for both parts, $D = d$
- **Tolerance:** difference between max & min size
- **Deviation:** difference between a size and the basic size

Mohammad Suliman Abuhaiba, Ph.D., PE
Nomenclature for Cylindrical Fit

- **Upper deviation** = max limit - basic size
- **Lower deviation** = min limit - basic size
- **Fundamental deviation**: either the upper or the lower deviation, depending on which is closer to the basic size
Nomenclature for Cylindrical Fit

- **Hole basis**: a system of fits corresponding to a basic hole size. Fundamental deviation is H

- **Shaft basis**: a system of fits corresponding to a basic shaft size. Fundamental deviation is h
Tolerance Grade Number

- **International tolerance grade numbers**: groups of tolerances such that the tolerances for a particular IT number have the same relative level of accuracy but vary depending on the basic size.
Tolerance Grade Number

- **IT** grades range from IT0 to IT16, but only IT6 to IT11 are generally needed.

- Specifications for **IT** grades:
  - **Table A–11**: metric series
  - **Table A–13**: inch series
# Table A–11

Tolerance Grades – Metric Series

<table>
<thead>
<tr>
<th>Basic Sizes</th>
<th>IT6</th>
<th>IT7</th>
<th>Tolerance Grades</th>
<th>IT8</th>
<th>IT9</th>
<th>IT10</th>
<th>IT11</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–3</td>
<td>0.006</td>
<td>0.010</td>
<td>0.014</td>
<td>0.025</td>
<td>0.040</td>
<td>0.060</td>
<td></td>
</tr>
<tr>
<td>3–6</td>
<td>0.008</td>
<td>0.012</td>
<td>0.018</td>
<td>0.030</td>
<td>0.048</td>
<td>0.075</td>
<td></td>
</tr>
<tr>
<td>6–10</td>
<td>0.009</td>
<td>0.015</td>
<td>0.022</td>
<td>0.036</td>
<td>0.058</td>
<td>0.090</td>
<td></td>
</tr>
<tr>
<td>10–18</td>
<td>0.011</td>
<td>0.018</td>
<td>0.027</td>
<td>0.043</td>
<td>0.070</td>
<td>0.110</td>
<td></td>
</tr>
<tr>
<td>18–30</td>
<td>0.013</td>
<td>0.021</td>
<td>0.033</td>
<td>0.052</td>
<td>0.084</td>
<td>0.130</td>
<td></td>
</tr>
<tr>
<td>30–50</td>
<td>0.016</td>
<td>0.025</td>
<td>0.039</td>
<td>0.062</td>
<td>0.100</td>
<td>0.160</td>
<td></td>
</tr>
<tr>
<td>50–80</td>
<td>0.019</td>
<td>0.030</td>
<td>0.046</td>
<td>0.074</td>
<td>0.120</td>
<td>0.190</td>
<td></td>
</tr>
<tr>
<td>80–120</td>
<td>0.022</td>
<td>0.035</td>
<td>0.054</td>
<td>0.087</td>
<td>0.140</td>
<td>0.220</td>
<td></td>
</tr>
<tr>
<td>120–180</td>
<td>0.025</td>
<td>0.040</td>
<td>0.063</td>
<td>0.100</td>
<td>0.160</td>
<td>0.250</td>
<td></td>
</tr>
<tr>
<td>180–250</td>
<td>0.029</td>
<td>0.046</td>
<td>0.072</td>
<td>0.115</td>
<td>0.185</td>
<td>0.290</td>
<td></td>
</tr>
<tr>
<td>250–315</td>
<td>0.032</td>
<td>0.052</td>
<td>0.081</td>
<td>0.130</td>
<td>0.210</td>
<td>0.320</td>
<td></td>
</tr>
<tr>
<td>315–400</td>
<td>0.036</td>
<td>0.057</td>
<td>0.089</td>
<td>0.140</td>
<td>0.230</td>
<td>0.360</td>
<td></td>
</tr>
</tbody>
</table>

Mohammad Suliman Abuhaiba, Ph.D., PE
Fundamental Deviation Letter Codes

Shafts with clearance fits

- Letter codes c, d, f, g, and h
- Upper deviation = fundamental deviation
- Lower deviation = upper deviation – tolerance grade

Mohammad Suliman Abuhaiba, Ph.D., PE
Fundamental Deviation Letter Codes

**Shafts with transition or interference fits**

- Letter codes k, n, p, s, and u
- Lower deviation = fundamental deviation
- Upper deviation = lower deviation + tolerance grade
Fundamental Deviation Letter Codes

- **Hole**
  - *Hole based standard, letter code H*
  - Lower deviation = 0 \((D_{\text{min}} = D)\)
  - Upper deviation = tolerance grade

- **Fundamental deviations for letter codes:**
  - Table A–12: metric series
  - Table A–14: inch series
## Table A–12
Fundamental Deviations – Metric series

<table>
<thead>
<tr>
<th>Basic Sizes</th>
<th>c</th>
<th>d</th>
<th>f</th>
<th>g</th>
<th>h</th>
<th>k</th>
<th>n</th>
<th>p</th>
<th>s</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–3</td>
<td>-0.060</td>
<td>-0.020</td>
<td>-0.006</td>
<td>-0.002</td>
<td>0</td>
<td>0</td>
<td>+0.004</td>
<td>+0.006</td>
<td>+0.014</td>
<td>+0.018</td>
</tr>
<tr>
<td>3–6</td>
<td>-0.070</td>
<td>-0.030</td>
<td>-0.010</td>
<td>-0.004</td>
<td>0</td>
<td>+0.001</td>
<td>+0.008</td>
<td>+0.012</td>
<td>+0.019</td>
<td>+0.023</td>
</tr>
<tr>
<td>6–10</td>
<td>-0.080</td>
<td>-0.040</td>
<td>-0.013</td>
<td>-0.005</td>
<td>0</td>
<td>+0.001</td>
<td>+0.010</td>
<td>+0.015</td>
<td>+0.023</td>
<td>+0.028</td>
</tr>
<tr>
<td>10–14</td>
<td>-0.095</td>
<td>-0.050</td>
<td>-0.016</td>
<td>-0.006</td>
<td>0</td>
<td>+0.001</td>
<td>+0.012</td>
<td>+0.018</td>
<td>+0.028</td>
<td>+0.033</td>
</tr>
<tr>
<td>14–18</td>
<td>-0.095</td>
<td>-0.050</td>
<td>-0.016</td>
<td>-0.006</td>
<td>0</td>
<td>+0.001</td>
<td>+0.012</td>
<td>+0.018</td>
<td>+0.028</td>
<td>+0.033</td>
</tr>
<tr>
<td>18–24</td>
<td>-0.110</td>
<td>-0.065</td>
<td>-0.020</td>
<td>-0.007</td>
<td>0</td>
<td>+0.002</td>
<td>+0.015</td>
<td>+0.022</td>
<td>+0.035</td>
<td>+0.041</td>
</tr>
<tr>
<td>24–30</td>
<td>-0.110</td>
<td>-0.065</td>
<td>-0.020</td>
<td>-0.007</td>
<td>0</td>
<td>+0.002</td>
<td>+0.015</td>
<td>+0.022</td>
<td>+0.035</td>
<td>+0.048</td>
</tr>
<tr>
<td>30–40</td>
<td>-0.120</td>
<td>-0.080</td>
<td>-0.025</td>
<td>-0.009</td>
<td>0</td>
<td>+0.002</td>
<td>+0.017</td>
<td>+0.026</td>
<td>+0.043</td>
<td>+0.060</td>
</tr>
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<td>40–50</td>
<td>-0.130</td>
<td>-0.080</td>
<td>-0.025</td>
<td>-0.009</td>
<td>0</td>
<td>+0.002</td>
<td>+0.017</td>
<td>+0.026</td>
<td>+0.043</td>
<td>+0.070</td>
</tr>
<tr>
<td>50–65</td>
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<td>-0.100</td>
<td>-0.030</td>
<td>-0.010</td>
<td>0</td>
<td>+0.002</td>
<td>+0.020</td>
<td>+0.032</td>
<td>+0.053</td>
<td>+0.087</td>
</tr>
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<td>-0.030</td>
<td>-0.010</td>
<td>0</td>
<td>+0.002</td>
<td>+0.020</td>
<td>+0.032</td>
<td>+0.059</td>
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<td>-0.012</td>
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<td>+0.023</td>
<td>+0.037</td>
<td>+0.071</td>
<td>+0.124</td>
</tr>
<tr>
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<td>-0.180</td>
<td>-0.120</td>
<td>-0.036</td>
<td>-0.012</td>
<td>0</td>
<td>+0.003</td>
<td>+0.023</td>
<td>+0.037</td>
<td>+0.079</td>
<td>+0.144</td>
</tr>
<tr>
<td>120–140</td>
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<td>-0.145</td>
<td>-0.043</td>
<td>-0.014</td>
<td>0</td>
<td>+0.003</td>
<td>+0.027</td>
<td>+0.043</td>
<td>+0.092</td>
<td>+0.170</td>
</tr>
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<td>140–160</td>
<td>-0.210</td>
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<td>-0.043</td>
<td>-0.014</td>
<td>0</td>
<td>+0.003</td>
<td>+0.027</td>
<td>+0.043</td>
<td>+0.100</td>
<td>+0.190</td>
</tr>
<tr>
<td>160–180</td>
<td>-0.230</td>
<td>-0.145</td>
<td>-0.043</td>
<td>-0.014</td>
<td>0</td>
<td>+0.003</td>
<td>+0.027</td>
<td>+0.043</td>
<td>+0.108</td>
<td>+0.210</td>
</tr>
<tr>
<td>180–200</td>
<td>-0.240</td>
<td>-0.170</td>
<td>-0.050</td>
<td>-0.015</td>
<td>0</td>
<td>+0.004</td>
<td>+0.031</td>
<td>+0.050</td>
<td>+0.122</td>
<td>+0.236</td>
</tr>
<tr>
<td>200–225</td>
<td>-0.260</td>
<td>-0.170</td>
<td>-0.050</td>
<td>-0.015</td>
<td>0</td>
<td>+0.004</td>
<td>+0.031</td>
<td>+0.050</td>
<td>+0.130</td>
<td>+0.258</td>
</tr>
<tr>
<td>225–250</td>
<td>-0.280</td>
<td>-0.170</td>
<td>-0.050</td>
<td>-0.015</td>
<td>0</td>
<td>+0.004</td>
<td>+0.031</td>
<td>+0.050</td>
<td>+0.140</td>
<td>+0.284</td>
</tr>
<tr>
<td>250–280</td>
<td>-0.300</td>
<td>-0.190</td>
<td>-0.056</td>
<td>-0.017</td>
<td>0</td>
<td>+0.004</td>
<td>+0.034</td>
<td>+0.056</td>
<td>+0.158</td>
<td>+0.315</td>
</tr>
<tr>
<td>280–315</td>
<td>-0.330</td>
<td>-0.190</td>
<td>-0.056</td>
<td>-0.017</td>
<td>0</td>
<td>+0.004</td>
<td>+0.034</td>
<td>+0.056</td>
<td>+0.170</td>
<td>+0.350</td>
</tr>
<tr>
<td>315–355</td>
<td>-0.360</td>
<td>-0.210</td>
<td>-0.062</td>
<td>-0.018</td>
<td>0</td>
<td>+0.004</td>
<td>+0.037</td>
<td>+0.062</td>
<td>+0.190</td>
<td>+0.390</td>
</tr>
<tr>
<td>355–400</td>
<td>-0.400</td>
<td>-0.210</td>
<td>-0.062</td>
<td>-0.018</td>
<td>0</td>
<td>+0.004</td>
<td>+0.037</td>
<td>+0.062</td>
<td>+0.208</td>
<td>+0.435</td>
</tr>
</tbody>
</table>
Specification of Fit

- A particular fit is specified by giving the **basic size** followed by **letter code** and **IT grades for hole and shaft**.
Specification of Fit

- Ex: a sliding fit of a nominally 32 mm diameter shaft & hub would be specified as **32H7/g6**
  - 32 mm basic size
  - hole with IT grade of 7 (ΔD in Table A–11)
  - shaft with fundamental deviation specified by letter code g (fundamental deviation δF in Table A–12)
  - shaft with IT grade of 6 (tolerance Δd in Table A–11)
# Preferred Fits (Clearance)

## Table 7–9

<table>
<thead>
<tr>
<th>Type of Fit</th>
<th>Description</th>
<th>Symbol</th>
</tr>
</thead>
</table>
| Clearance     | *Loose running fit:* for wide commercial tolerances or allowances on external members  

*Free running fit:* not for use where accuracy is essential, but good for large temperature variations, high running speeds, or heavy journal pressures  

*Close running fit:* for running on accurate machines and for accurate location at moderate speeds and journal pressures  

*Sliding fit:* where parts are not intended to run freely, but must move and turn freely and locate accurately  

*Locational clearance fit:* provides snug fit for location of stationary parts, but can be freely assembled and disassembled                                                                                                   | H11/c11 | H9/d9  |
|               |                                                                                                                                                                                                                                                                                                                                                                                                     |         | H8/f7  |
|               |                                                                                                                                                                                                                                                                                                                                                                                                     |         | H7/g6  |
|               |                                                                                                                                                                                                                                                                                                                                                                                                     |         | H7/h6  |
# Table 7–9

## Preferred Fits (Transition & Interference)

<table>
<thead>
<tr>
<th>Type of Fit</th>
<th>Description</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transition</td>
<td><em>Locational transition fit</em>: for accurate location, a compromise between clearance and interference</td>
<td>H7/k6</td>
</tr>
<tr>
<td></td>
<td><em>Locational transition fit</em>: for more accurate location where greater interference is permissible</td>
<td>H7/n6</td>
</tr>
<tr>
<td>Interference</td>
<td><em>Locational interference fit</em>: for parts requiring rigidity and alignment with prime accuracy of location but without special bore pressure requirements</td>
<td>H7/p6</td>
</tr>
<tr>
<td></td>
<td><em>Medium drive fit</em>: for ordinary steel parts or shrink fits on light sections, the tightest fit usable with cast iron</td>
<td>H7/s6</td>
</tr>
<tr>
<td></td>
<td><em>Force fit</em>: suitable for parts that can be highly stressed or for shrink fits where the heavy pressing forces required are impractical</td>
<td>H7/u6</td>
</tr>
</tbody>
</table>

Mohammad Suliman Abuhaiba, Ph.D., PE
Procedure to Size for Specified Fit

- Select description of desired fit: Table 7–9
- Obtain letter codes & IT grades from symbol for desired fit: Table 7–9
- Table A–11 (metric) or A–13 (inch) with IT grade numbers to obtain $\Delta D$ for hole & $\Delta d$ for shaft
- Table A–12 (metric) or A–14 (inch) with shaft letter code to obtain $\delta_F$ for shaft
Procedure to Size for Specified Fit

- For hole

\[
D_{\text{max}} = D + \Delta D \quad D_{\text{min}} = D \quad (7-36)
\]

- For shafts with clearance fits c, d, f, g, & h

\[
d_{\text{max}} = d + \delta_F \quad d_{\text{min}} = d + \delta_F - \Delta d \quad (7-37)
\]

- For shafts with interference fits k, n, p, s, & u

\[
d_{\text{min}} = d + \delta_F \quad d_{\text{max}} = d + \delta_F + \Delta d \quad (7-38)
\]
Example 7-7

Find the shaft and hole dimensions for a loose running fit with a 34-mm basic size.
Example 7-8

Find the hole and shaft limits for a medium drive fit using a basic hole size of 2 in.
Stress in Interference Fits (IF)

- IF generates pressure at interface
- Treat shaft as cylinder with uniform external pressure
- Treat hub as hollow cylinder with uniform internal pressure
Stress in Interference Fits

- The pressure at interface, (Eq.3–56) converted into terms of diameters,

\[ p = \frac{\delta}{d} \left( \frac{d^2 + d^2}{d_o^2 - d^2} + v_o \right) + \frac{d}{E_i} \left( \frac{d^2 + d_i^2}{d^2 - d_i^2} - v_i \right) \]  

(7-39)

- If both members are of same material,

\[ p = \frac{E \delta}{2d^3} \left[ \frac{(d_o^2 - d^2)(d^2 - d_i^2)}{d_o^2 - d_i^2} \right] \]  

(7-40)
Stress in Interference Fits

- $\delta$ is *diametral* interference

\[
\delta = d_{\text{shaft}} - d_{\text{hub}} \tag{7-41}
\]

- Taking into account the tolerances,

\[
\delta_{\text{min}} = d_{\text{min}} - D_{\text{max}} \tag{7-42}
\]

\[
\delta_{\text{max}} = d_{\text{max}} - D_{\text{min}} \tag{7-43}
\]
Stress in Interference Fits

From Eqs. (3–58) & (3–59), with radii converted to diameters, tangential stresses at the interface are

\[ \sigma_{t, \text{shaft}} = -p \frac{d^2 + d_i^2}{d^2 - d_i^2} \]  
(7–44)

\[ \sigma_{t, \text{hub}} = p \frac{d_o^2 + d^2}{d_o^2 - d^2} \]  
(7–45)
Stress in Interference Fits

- Radial stresses at the interface are

\[
\sigma_r, \text{ shaft } = -p \tag{7-46}
\]

\[
\sigma_r, \text{ hub } = -p \tag{7-47}
\]

- Tangential & radial stresses are orthogonal and can be combined using a failure theory
Torque Transmission from IF

- Estimate torque transmitted through IF by friction analysis at interface

\[ F_f = fN = f(pA) = f[p2\pi(d/2)l] = \pi fpld \]  \hspace{1cm} (7–48)

\[ T = \frac{F_f d}{2} = \pi fpld(d/2) \]

\[ T = \left(\frac{\pi}{2}\right) fpld^2 \]  \hspace{1cm} (7–49)

- Use min interference to determine min pressure to find max torque that the joint should be expected to transmit.

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Project

- Crushing and steel cutting machine
- Design considerations: Due Monday 13/10/2014
- Design specifications: Due Monday 13/10/2014
- action plan: Due Monday 13/10/2014

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Design considerations

- Cut steel with different shape and size into small pieces (with different mechanical properties)
- Expected capacity: 20 to 50 ton per day
Design specifications

- Cut steel with different shape
- size
- small pieces: 5*5*5cm max
- different mechanical properties: specify most critical mech properties: hardness, Ultimate, yield, ...
- Expected capacity: 30+-5
Action Plan

- Literature review: books, papers, videos, manufactures
- Concept design
- Detailed design
  ✓ Machine layout
  ✓ Design for strength
  ✓ For deflection
  ✓ For resonance
  ✓ Design for ease of manufacturing and assembly

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