Chapter 16

Turning Moment Diagrams and Flywheel

11/27/2014

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16.1. Introduction

- **Turning moment diagram** = graphical representation of turning moment or crank-effort for various positions of the crank
16.2. Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine

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16.2. Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine

- Turning moment on the crankshaft,

\[ T = F_p \times r \left( \sin \theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2 \theta}} \right) \]

- \( F_p \) = Piston effort
- Out stroke = curve abc
- In stroke = curve cde
16.2. Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine

- Work done = turning moment × angle turned
- work done = area of turning moment diagram per revolution
- In actual practice, engine is assumed to work against the mean resisting torque
- Area of rectangle aAFε is proportional to work done against the mean resisting torque
16.2. Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine

- When the turning moment is positive (when the engine torque is more than the mean resisting torque) as shown between points B and C (or D and E) in Fig. 16.1, the crankshaft accelerates and the work is done by the steam.
16.2. Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine

- When the turning moment is negative (when the engine torque is less than the mean resisting torque) as shown between points C and D in Fig. 16.1, the crankshaft retards and the work is done on the steam.
16.2. Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine

- $T = \text{Torque on the crankshaft at any instant}$
- $T_{\text{mean}} = \text{Mean resisting torque}$
- Accelerating torque on rotating parts of the engine = $T - T_{\text{mean}}$
- If $(T - T_{\text{mean}})$ is positive, the flywheel accelerates
- If $(T - T_{\text{mean}})$ is negative, the flywheel retards
16.3. Turning Moment Diagram for a 4 Stroke Cycle ICE

\[ T_{\text{max}} \]
\[ T_{\text{mean}} \]

- Positive loop
- Negative loop

Mean resisting torque

\[ 0 \quad \pi \quad 2\pi \quad 3\pi \quad 4\pi \]

- Suction
- Compression
- Working
- Exhaust

Crank angle

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16.3. Turning Moment Diagram for a 4 Stroke Cycle ICE

- Pressure inside engine cylinder is less than atmospheric pressure during the suction stroke, therefore a negative loop is formed.

- During compression stroke, work is done on gases, therefore a higher negative loop is obtained.

- During expansion or working stroke, fuel burns and gases expand, therefore a large positive loop is obtained. In this stroke, work is done by the gases.

- During exhaust stroke, work is done on gases, therefore a negative loop is formed.
16.4. Turning Moment Diagram for a Multi-cylinder Engine

![Diagram of turning moment for a multi-cylinder engine](image)

- **Resultant turning moment**
- **Mean torque**

- **Cylinder 1**
- **Cylinder 2**
- **Cylinder 3**

**Crank angle**

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16.4. Turning Moment Diagram for a Multi-cylinder Engine

- Resultant TMD = sum of turning moment diagrams for the three cylinders.
- 1<sup>st</sup> cylinder = high pressure cylinder
- 2<sup>nd</sup> cylinder = intermediate cylinder
- 3<sup>rd</sup> cylinder = low pressure cylinder
- Cranks, in case of three cylinders, are usually placed at 120° to each other.

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16.5. Fluctuation of Energy

The diagram illustrates the fluctuation of energy with respect to the crank angle. The turning moment is shown as $T_{max}$ and $T_{mean}$, with key points labeled A to F corresponding to the crank angle from 0° to 360°. The mean resisting torque is indicated by the highlighted section between B and D.

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16.5. Fluctuation of Energy

- **Fig. 16.1:** TMD for a single cylinder double acting steam engine

- When the crank moves from \( a \) to \( p \), work done by engine = area \( aBp \), whereas energy required is represented by area \( aABp \)

- The engine has done less work (equal to area \( aAB \)) than the requirement.

- This amount of energy is taken from flywheel and hence speed of the flywheel decreases.

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16.5. Fluctuation of Energy

- From \( p \) to \( q \), work done by engine = area \( pBbCq \), whereas requirement of energy is represented by area \( pBCq \)
- Therefore, engine has done more work than the requirement
- This excess work (area \( BbC \)) is stored in the flywheel and hence speed of flywheel increases while crank moves from \( p \) to \( q \).
16.5. Fluctuation of Energy

- Similarly, when the crank moves from \( q \) to \( r \), more work is taken from the engine than is developed. This loss of work is represented by area \( CcD \). To supply this loss, the flywheel gives up some of its energy and thus the speed decreases while the crank moves from \( q \) to \( r \).
16.5. Fluctuation of Energy

- As the crank moves from $r$ to $s$, excess energy is again developed given by area $DdE$ and the speed again increases.
- As the piston moves from $s$ to $e$, again there is a loss of work and the speed decreases.
16.5. Fluctuation of Energy

- **Fluctuations of energy**: variations of energy above and below the mean resisting torque line.
- Areas \( BbC, \ CcD, \ DdE, \) etc. represent fluctuations of energy.
16.5. Fluctuation of Energy

- The engine has a maximum speed either at \( q \) or at \( s \). This is due to the fact that the flywheel absorbs energy while the crank moves from \( p \) to \( q \) and from \( r \) to \( s \).
16.5. Fluctuation of Energy

- The engine has a minimum speed either at $p$ or at $r$. The reason is that the flywheel gives out some of its energy when the crank moves from $a$ to $p$ and $q$ to $r$.

- Maximum fluctuation of energy: difference between maximum and minimum energies
16.6. Determination of Maximum Fluctuation of Energy

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16.6. Determination of Maximum Fluctuation of Energy

- **Fig. 16.4**: TMD for a multi-cylinder engine
- Line AG: mean torque line
- $a_1, a_3, a_5 = \text{areas above mean torque line}$
- $a_2, a_4, a_6 = \text{areas below mean torque line}$
16.6. Determination of Maximum Fluctuation of Energy

- Let energy in the flywheel at $A = E$
- Energy at $B = E + a_1$
- Energy at $C = E + a_1 - a_2$
- Energy at $D = E + a_1 - a_2 + a_3$
- Energy at $E = E + a_1 - a_2 + a_3 - a_4$
- Energy at $F = E + a_1 - a_2 + a_3 - a_4 + a_5$
- Energy at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6 = E$

Energy at $A$ (cycle repeats after $G$)
16.6. Determination of Maximum Fluctuation of Energy

- Suppose that the greatest of these energies is at $B$ and least at $E$
- Max energy in flywheel = $E + a_1$
- Min energy in flywheel = $E + a_1 - a_2 + a_3 - a_4$
- Max fluctuation of energy, $E = \text{Max energy} - \text{Min energy} = (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$
16.7. Coefficient of Fluctuation of Energy

\[ C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}} \]

Work done per cycle = \[ T_{\text{mean}} \times \theta \]

- \[ \theta = \text{Angle turned in one revolution} \]
- \[ \theta = 2\pi \text{ in case of steam engine and two stroke ICE} \]
- \[ \theta = 4\pi \text{ in case of four stroke ICE} \]

\[ T_{\text{mean}} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega} \]

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16.7. Coefficient of Fluctuation of Energy

\[
\text{Work done per cycle} = \frac{P \times 60}{n}
\]

- \( n \) = Number of working strokes per minute
- \( n = N \), in case of steam engines & two stroke ICE
- \( n = N /2 \), in case of four stroke ICE

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## 16.7. Coefficient of Fluctuation of Energy

**Table 16.1:** $C_E$ for steam and ICEs

<table>
<thead>
<tr>
<th>Type of engine</th>
<th>$C_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single cylinder, double acting steam engine</td>
<td>0.21</td>
</tr>
<tr>
<td>Cross-compound steam engine</td>
<td>0.096</td>
</tr>
<tr>
<td>Single cylinder, single acting, four stroke gas engine</td>
<td>1.93</td>
</tr>
<tr>
<td>Four cylinders, single acting, four stroke gas engine</td>
<td>0.066</td>
</tr>
<tr>
<td>Six cylinders, single acting, four stroke gas engine</td>
<td>0.031</td>
</tr>
</tbody>
</table>

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16.8. Flywheel

- A flywheel serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.

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16.8. Flywheel

- In case of steam engines, ICEs, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke.

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16.8. Flywheel

- In ICE, energy is developed only during expansion or power stroke which is much more than engine load and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines.
16.8. Flywheel

- The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed.

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16.8. Flywheel

- When the flywheel absorbs energy, its speed increases and when it releases energy, the speed decreases.
- A flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed.
16.8. Flywheel

A flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.
16.8. Flywheel

- In machines where the operation is intermittent like crushers, the flywheel stores energy from the power source during the greater portion of the operating cycle and gives it up during a small period of the cycle.

- The energy from the power source to the machines is supplied practically at a constant rate throughout the operation.
16.8. Flywheel

- The governor in an engine regulates the mean speed of an engine when there are variations in the load, e.g., when the load on the engine increases, it becomes necessary to increase the supply of working fluid.

- When the load decreases, less working fluid is required.
16.8. Flywheel

- The governor automatically controls the supply of working fluid to the engine with the varying load condition and keeps the mean speed of the engine within certain limits.
16.8. Flywheel

- The flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed.
- It does not control the speed variations caused by the varying load.
16.9. Coefficient of Fluctuation of Speed

- **Max fluctuation of speed**: difference between max and min speeds during a cycle
- **Coefficient of fluctuation of speed**: ratio of max fluctuation of speed to mean speed
- \( N_1 \) & \( N_2 \) = Max & min speeds in rpm during the cycle
- \( N = (N_1 + N_2)/2 \)

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16.9. Coefficient of Fluctuation of Speed

\[ C_s = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2} \]

\[ = \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \]

\[ = \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \]

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16.9. Coefficient of Fluctuation of Speed

- $C_s$ is a limiting factor in the design of flywheel.
- It varies depending upon the nature of service to which the flywheel is employed.
- *Coefficient of steadiness* = reciprocal of $C_s$

$$m = \frac{1}{C_s} = \frac{N}{N_1 - N_2}$$
16.10. Energy Stored in a Flywheel

- When a flywheel absorbs energy, its speed increases and when it gives up energy, its speed decreases.
- $m =$ Mass of flywheel
- $k =$ Radius of gyration of flywheel
- $I =$ Mass moment of inertia of flywheel about its axis of rotation $= m.k^2$
16.10. Energy Stored in a Flywheel

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16.10. Energy Stored in a Flywheel

\[ N = \text{Mean speed during the cycle in r.p.m.} = \frac{N_1 + N_2}{2}, \]

\[ \omega = \text{Mean angular speed during the cycle in rad/s} = \frac{\omega_1 + \omega_2}{2}, \]

\[ C_s = \text{Coefficient of fluctuation of speed,} = \frac{N_1 - N_2}{N} \quad \text{or} \quad \frac{\omega_1 - \omega_2}{\omega} \]
16.10. Energy Stored in a Flywheel

- Mean kinetic energy of the flywheel

\[ E = \frac{1}{2} \times I \cdot \omega^2 = \frac{1}{2} \times m.k^2 \cdot \omega^2 \]

- Maximum fluctuation of energy

\[ \Delta E = \text{Maximum K.E.} - \text{Minimum K.E.} \]

\[ = \frac{1}{2} \times I \left( \omega_1 \right)^2 - \frac{1}{2} \times I \left( \omega_2 \right)^2 = \frac{1}{2} \times I \left[ \left( \omega_1 \right)^2 - \left( \omega_2 \right)^2 \right] \]

\[ = \frac{1}{2} \times I \left( \omega_1 + \omega_2 \right) \left( \omega_1 - \omega_2 \right) = I \cdot \omega \left( \omega_1 - \omega_2 \right) \]
16.10. Energy Stored in a Flywheel

\[ \Delta E = I \omega^2 \left( \frac{\omega_1 - \omega_2}{\omega} \right) \]

\[ = I \omega^2 C_S = m k^2 \omega^2 C_S \]

\[ = 2.6 C_S \text{ (in N} \cdot \text{m or joules)} \]

- Radius of gyration \((k)\) may be taken equal to mean radius of the rim \((R)\),

\[ \Delta E = m R^2 \omega^2 C_S = m v^2 C_S \]
16.10. Energy Stored in a Flywheel

\[ \Delta E = I \times \frac{2\pi N}{60} \left( \frac{2\pi N_1}{60} - \frac{2\pi N_2}{60} \right) = \frac{4\pi^2}{3600} \times I \times N \left( N_1 - N_2 \right) \]

\[ = \frac{\pi^2}{900} \times m.k^2.N \left( N_1 - N_2 \right) \]

\[ = \frac{\pi^2}{900} \times m.k^2.N^2.C_s \]

\[ \ldots \left( \therefore C_s = \frac{N_1 - N_2}{N} \right) \]
Example 16.1

The mass of flywheel of an engine is 6.5 tons and the radius of gyration is 1.8 m. It is found from the turning moment diagram that the fluctuation of energy is 56 kN.m. If the mean speed of the engine is 120 rpm, find the maximum and minimum speeds.
Example 16.2

The flywheel of a steam engine has a radius of gyration of 1 m and mass 2500 kg. The starting torque of the steam engine is 1500 N.m and may be assumed constant. Determine:

1. The angular acceleration of the flywheel
2. The kinetic energy of the flywheel after 10 seconds from the start.
Example 16.3

A horizontal cross compound steam engine develops 300 kW at 90 rpm. The coefficient of fluctuation of energy as found from the turning moment diagram is to be 0.1 and the fluctuation of speed is to be kept within ± 0.5% of the mean speed. Find the weight of the flywheel required, if the radius of gyration is 2 m.
Example 16.4

The turning moment diagram for a petrol engine is drawn to the following scales: Turning moment, 1 mm = 5 N.m; crank angle, 1 mm = 1°. The turning moment diagram repeats itself at every half revolution of the engine and the areas above and below the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm². The rotating parts are equivalent to a mass of 36 kg at a radius of gyration of 150 mm. Determine the coefficient of fluctuation of speed when the engine runs at 1800 rpm.

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Example 16.4
Example 16.5

The TMD for a multicylinder engine has been drawn to a scale 1 mm = 600 N.m vertically and 1 mm = 3° horizontally. The intercepted areas between the output torque curve and the mean resistance line, taken in order from one end, are as follows: + 52, − 124, + 92, − 140, + 85, − 72 and + 107 mm², when the engine is running at a speed of 600 rpm. If the total fluctuation of speed is not to exceed ± 1.5% of the mean, find the necessary mass of the flywheel of radius 0.5 m.

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Example 16.5

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Example 16.6

A shaft fitted with a flywheel rotates at 250 rpm and drives a machine. The torque of machine varies in a cyclic manner over a period of 3 revolutions. The torque rises from 750 N.m to 3000 N.m uniformly during 1/2 revolution and remains constant for the following revolution. It then falls uniformly to 750 N.m during the next 1/2 revolution and remains constant for one revolution, the cycle being repeated thereafter. Determine the power required to drive the machine and percentage fluctuation in speed, if the driving torque applied to the shaft is constant and the mass of the flywheel is 500 kg with radius of gyration of 600 mm.
Example 16.6

Turning moment (N-m)

Delta E

1/2 Rev. 1 Rev. 1/2 Rev.

Crank angle

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Example 16.7

During forward stroke of the piston of the double acting steam engine, the turning moment has the maximum value of 2000 N.m when the crank makes an angle of 80° with the IDC. During the backward stroke, the maximum turning moment is 1500 N.m when the crank makes an angle of 80° with the ODC. The turning moment diagram for the engine may be assumed for simplicity to be represented by two triangles. If the crank makes 100 rpm and the radius of gyration of the flywheel is 1.75 m, find the coefficient of fluctuation of energy and the mass of the flywheel to keep the speed within ± 0.75% of the mean speed. Also determine the crank angle at which the speed has its minimum and maximum values.
Example 16.7

\[ T_{mean} \]

 Turning moment (N-m)

 I.D.C. \( \theta_c \) \( \frac{4\pi}{9} \) \( \theta_d \) \( \frac{\pi}{9} \) O.D.C. \( \frac{13\pi}{9} \) 2\( \pi \)

\( \Delta E \)

Forward stroke \( \rightarrow \) Backward stroke \( \rightarrow \)

Crank angle

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Example 16.8

A three cylinder single acting engine has its cranks set equally at 120° and it runs at 600 rpm. The torque-crank angle diagram for each cycle is a triangle for the power stroke with a maximum torque of 90 N.m at 60° from IDC of corresponding crank. The torque on the return stroke is sensibly zero. Determine:

1. power developed
2. coefficient of fluctuation of speed, if the mass of the flywheel is 12 kg and has a radius of gyration of 80 mm
3. coefficient of fluctuation of energy
4. maximum angular acceleration of the flywheel.

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Example 16.8

A graph showing the turning moment (N-m) over the crank angle from 0 to 360°. The graph indicates three distinct phases labeled 1, 2, and 3, with corresponding turning moments of 90, 45, and unspecified values, respectively. The graph also highlights the cylinder's position at each phase of the crank angle.
Example 16.8

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Example 16.9

A single cylinder, single acting, four stroke gas engine develops 20 kW at 300 rpm. The work done by the gases during the expansion stroke is three times the work done on the gases during the compression stroke, the work done during the suction and exhaust strokes being negligible. If the total fluctuation of speed is not to exceed ± 2 % of the mean speed and the TMD during compression and expansion is assumed to be triangular in shape, find the moment of inertia of the flywheel.

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Example 16.9

![Diagram showing turning moment and crank angle with sections labeled Suction, Compression, Expansion, and Exhaust.]

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Example 16.10

The TMD for a four stroke gas engine may be assumed for simplicity to be represented by four triangles, the areas of which from the line of zero pressure are as follows: Suction stroke = \(0.45 \times 10^{-3} \text{ m}^2\); Compression stroke = \(1.7 \times 10^{-3} \text{ m}^2\); Expansion stroke = \(6.8 \times 10^{-3} \text{ m}^2\); Exhaust stroke = \(0.65 \times 10^{-3} \text{ m}^2\). Each \(\text{m}^2\) of area represents 3 MN.m of energy. Assuming the resisting torque to be uniform, find the mass of the rim of a flywheel required to keep the speed between 202 and 198 rpm. The mean radius of the rim is 1.2 m.
Example 16.10

The diagram illustrates the turning moment throughout the engine cycle. The turning moment is divided into four sections: Suction (0 to 2π), Compression (2π to 3π), Expansion (3π to 4π), and Exhaust (4π to 0). Each section is represented by a triangle labeled with subscripts 1, 2, 3, and 4, indicating the specific moment changes during these phases. The diagram also shows the line of zero pressure and the maximum turning moment (T_max) and mean turning moment (T_mean).
Example 16.11
The turning moment curve for an engine is represented by the equation, 
\[ T = (20000 + 9500 \sin 2\theta - 5700 \cos 2\theta) \text{ N.m}, \] 
where \( \theta \) is angle moved by the crank from IDC. If the resisting torque is constant, find:

1. **Power developed by the engine**

2. **Moment of inertia of flywheel in kg-m\(^2\), if total fluctuation of speed is not exceed 1% of mean speed which is 180 rpm**

3. **Angular acceleration of flywheel when crank has turned through 45° from IDC**
Example 16.11

![Diagram showing the turning moment and crank angle relationship with shaded areas representing different energy changes.](Image)
Example 16.12

A certain machine requires a torque of \((5000 + 500 \sin \theta)\) N.m to drive it, where \(\theta\) is angle of rotation of shaft measured from certain datum. The machine is directly coupled to an engine which produces a torque of \((5000 + 600 \sin 2\theta)\) N.m. The flywheel and the other rotating parts attached to the engine has a mass of 500 kg at a radius of gyration of 0.4 m. If the mean speed is 150 rpm, find:

1. fluctuation of energy
2. total percentage fluctuation of speed
3. maximum and minimum angular acceleration of the flywheel and the corresponding shaft position.
Example 16.12

The diagram illustrates the turning moment over a cycle from 0 to 360°. The shaded area represents the change in energy, denoted as $\Delta E$. The turning moment is marked at points A, B, C, D, and E, with angular positions at $\theta_B$, 90°, 180°, 270°, and $\theta_D$, respectively.
Example 16.13

The equation of the turning moment curve of a three crank engine is \((5000 + 1500 \sin 3\theta)\) N.m, where \(\theta\) is the crank angle in radians. The moment of inertia of the flywheel is 1000 kg.m² and the mean speed is 300 rpm. Calculate:

1. power of the engine
2. maximum fluctuation of the speed of the flywheel in percentage when (i) the resisting torque is constant, and (ii) the resisting torque is \((5000 + 600 \sin \theta)\) N.m.
Example 16.13
16.11. Dimensions of the Flywheel Rim

- $D =$ Mean diameter of rim
- $R =$ Mean radius of rim
- $A =$ x-sectional area of rim
- $\rho =$ Density of rim material
- $N =$ Speed of flywheel in rpm
- $\omega =$ Angular velocity of the flywheel in rad/s
- $v =$ Linear velocity at mean radius = $\omega R$
- $\sigma =$ hoop stress due to centrifugal force
16.11. Dimensions of the Flywheel Rim

- Volume of the small element $= A \times R \cdot \delta \theta$

  \[
  dm = \text{Density} \times \text{volume} = \rho \cdot A \cdot R \cdot \delta \theta
  \]

  \[
  dF = dm \cdot \omega^2 \cdot R = \rho \cdot A \cdot R^2 \cdot \omega^2 \cdot \delta \theta
  \]

  \[
  dF \cdot \sin \theta = \rho \cdot A \cdot R^2 \cdot \omega^2 \cdot \delta \theta \cdot \sin \theta
  \]

- Total vertical upward force tending to burst the rim across the diameter $X Y$

  \[
  = \rho \cdot A \cdot R^2 \cdot \omega^2 \int_{0}^{\pi} \sin \theta \cdot d\theta = \rho \cdot A \cdot R^2 \cdot \omega^2 \left[ - \cos \theta \right]_{0}^{\pi}
  \]
16.11. Dimensions of the Flywheel Rim

\[ 2 \cdot \rho \cdot A \cdot R^2 \cdot \omega^2 = 2\sigma \cdot A \]
\[ \sigma = \rho \cdot R^2 \cdot \omega^2 = \rho \cdot v^2 \]
\[ v = \sqrt{\frac{\sigma}{\rho}} \]

\[ m = \text{Volume} \times \text{density} = \pi D A \rho \]

\[ A = \frac{m}{\pi D \rho} \]
Example 16.14

The turning moment diagram for a multi-cylinder engine has been drawn to a scale of 1 mm to 500 N.m torque and 1 mm to 6° of crank displacement. The intercepted areas between output torque curve and mean resistance line taken in order from one end, in are – 30, + 410, – 280, + 320, – 330, + 250, – 360, + 280, – 260 mm², when the engine is running at 800 rpm. The engine has a stroke of 300 mm and the fluctuation of speed is not to exceed ± 2% of the mean speed. Determine a suitable diameter and cross-section of the flywheel rim for a limiting value of the safe centrifugal stress of 7 MPa. The material density may be assumed as 7200 kg/m³. The width of the rim is to be 5 times the thickness.
Example 16.14

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Example 16.15

A single cylinder double acting steam engine develops 150 kW at a mean speed of 80 rpm. The coefficient of fluctuation of energy is 0.1 and the fluctuation of speed is $\pm 2\%$ of mean speed. If the mean diameter of the flywheel rim is 2 m and the hub and spokes provide 5% of the rotational inertia of the flywheel, find the mass and cross-sectional area of the flywheel rim. Assume the density of the flywheel material (which is cast iron) as 7200 kg/m$^3$.

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Example 16.16

A multi-cylinder engine is to run at a speed of 600 rpm. On drawing the turning moment diagram to a scale of 1 mm = 250 N.m and 1 mm = 3 °, the areas above and below the mean torque line in mm² are: + 160, – 172, + 168, – 191, + 197, – 162. The speed is to be kept within ± 1% of the mean speed of the engine. Calculate the necessary moment of inertia of the flywheel. Determine the suitable dimensions of a rectangular flywheel rim if the breadth is twice its thickness. The density of the cast iron is 7250 kg/m³ and its hoop stress is 6 MPa. Assume that the rim contributes 92% of the flywheel effect.
Example 16.16

![Graph showing turning moment and crank angle with points labeled A to G(A) and values 160, 172, 168, 191, 197, 162. The mean torque line is indicated.](image)

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Example 16.17

The TMD of a four stroke engine may be assumed for the sake of simplicity to be represented by four triangles in each stroke. The areas of these triangles are as follows:

Suction stroke = $5 \times 10^{-5} \, m^2$; Compression stroke = $21 \times 10^{-5} \, m^2$; Expansion stroke = $85 \times 10^{-5} \, m^2$; Exhaust stroke = $8 \times 10^{-5} \, m^2$. All the areas excepting expression stroke are negative. Each $m^2$ of area represents 14 MN.m of work. Assuming the resisting torque to be constant, determine the moment of inertia of the flywheel to keep the speed between 98 rpm and 102 rpm. Also find the size of a rim-type flywheel based on the minimum material criterion, given that density of flywheel material is 8150 kg/m$^3$; the allowable tensile stress of the flywheel material is 7.5 MPa. The rim cross-section is rectangular, one side being four times the length of the other.
Example 16.17

Turning moment

$T_{max}$

$T_{mean}$

Line of zero pressure

Suction

Comp

Expansion

Exhaust

Crank angle

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Example 16.18

An otto cycle engine develops 50 kW at 150 rp. with 75 explosions per minute. The change of speed from the commencement to the end of power stroke must not exceed 0.5% of mean on either side. Find the mean diameter of the flywheel and a suitable rim cross section having width four times the depth so that the hoop stress does not exceed 4 MPa. Assume that the flywheel stores 16/15 times the energy stored by the rim and the work done during power stroke is 1.40 times the work done during the cycle. Density of rim material is 7200 kg/m³.

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Example 16.18

$T_{max}$

$T_{mean}$

Suction → Compression → Power of working → Exhaust

Crank angle

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