Chapter (3)
Water Flow in Pipes
Bernoulli Equation
Recall fluid mechanics course, the Bernoulli equation is:

\[ \frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 - h_P + h_T + \sum h_L \]

Here, we want to study how to calculate the total losses (\( \sum h_L \)):

\[ \sum h_L = \text{Major Losses} + \text{Minor Losses} \]

Major Losses
Occurs mainly due to the pipe friction and viscous dissipation in the flowing water.
The major head loss is termed by (\( h_f \)).

There are several formulas have been developed to calculate major losses:

1. Darcy-Weisbach Formula:
Is the most popular formula used to calculate major losses and it has the following form:

\[ h_f = f \left( \frac{L}{D} \right) \left( \frac{V^2}{2g} \right) \quad , \quad V = \frac{Q}{A} = \frac{Q}{\pi D^2} \quad \Rightarrow \quad h_f = \frac{8 f L Q^2}{\pi^2 g D^5} \]

\( L = \) length of pipe (m)
\( D = \) diameter of pipe (m)
\( \frac{V^2}{2g} = \) velocity head (m)
\( Q = \) flow rate (m\(^3\)/s)
\( f = \) friction factor

Friction Factor (\( f \)):
For laminar flow (\( R_e < 2000 \)) the friction factor depends only on \( R_e \):

\[ f = \frac{64}{R_e} \] (smooth pipe)

If the pipe is smooth (\( e = 0 \)) and the flow is turbulent with (\( 4000 < R_e < 10^5 \))

\( \Rightarrow \) The friction factor depends only on \( R_e \):

\[ f = \frac{0.316}{R_e^{0.25}} \]
For **turbulent flow** \((R_e > 4000)\) the friction factor can be founded by **Moody diagram**.

**To use Moody diagram you need the followings:**
- The Reynolds number: \(R_e\)
- The relative roughness: \(\frac{e}{D}\)

**Reynolds Number:** \(R_e\)
\[
R_e = \frac{\rho \, V \, D}{\mu} \quad \text{but} \quad v = \frac{\mu}{\rho} \quad \Rightarrow \quad R_e = \frac{V \, D}{v}
\]

\(\mu\) = dynamic viscosity (Pa.s) or kg/m. s
\(v\) = kinematic viscosity (m²/s)

\(V\) = mean velocity in the pipe
\(D\) = pipe diameter

**Note:**

Kinematic viscosity depends on the fluid temperature and can be calculated from the following formula:
\[
v = 497 \times 10^{-6} \left(\frac{T}{T + 42.5}\right)^{1.5} \quad \text{T: is fluid temperature in Celsius}
\]

**Relative Roughness:** \(\frac{e}{D}\)

\(e\) (mm) = Roughness height (internal roughness of the pipe) and it depends mainly on the pipe material.

Table (3.1) in slides of Dr.Khalil shows the value of \(e\) for different pipe materials.

**The following figure exhibits Moody diagram:**
Note:
Moody diagram is a graphical representation of Colebrook-White formula:
\[
\frac{1}{\sqrt{f}} = -2 \log \left( \frac{e/D}{3.7} + \frac{2.51}{R_e\sqrt{f}} \right)
\]

Empirical Formulas for Friction Head Losses
These formulas gives exact value for friction head losses and each formula extensively used in a specific field, for example, Hazen-Williams formula is used extensively in water supply systems and most software’s (like WaterCAD) used it in analyzing and design of water networks. However, Manning equation us extensively used in open channel, and in designing of waste water networks.
See these formulas from the slides of Dr.Khalil.
**Minor Losses**

Occurs due to the change of the velocity of the flowing fluid in the magnitude or in the direction. So, the minor losses at:

- Valves.
- Tees.
- Bends.
- Contraction and Expansion.

The minor losses are termed by \( h_m \) and have a common form:

\[
h_m = K_L \frac{V^2}{2g}
\]

\( K_L \) = minor losses coefficient and it depends on the type of fitting

**Minor Losses Formulas**

<table>
<thead>
<tr>
<th>Minor Losses</th>
<th>Entrance of a pipe: ( h_{ent} = K_{entr} \frac{v^2}{2g} )</th>
<th>Exit of a pipe: ( h_{exit} = K_{exit} \frac{v^2}{2g} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sudden Contraction: ( h_{sc} = K_{sc} \frac{v_2^2}{2g} )</td>
<td>Sudden Expansion: ( h_{se} = K_{se} \frac{v_1^2}{2g} ) or ( h_{se} = \frac{(v_1 - v_2)^2}{2g} )</td>
<td></td>
</tr>
<tr>
<td>Gradual Enlargement: ( h_{ge} = K_{ge} \frac{(v_1^2 - v_2^2)}{2g} )</td>
<td>Gradual Contraction: ( h_{gc} = K_{gc} \frac{(v_2^2 - v_1^2)}{2g} )</td>
<td></td>
</tr>
<tr>
<td>Bends in pipes: ( h_{bend} = K_{bend} \frac{v^2}{2g} )</td>
<td>Pipe Fittings: ( h_v = K_v \frac{v^2}{2g} )</td>
<td></td>
</tr>
</tbody>
</table>

The value of \( K \) for each fitting can be estimated from tables exist in slides of Dr.Khalil.

You must save the following values of \( K \):

- For sharp edge entrance: \( K = 0.5 \)
- For all types of exists: \( K = 1 \).
Problems

1. In the shown figure below, the smaller tank is 50m in diameter. **Find the flow rate**, \( Q \). Assume **laminar flow** and neglect minor losses.

Take \( \mu = 1.2 \times 10^{-3} \) kg/m.s \( \rho = 788 \) kg/m^3

**Solution**

For laminar \( f = \frac{64}{R_e} \)

\[
R_e = \frac{\rho V D}{\mu} = \frac{788 \times 2 \times 10^{-3} \times V}{1.2 \times 10^{-3}}
\]

\( \rightarrow R_e = 1313.33 \) \( V \) (substitute in \( f \))

\[
f = \frac{64}{R_e} = \frac{64}{1313.33 \text{ V}} = \frac{0.0487}{V} \ll (1)
\]

Now by applying Bernoulli’s equation from the **free surface** of upper to lower reservoir (Points 1 and 2).

\[
\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + \sum h_L
\]

\( P_1 = P_2 = V_1 = V_2 = 0.0 \)

Assume the datum at point (2) \( \rightarrow \)

\( 0 + 0 + (0.4 + 0.6) = 0 + 0 + 0 + \sum h_L \rightarrow \sum h_L = 1 \text{m} \)

\( \sum h_L = 1 \text{m} = h_f \) in the pipe that transport fluid from reservoir 1 to 2

\( h_f = f \left( \frac{L}{D} \right) \left( \frac{V^2}{2g} \right) \quad L = (0.4 + 0.8) = 1.2 \text{m} \quad D = 0.002 \text{m} \)

\( 1 = f \left( \frac{1.2}{0.002} \right) \left( \frac{V^2}{19.62} \right) \gg 2 \rightarrow \text{Substitute from (1)in (2) } \rightarrow \)

\( 1 = \frac{0.0487}{V} \times \left( \frac{1.2}{0.002} \right) \left( \frac{V^2}{19.62} \right) \rightarrow V = 0.671 \text{ m/s} \)

\( Q = A \times V = \frac{\pi}{4} \times 0.002^2 \times 0.671 = 2.1 \times 10^{-6} \text{ m}^3/\text{s} \).
2.
A uniform pipeline, 5000m long, 200mm in diameter and roughness size of 0.03mm, conveys water at 20°C ($v = 1.003 \times 10^{-6} \text{ m}^2/\text{s}$) between two reservoirs as shown in the figure. The difference in water level between the reservoirs is 50m. Include all minor losses in your calculations, determine the discharge.

*Note:* the valve produces a head loss of $(10V^2/2g)$ and the entrance to and exit from the pipe are sharp.

![Diagram of water flow in pipes](image)

**Solution**

Applying Bernoulli’s equation between the two reservoirs

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + \sum h_L$$

$P_1 = P_2 = V_1 = V_2 = 0.0$

Assume the datum at lower reservoir (point 2) →

$0 + 0 + 50 = 0 + 0 + 0 + \sum h_L \rightarrow \sum h_L = 50m$

$$\sum h_L = h_f + h_m = 50$$

**Major Losses:**

$$h_f = f \left( \frac{L}{D} \right) \left( \frac{V^2}{2g} \right) = h_f = f \times \left( \frac{5000}{0.2} \right) \left( \frac{V^2}{19.62} \right) = 1274.21 \text{ fV}^2$$

**Minor losses:**

For Entrance:

$$h_{ent} = K_{entr} \frac{V^2}{2g} \text{ (For sharp edge entrance, the value of } K_{entr} = 0.5) \rightarrow$$
\[ h_{\text{ent}} = 0.5 \times \frac{V^2}{19.62} = 0.0255 \, V^2 \]

For Exit:
\[ h_{\text{exit}} = K_{\text{exit}} \frac{V^2}{2g} \quad (\text{For exit: } K_{\text{exit}} = 1) \rightarrow \]
\[ h_{\text{exit}} = 1 \times \frac{V^2}{19.62} = 0.051 \, V^2 \]

For Valve:
\[ h_{\text{valve}} = 10 \times \frac{V^2}{2g} = 0.51V^2 \]
\[ h_m = (0.0255 + 0.051 + 0.51)V^2 = 0.5865 \, V^2 \]

\[ \sum h_L = 1274.21 \, f \, V^2 + 0.5865 \, V^2 = 50 \rightarrow V^2 = \frac{50}{1274.21 \, f + 0.5865} \quad (1) \]
\[ \frac{e}{D} = \frac{0.03}{200} = 0.00015 \]

In all problems like this (flow rate or velocity is unknown), the best initial value for \( f \) can be found as following:

Draw a horizontal line (from left to right) starts from the value of \( \frac{e}{D} \) till intercept with the vertical axis of the Moody chart and the initial \( f \) value is the intercept as shown in the following figure:
So as shown in the above figure, the initial value of $f = 0.0128$

Substitute in Eq. (1) → $V^2 = \frac{50}{1274.21 \times 0.0128 + 0.5865} = 2.96$

$→ V = 1.72 \text{ m/s}$

$→ R_e = \frac{V D}{\nu} = \frac{1.72 \times 0.2}{1.003 \times 10^{-6}} = 3.4 \times 10^5$ and $\frac{e}{D} = 0.00015$ → Moody

$f = 0.016 → \text{Substitute in Eq. (1) → } V = 1.54 → R_e = 3.1 \times 10^5$

$→ \text{Moody } f \cong 0.016$

So, the velocity is $V = 1.54 \text{ m/s}$

$Q = A \times V = \frac{\pi}{4} \times 0.2^2 \times 1.54 = 0.0483 \text{ m}^3/\text{s}$ ✓.

3.

The pipe shown in the figure below contains water flowing at a flow rate of $0.0065 \text{ m}^3/\text{s}$. The difference in elevation between points A and B is 11m.

For the pressure measurement shown,

a) What is the direction of the flow?

b) What is the total head loss between points A and B?

c) What is the diameter of the pipe?

**Given data:**

$v = 10^{-6} \text{ m}^2/\text{s} \ , \ e = 0.015 \text{mm} \ , \ \text{Pipe length} = 50\text{m} \ , \ 1\text{atm} = 10^5 \text{ Pa}.$
Solution

a) Direction of flow??
To know the direction of flow, we calculate the total head at each point, and then the fluid will move from the higher head to lower head.
Assume the datum is at point A:
Total head at point A:
\[
\frac{P_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{2.5 \times 10^5}{9810} + \frac{v_A^2}{2g} + 0 = 25.484 + \frac{v_A^2}{2g}
\]
Total head at point BA:
\[
\frac{P_B}{\rho g} + \frac{v_B^2}{2g} + z_B = \frac{2 \times 10^5}{9810} + \frac{v_B^2}{2g} + 11 = 31.39 + \frac{v_B^2}{2g}
\]
But, \(V_A = V_B\) (Since there is the same diameter and flow at A and B)
So, the total head at B is larger than total head at A >> the water is flowing from B (upper) to A (lower) ✓.

b) Head loss between A and B??
Apply Bernoulli’s equation between B and A (from B to A)
\[
\frac{P_B}{\rho g} + \frac{v_B^2}{2g} + z_B = \frac{P_A}{\rho g} + \frac{v_A^2}{2g} + z_A + \sum h_L
\]
\[
31.39 + \frac{v_B^2}{2g} = 25.484 + \frac{v_A^2}{2g} + \sum h_L\]
But \(V_A = V_B\) \(\rightarrow\) \(\sum h_L = 5.9\) m ✓.

c) Diameter of the pipe??
\[
\sum h_L = h_f + h_m = 5.9\) m (no minor losses) \(\rightarrow\) \(\sum h_L = h_f = 5.9\) m
\[
h_f = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)
\]
\[
5.9 = f\left(\frac{50}{D}\right)\times\left(\frac{V^2}{19.62}\right)
\]
\[
V = \frac{Q}{A} = \frac{0.0065}{\frac{\pi}{4} \times D^2} = \frac{0.00827}{D^2} \rightarrow V^2 = \frac{6.85 \times 10^{-5}}{D^4} \rightarrow
\]
\[
5.9 = f\left(\frac{50}{D}\right)\times\left(\frac{6.85 \times 10^{-5}}{19.62 D^4}\right) \rightarrow 5.9 = f \times \frac{0.000174}{D^5}
\]
→ \( D^5 = 2.96 \times 10^{-5} f \) → \( D = (2.96 \times 10^{-5})^{\frac{1}{5}} \times f^{\frac{1}{5}} \) → \( D = 0.124 f^{\frac{1}{5}} \)

\[ R_e = \frac{D^2}{10^{-6}} \times \frac{0.00827}{D} = \frac{0.00827}{10^{-6} D} \]

Now, assume the initial value for \( f \) is 0.02 (random guess)

→ \( D = 0.124 \times 0.02^{\frac{1}{5}} = 0.0567 \) m → \( R_e = \frac{0.00827}{10^{-6} \times 0.0567} = 1.46 \times 10^5 \)

→ \( e = \frac{0.015}{56.7} = 0.00026 \) → Moody → \( f \approx 0.019 \)

→ \( D = 0.124 \times 0.019^{\frac{1}{5}} = 0.056 \) m → \( R_e = \frac{0.00827}{10^{-6} \times 0.056} = 1.47 \times 10^5 \)

→ \( e = \frac{0.015}{56} = 0.00027 \) → Moody → \( f \approx 0.019 \)

So, the diameter of the pipe is 0.056 m = 56 mm ✓.

4.

In the shown figure, the connecting pipe is commercial steel 6 cm in diameter having a roughness height of 0.045mm. Determine the direction of flow, and then calculate the flow rate. The fluid is water \( (\mu = 1 \times 10^{-3} \text{ kg/m.s}) \). Neglect Minor losses.

Solution
Assume the datum is at point 1:
Total head at point A:
\[ P_1 + \frac{v_1^2}{2g} + z_1 = \frac{200 \times 10^3}{9810} + 0 + 0 = 20.387 \text{ m} \]
Total head at point 2:
\[ P_2 + \frac{v_2^2}{2g} + z_2 = 0 + 0 + 15 = 15 \text{ m} \]
So, the total head at 1 is larger than total head at 2>> the water is flowing from 1 to 2 ✓.

Applying Bernoulli’s equation between 1 and 2:
\[ P_1 + \frac{v_1^2}{2g} + z_1 = P_2 + \frac{v_2^2}{2g} + z_2 + \sum h_L \]
\[ 20.387 = 15 + \sum h_L \rightarrow \sum h_L = 5.38 \text{ m} \]
\[ \sum h_L = h_f + h_m = 5.38 \text{ m} \]
\[ h_m = 0 \text{ (given)} \rightarrow h_f = 5.38 \text{ m} \]

\[ 5.38 = f \left( \frac{L}{D} \right) \left( \frac{V^2}{2g} \right) = h_f = f \times \left( \frac{0.06}{0.1962} \right) \left( \frac{V^2}{19.62} \right) = 42.47 f V^2 \]
\[ \rightarrow V^2 = \frac{0.126}{f} \rightarrow \text{Eq. (1)} \]
\[ R_e = \frac{\rho V D}{\mu} = \frac{1000 \times V \times 0.06}{1 \times 10^{-3}} = 60,000V \rightarrow \text{Eq. (2)} \]
\[ e \frac{0.045}{D} = 0.00075 \]
Assume fully turbulent flow \( f = 0.018 \) → Sub.in (1) →
\[ V = 2.64 \text{ m/s} \rightarrow \text{Sub. in (2)} \rightarrow R_e = 1.58 \times 10^5 \rightarrow \text{Moody} \]
\[ \rightarrow f = 0.02 \rightarrow V = 2.51 \rightarrow R_e = 1.5 \times 10^5 \rightarrow \text{Moody} \]
\[ \rightarrow f \cong 0.02 \rightarrow V = 2.51 \text{ m/s} \]
\[ Q = A \times V = \frac{\pi}{4} \times 0.06^2 \times 2.51 = 0.0071 \text{ m}^3/\text{s} \checkmark. \]
5.
Two reservoirs having a constant difference in water level of 66 m are connected by a pipe having a diameter of 225 mm and a length of 4km. The pipe is tapped at point C which is located 1.6km from the upper reservoir, and water drawn off at the rate of 0.0425 m³/s. **Determine** the flow rate at which water enters the lower reservoir. Use a friction coefficient of \( f = 0.036 \) for all pipes. Use the following K values for the minor losses:
\[ K_{\text{ent}} = 0.5, \quad K_{\text{exit}} = 1, \quad K_v = 5 \]

![Image of water flow in pipes diagram]

**Solution**

Applying Bernoulli’s equation between points A and E:
\[
\frac{P_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{P_E}{\rho g} + \frac{v_E^2}{2g} + z_E + \sum h_L
\]
\[ P_A = P_E = V_A = V_E = 0.0 \]
Assume the datum at point E →
\[ 0 + 0 + 66 = 0 + 0 + 0 + \sum h_L \rightarrow \sum h_L = 66 \text{m} \]
\[ \sum h_L = h_f + h_m = 66 \]

**Major Losses:**
Since the pipe is tapped, we will divide the pipe into two parts with the same diameter; (1) 1.6 km length from B to C, and (2) 2.4 km length from C to D.
\[ h_f = f \left( \frac{L}{D} \right) \left( \frac{V^2}{2g} \right) \]

For part (1)
\[ h_{f,1} = 0.036 \times \left( \frac{1600}{0.225} \right) \left( \frac{V_1^2}{19.62} \right) = 13.05 \ V_1^2 \]

For part (2)
\[ h_{f,2} = 0.036 \times \left( \frac{2400}{0.225} \right) \left( \frac{V_2^2}{19.62} \right) = 19.6 \ V_2^2 \]
\[ h_f = 13.05 \ V_1^2 + 19.6 \ V_2^2 \]

**Minor losses:**

For Entrance: (due to part 1)
\[ h_{ent} = K_{entr} \frac{V_1^2}{2g} \quad (K_{entr} = 0.5) \rightarrow \]
\[ h_{ent} = 0.5 \times \frac{V_1^2}{19.62} = 0.0255 \ V_1^2 \]

For Exit: (due to part 2)
\[ h_{exit} = K_{exit} \frac{V_2^2}{2g} \quad (K_{exit} = 1) \rightarrow \]
\[ h_{exit} = 1 \times \frac{V_2^2}{19.62} = 0.051 \ V_2^2 \]

For Valve: (valve exists in part 2)
\[ h_{valve} = 5 \times \frac{V_2^2}{2g} = 0.255V_2^2 \]
\[ h_m = 0.0255 \ V_1^2 + 0.051 \ V_2^2 + 0.255V_2^2 = 0.0255 \ V_1^2 + 0.306V_2^2 \]
\[ \sum h_L = (13.05 \ V_1^2 + 19.6 \ V_2^2) + (0.0255 \ V_1^2 + 0.306V_2^2) = 66 \]
\[ \rightarrow 66 = 13.0755 \ V_1^2 + 19.906 \ V_2^2 \gg Eq. \ (1) \]

**Continuity Equation:**
\[ \frac{\pi}{4} \times 0.225^2 \times V_1 = 0.0425 + \frac{\pi}{4} \times 0.225^2 \times V_2 \]
\[ V_1 = 1.068 + V_2 \quad (\text{substitute in Eq.} \ (1)) \rightarrow \]
\[ 66 = 13.0755 \left( 1.068 + V_2 \right)^2 + 19.906 \, V_2^2 \rightarrow V_2 = 0.89 \, \text{m/s} \]

\[ Q_2 = \frac{\pi}{4} \times 0.225^2 \times V_2 = \frac{\pi}{4} \times 0.225^2 \times 0.89 = 0.0354 \, \text{m}^3/\text{s} \]
Chapter (4)

Pipelines and Pipe Networks
Flow through series pipes

Is the same in case of single pipe (Ch.3), but here the total losses occur by more than 1 pipe in series. (See examples 3.9 and 4.1 in text book).

Flow through Parallel pipes

Here, the main pipe divides into two or more branches and again join together downstream to form single pipe.
The discharge will be divided on the pipes:

\[ Q = Q_1 + Q_2 + Q_3 + \cdots \]

But, the head loss in each branch is the same, because the pressure at the beginning and the end of each branch is the same (all pipes branching from the same point and then collecting to another one point).

\[ h_L = h_{f,1} = h_{f,2} = h_{f,3} = \cdots \]

Pipelines with Negative Pressure (Siphon Phenomena)

When the pipe line is raised above the hydraulic grade line, the pressure (gauge pressure) at the highest point of the siphon will be negative.
The highest point of the siphon is called Summit (S).
If the negative gauge pressure at the summit exceeds a specified value, the water will starts liberated and the flow of water will be obstructed.
The allowed negative pressure on summit is \(-10.3\) m (theoretically), but in practice this value is \(-7.6\) m.
If the pressure at S is less than or equal (at max.) \(-7.6\), we can say the water will flow through the pipe, otherwise \((P_s > -7.6)\) the water will not flow and the pump is needed to provide an additional head.

Note

\[ P_{abs} = P_{atm} + P_{gauge} \]

The value of \(P_{atm} = 101.3\) kPa \(= \frac{101.3 \times 10^3}{9810} = 10.3\) m

\[ P_{abs} = 10.3 + P_{gauge} \]

So always we want to keep \(P_{abs}\) positive \((P_{gauge} = -10.3\) m as max, theo.)
To maintain the flow without pump.
Problems:

1. Three pipes A, B and C are interconnected as shown. The pipe are as follows:

<table>
<thead>
<tr>
<th>Pipe</th>
<th>D (m)</th>
<th>L (m)</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.15</td>
<td>600</td>
<td>0.02</td>
</tr>
<tr>
<td>B</td>
<td>0.1</td>
<td>480</td>
<td>0.032</td>
</tr>
<tr>
<td>C</td>
<td>0.2</td>
<td>1200</td>
<td>0.024</td>
</tr>
</tbody>
</table>

Find:

1) The rate at which water will flows in each pipe.
2) The pressure at point M.

Hint: Neglect minor losses.

Solution

Applying Bernoulli’s equation between the points 1 and 2.

\[
\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum h_L
\]

\[
P_1 = V_1 = P_2 = 0.0 \quad , \quad V_1 = V_C = ?? \quad , \quad \sum h_L = ??
\]

\[
0 + 0 + 200 = 0 + \frac{V_C^2}{19.62} + 50 + \sum h_L
\]

\[
0.0509 V_C^2 + \sum h_L = 150 m \rightarrow \text{Eq. (1)}
\]

\[
h_f = f \left( \frac{L}{D} \right) \left( \frac{V^2}{2g} \right)
\]

\[
h_{f,A} = h_{f,B} \quad \text{(Parallel Pipes)} \quad \text{(take pipe A)}
\]

\[
\sum h_L = h_{f,A} + h_{f,C}
\]

\[
h_{f,A} = 0.02 \left( \frac{600}{0.15} \right) \left( \frac{V_A^2}{2g} \right) = 4.07 V_A^2
\]
Hydraulics

\[ h_{fC} = 0.024 \left( \frac{1200}{0.2} \right) \left( \frac{V_C^2}{2g} \right) = 7.34V_C^2 \]

\[ \sum h_L = 4.07V_A^2 + 7.34V_C^2 \]  (substitute in Eq. (1)) →

\[ 0.0509 V_C^2 + 4.07V_A^2 + 7.34V_C^2 = 150m \]

→ \[ 7.39V_C^2 + 4.07V_A^2 = 150 \rightarrow \text{Eq. (2)} \]

How we can find other relation between \( V_A \) and \( V_C \)?

**Continuity Equation**

\[ Q_A + Q_B = Q_C \]

\[ \frac{\pi}{4} \times 0.15^2 V_A + \frac{\pi}{4} \times 0.1^2 V_B = \frac{\pi}{4} \times 0.2^2 V_C \]

→ \[ 0.025V_A + 0.01V_B = 0.04V_C \rightarrow \text{Eq. (3)} \]

But, \( h_{fA} = h_{fB} \rightarrow \)

\[ h_{fA} = 4.07V_A^2 \] (calculated above)

\[ h_{fB} = 0.032 \left( \frac{480}{0.1} \right) \left( \frac{V_B^2}{2g} \right) = 7.82V_B^2 \]

\[ 4.07V_A^2 = 7.82V_B^2 \rightarrow V_B^2 = 0.52V_A^2 \]

→ \[ V_B = 0.72V_A \] (substitute in Eq. (3))

→ \[ 0.025V_A + 0.01(0.72V_A) = 0.04V_C \rightarrow V_C = 0.805V_A \] (Subs. in Eq. 2)

→ \[ 7.39(0.805V_A)^2 + 4.07V_A^2 = 150m \rightarrow V_A = 4.11m/s \]

→ \[ V_B = 0.72 \times 4.11 = 2.13m/s \]

→ \[ V_C = 0.805 \times 4.11 = 3.3m/s \]

\[ Q_A = \frac{\pi}{4} \times 0.15^2 \times 4.11 = 0.0726m^3/s \checkmark. \]

\[ Q_B = \frac{\pi}{4} \times 0.1^2 \times 2.13 = 0.0167m^3/s \checkmark. \]

\[ Q_C = \frac{\pi}{4} \times 0.2^2 \times 3.3 = 0.103m^3/s \checkmark. \]

**Pressure at M:** → Bernoulli’s equation between the points M and 2.

\[ \frac{P_M}{\rho g} + \frac{V_M^2}{2g} + z_M = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum h_{L(M \rightarrow 2)} \]

\[ V_M = V_2 \]

\[ \sum h_{L(M \rightarrow 2)} = h_{fC} = 7.34 \times 3.3^2 = 79.93m \]

→ \[ \frac{P_M}{9810} + 120 = 0 + 50 + 79.93 \rightarrow P_M = 97413.3\, \text{Pa}. \checkmark. \]
2.
Three pipes A, B and C are interconnected as shown. The pipe are as follows:

Find:
1) The rate at which water will flows in each pipe.
2) The pressure at point M.

Solve the problem in the following two cases:

a) The valve (V) is closed.
b) The valve (V) is opened with $K = 5$

<table>
<thead>
<tr>
<th>Pipe</th>
<th>D (m)</th>
<th>L (m)</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.15</td>
<td>600</td>
<td>0.02</td>
</tr>
<tr>
<td>B</td>
<td>0.1</td>
<td>480</td>
<td>0.032</td>
</tr>
<tr>
<td>C</td>
<td>0.2</td>
<td>1200</td>
<td>0.024</td>
</tr>
</tbody>
</table>

Solution

Case (a): Valve is closed
When the valve is closed, no water will flowing in pipe B because the valve prevents water to pass through the pipe. Thus the water will flowing throughout pipe A and pipe C (in series) and the system will be as follows:
Now, you can solve the problem as any problem (Pipes in series).
And if you are given the losses of the enlargement take it, if not, neglect it.

**Case (b): Valve is Open**
Here the solution procedures will be exactly the same as problem (1) >> water will flow through pipe A and B and C, and pipes A and B are parallel to each other.

**But the only difference with problem (1) is:**
Total head loss in pipe A = Total head loss in pipe B
Total head loss in pipe A = $h_{fA}$
Total head loss in pipe B = $h_{fB} + h_{m,\text{valve}}$

$$h_{m,\text{valve}} = K_{\text{valve}} \frac{V_B^2}{2g} = 5 \frac{V_B^2}{2g}$$

Now, we can complete the problem, as problem 1.

3.

The flow rate between tank A and tank B shown in the figure below is to be increased by 30% (i.e., from Q to 1.30Q) by the addition of a second pipe in parallel (indicated by the dotted lines) running from node C to tank B. If the elevation of the free surface in tank A is 7.5m above that in tank B, determine the diameter, D, of this new pipe. Neglect minor losses and assume that the friction factor for each pipe is 0.02

![Diagram of pipeline network](image_url)

**Solution**
Before addition of pipe:

Apply Bernoulli’s equation between reservoirs A and B.

\[
\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B + \sum h_{L(A\rightarrow B)}
\]

\[ 0 + 0 + 7.5 = 0 + 0 + 0 + \sum h_{L(A\rightarrow B)} \Rightarrow \sum h_{L(A\rightarrow B)} = 7.5\ m \]

\[ \sum h_{L(A\rightarrow B)} = 7.5\ m = 0.02 \times \frac{180 + 150}{0.15} \times \frac{V^2}{2g} \Rightarrow V = 1.828\ m/s \]

\[ \Rightarrow Q_{before} = A \times V = \frac{\pi}{4} \times 0.15^2 \times 1.828 = 0.0323\ m^3/s \]

After addition of pipe:

\[ Q_{after,1} = 1.3\ Q_{before} = 1.3 \times 0.0323 = 0.0412\ m^3/s \]

\[ V_1 = \frac{\pi}{4} \times 0.15^2 = 2.33\ m/s \]

Apply Bernoulli’s equation between reservoirs A and B.

\[ 0 + 0 + 7.5 = 0 + 0 + 0 + \sum h_{L(A\rightarrow B)} \Rightarrow \sum h_{L(A\rightarrow B)} = 7.5\ m \]

\[ \sum h_{L(A\rightarrow B)} = h_{f1} + h_{f2} \]

\[ 7.5 = 0.02 \times \frac{180}{0.15} \times \frac{2.33^2}{2g} + 0.02 \times \frac{150}{0.15} \times \frac{V_2^2}{2g} \Rightarrow V_2 = 0.918\ m/s \]

\[ \Rightarrow Q_2 = A_2 \times V_2 = \frac{\pi}{4} \times 0.15^2 \times 0.918 = 0.0162\ m^3/s \]

\[ \Rightarrow Q_3 = Q_1 - Q_2 = 0.0412 - 0.0162 = 0.025\ m^3/s \]

\[ h_{f2} = h_{f3} \]

\[ 0.02 \times \frac{150}{0.15} \times \frac{0.918^2}{2g} = 0.02 \times \frac{150}{D_3} \times \frac{V_3^2}{2g} \Rightarrow D_3 = 0.178V_3^2 \]

\[ Q_3 = A_3 \times V_3 \Rightarrow 0.025 = \frac{\pi}{4} \times D_3^2 \times V_3 \Rightarrow 0.025 = \frac{\pi}{4} \times (0.178V_3^2)^2 \times V_3 \]

\[ \Rightarrow V_3 = 0.99\ m/s \Rightarrow D_3 = 0.178 \times 0.99^2 = 0.174\ m = 174\ mm \checkmark. \]
4.
A 500 mm diameter siphon pipeline discharges water from a large reservoir. Determine:

a. The maximum possible elevation of its summit, B, for a discharge of 2.15 m³/s without the pressure becoming less than 20 kN/m² absolute.
b. The corresponding elevation of its discharge end (Z_C).

Neglect all losses.

**Solution**

\[ P_{\text{abs}} = P_{\text{atm}} + P_{\text{gauge}} \]

\[ P_{\text{abs,B}} = \frac{20 \times 10^3}{9810} = 2.038 \text{ m} \]

\[ P_{\text{atm}} = 10.3 \text{ m} \]

\[ P_{\text{gauge,B}} = 2.038 - 10.3 = -8.26 \text{ m} \]

\[ Q = AV \rightarrow V = \frac{Q}{A} = \frac{2.15}{\frac{\pi}{4} \times 0.5^2} = 10.95 \text{ m/s} \]

Applying Bernoulli’s equation between the points A and B.

\[ \frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B + \sum h_L \]

(Datum at A), \( h_L = 0 \) (given)

\[ 0 + 0 + 0 = -8.26 + \frac{10.95^2}{19.62} + z_B + 0 \rightarrow z_B = 2.15 \text{ m} \checkmark. \]

**Calculation of Z_C:**

Applying Bernoulli’s equation between the points A and C.

\[ \frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_C}{\rho g} + \frac{V_C^2}{2g} + z_C + \sum h_L \]

(Datum at A), \( h_L = 0 \) (given)

\[ 0 + 0 + 0 = 0 + \frac{10.95^2}{19.62} + (-z_C) \rightarrow z_C = 6.11 \text{ m} \checkmark. \]
5.

In the shown figure, if the length of the pipe between reservoir A and point B is 180 m and the absolute pressure at point B is 7.3 mwc. **Calculate** the minor loss coefficient (K) of the valve at point C.

**Given data:**
The entrance is sharp edged.
Total length of pipe (L= 730 m), Pipe diameter (d = 150mm), friction factor (f = 0.02).

**Solution**

\[
p_{\text{abs,}S} = 7.3 \text{m} , \; p_{\text{atm}} = 10.3 \text{m} \rightarrow p_{\text{gauge,}S} = 7.3 - 10.3 = -3 \text{m}
\]

Apply Bernoulli’s equation between the points A and D.
Datum at (D):
\[
\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_D}{\rho g} + \frac{V_D^2}{2g} + z_D + \sum h_{L(1\rightarrow D)}
\]

\[
0 + 0 + 16 = 0 + \frac{V^2}{2g} + 0 + \sum h_{L(1\rightarrow D)}
\]

\[
16 = \frac{V^2}{2g} + \sum h_{L(1\rightarrow D)} \rightarrow \text{Eq. (1)}
\]

\[
\sum h_{L(1\rightarrow D)} = h_f + h_m
\]

\[
h_f = 0.02 \left( \frac{730}{0.15} \right) \left( \frac{V^2}{2g} \right)
\]

\[
h_m = 0.5 \frac{V^2}{2g} + K_v \frac{V^2}{2g}
\]

\[
\sum h_{L(1\rightarrow D)} = 4.86V^2 + 0.0509K_vV^2 \rightarrow \text{Sub. in (1)} \rightarrow
\]

\[
16 = \frac{V^2}{2g} + 4.86V^2 + 0.0509K_vV^2 \quad \text{But} \; V = ??
\]

Apply Bernoulli’s equation between the points A and B.
Datum at (B):
6.
The difference in surface levels in two reservoirs connected by a siphon is 7.5m. The diameter of the siphon is 300 mm and its length 750 m. The friction coefficient is 0.025. If air is liberated from solution when the absolute pressure is less than 1.2 m of water, what will be the maximum length of the inlet leg (the portion of the pipe from the upper reservoir to the highest point of the siphon) in order the siphon is still run if the highest point is 5.4 m above the surface level of the upper reservoir? What will be the discharge.

Solution
The graph is not given so you should understand the problem, and then drawing the system as follows:

Applying Bernoulli’s equation between the points 1 and S.
Datum at (1):

\[ P_{\text{abs}} = P_{\text{atm}} + P_{\text{gauge}} \]
\[ P_{\text{abs},S} = 1.2 \text{m} \]
\[ P_{\text{atm}} = 10.3 \text{m} \]
\[ P_{\text{gauge},S} = 1.2 - 10.3 = -9.1 \text{m} \]
Hydraulics

\[
\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_S}{\rho g} + \frac{V_S^2}{2g} + z_S + \sum h_{L(1\rightarrow S)}
\]

\[
0 + 0 + 0 = -9.1 + \frac{V_S^2}{2g} + 5.4 + \sum h_{L(1\rightarrow S)} \rightarrow \text{Eq. (1)}
\]

\[
\sum h_{L(1\rightarrow S)} = h_{f(1\rightarrow S)} = 0.025 \left( \frac{X}{0.3} \right) \left( \frac{V^2}{19.62} \right) \quad \text{But } V = ??
\]

Applying Bernoulli’s equation between the points 1 and 2.

Datum at (2):

\[
\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum h_{L(1\rightarrow 2)}
\]

\[
0 + 0 + 7.5 = 0 + 0 + 0 + \sum h_{L(1\rightarrow 2)} \rightarrow \sum h_{L(1\rightarrow 2)} = 7.5m
\]

\[
\sum h_{L(1\rightarrow 2)} = 7.5m = h_{f(1\rightarrow 2)} = 0.025 \left( \frac{750}{0.3} \right) \left( \frac{V^2}{19.62} \right) \rightarrow V = 1.534m
\]

\[
h_{f(1\rightarrow S)} = 0.025 \left( \frac{X}{0.3} \right) \left( \frac{1.534^2}{19.62} \right) = 0.01X \quad \text{(Substitute in Eq. (1))}
\]

\[
0 + 0 + 0 = -9.1 + \frac{V_S^2}{2g} + 5.4 + 0.01X \quad (V_S = V = 1.534)
\]

\[
0 + 0 + 0 = -9.1 + \frac{1.534^2}{2g} + 5.4 + 0.01X \rightarrow X = 358m \checkmark.
\]
7.
For the three-reservoir system shown. If the flow in pipe 1(from reservoir A to junction J) is 0.4 m³/s and the friction factor for all pipes is 0.02, calculate:
- The flow in the other pipes (pipe 2, 3 and 4).
- The elevation of reservoir B (Z₉).

Neglect minor losses.

Solution

If we put a piezometer at point J, the will rise at height of Zₚ, which can be calculated by applying Bernoulli’s equation between A and J:

\[
P_A + \frac{V_A}{2g} + z_A = P_J + \frac{V_J}{2g} + z + \sum h_{L(A\rightarrow J)}\]

\[
0 + 0 + 100 = Z_P + \sum h_{L(A\rightarrow J)}\]

\[
\sum h_{L(A\rightarrow J)} = \frac{8fLQ^2}{\pi^2gD^5} = \frac{8 \times 0.02 \times 400 \times 0.4^2}{\pi^2g \times 0.4^5} = 10.32\,\text{m}\]

\[
\rightarrow 100 - 10.32 = Z_P = 89.68\,\text{m}\]

Note that Zₚ = 89.68 > Zₐ = 80, So the flow direction is from J to C, and to calculate this flow we apply Bernoulli’s equation between J and C:

\[
Z_P - Z_C = \sum h_{L(J\rightarrow C)}\]

\[
89.68 - 80 = \frac{8 \times 0.02 \times 300 \times Q_4^2}{\pi^2g \times 0.3^5} \rightarrow Q_4 = 0.2177\,\text{m}^3/\text{s}\]

Now, by applying continuity equation at Junction J:

\[
\sum Q_{@J} = 0.4 \rightarrow 0.4 = 0.2177 + (Q_2 + Q_3) \rightarrow Q_2 + Q_3 = 0.1823\,\text{m}^3/\text{s}\]
Since pipe 2 and 3 are in parallel, the head loss in these two pipes is the same:

\[ h_{L2} = h_{L3} \rightarrow \frac{8fL_2Q_2^2}{\pi^2gD_2^5} = \frac{8fL_3Q_3^2}{\pi^2gD_3^5} \]

\[ \rightarrow \frac{8 \times 0.02 \times 200 \times Q_2^2}{\pi^2g \times 0.2^5} = \frac{8 \times 0.02 \times 180 \times Q_3^2}{\pi^2g \times 0.15^5} \rightarrow Q_3 = 0.513Q_2 \]

But, \( Q_2 + Q_3 = 0.1823 \rightarrow Q_2 + 0.513Q_2 = 0.1823 \rightarrow Q_2 = 0.12 \text{ m}^3/\text{s} \checkmark \).

\( Q_3 = 0.513 \times 0.12 = 0.0618 \text{ m}^3/\text{s} \checkmark \).

Now we want to calculate the elevation \( Z_B \):

Apply Bernoulli’s equation between J and B:

\[ Z_P - Z_B = \sum h_{L(J\rightarrow B)} \text{ (take pipe 2)} \]

\[ 89.68 - Z_B = \frac{8 \times 0.02 \times 200 \times 0.12^2}{\pi^2g \times 0.2^5} \rightarrow Z_B = 74.8 \text{ m} \checkmark. \]

8.

In the shown figure, determine the total length of pipe 3 \( (L_3) \) and the head delivered by the pump \( (h_p) \). Neglect minor losses and take \( f = 0.032 \) for all pipes.
Solution

\[ h_{L1} = \frac{8fL_1Q_1^2}{\pi^2gD_1^5} \rightarrow 40 = \frac{8 \times 0.032 \times 950 \times Q_1^2}{\pi^2 \times 9.81 \times 0.45^5} \rightarrow Q_1 = 0.542 \text{ m}^3/\text{s} \]

\[ h_{L2} = \frac{8fL_2Q_2^2}{\pi^2gD_2^5} \rightarrow 40 = \frac{8 \times 0.032 \times 450 \times Q_2^2}{\pi^2 \times 9.81 \times 0.45^5} \rightarrow Q_2 = 0.352 \text{ m}^3/\text{s} \]

\[ \sum Q_{@J} = 0.0 \rightarrow Q_3 = 0.542 - 0.352 = 0.19 \text{ m}^3/\text{s} \]

\[ \rightarrow \text{(direction shown in the figure)} \]

To find the total head at point J (\(Z_P\)): apply Bernoulli equation between J and A:

\[ Z_P = 30 + \sum h_{L(J \rightarrow A)} \rightarrow Z_P = 30 + 8 = 38 \text{ m} \]

To find the length of pipe 3 (\(L_3\)): apply Bernoulli between J and B:

\[ Z_P = 25.5 + \sum h_{L(J \rightarrow B)} \rightarrow 38 - 25.5 = \frac{8 \times 0.032 \times L_3 \times 0.19^2}{\pi^2 \times 9.81 \times 0.3^5} \]

\[ \rightarrow L_3 = 318.22 \text{ m} \checkmark. \]

To find the pump head (\(h_P\)): apply Bernoulli between C and J:

\[ 18 = Z_P + h_{L1} - h_P \rightarrow h_P = 38 + 40 - 18 = 60 \text{ m} \checkmark. \]
9.
The network \( ABCD \) is supplied by water from reservoir \( E \) as shown in the figure. All pipes have a friction factor \( f = 0.02 \).

**Calculate:**

1. The flow rate in each pipe of the system using the Hardy Cross method. Consider the following:
   - **Assume for the first iteration that:**
     \( Q_{CB} = 0.15 \text{ m}^3/\text{s} \) from \( C \) to \( B \) and \( Q_{BA} = 0.05 \text{ m}^3/\text{s} \) from \( B \) to \( A \).
   - **Do only one iteration (stop after you correct \( Q \))

2. The pressure head at node \( A \).

**Given Also:**

<table>
<thead>
<tr>
<th>Nodes elevation</th>
<th>Node/Point</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Reservoir E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elevation(m)</td>
<td></td>
<td>20</td>
<td>25</td>
<td>20</td>
<td>30</td>
<td>50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pipes dimension</th>
<th>Pipe</th>
<th>AB</th>
<th>BC</th>
<th>CD</th>
<th>DA</th>
<th>BD</th>
<th>CE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (m)</td>
<td></td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>600</td>
<td>200</td>
</tr>
<tr>
<td>Diameter (m)</td>
<td></td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.40</td>
</tr>
</tbody>
</table>
Solution

Firstly, from the given flows, we calculate the initial flow in each pipe and the direction of flow through these pipes using continuity equation at each note.

![Diagram of a network of pipes with flows and directions](image)

We calculate the corrected flow in each pipe, from the following tables:

<table>
<thead>
<tr>
<th>Loop</th>
<th>Pipe</th>
<th>L</th>
<th>D</th>
<th>Q&lt;sub&gt;initial&lt;/sub&gt;</th>
<th>h&lt;sub&gt;f&lt;/sub&gt;</th>
<th>h&lt;sub&gt;f&lt;/sub&gt; / Q&lt;sub&gt;initial&lt;/sub&gt;</th>
<th>Δ</th>
<th>Q&lt;sub&gt;new&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>AB</td>
<td>400</td>
<td>0.3</td>
<td>-0.05</td>
<td>-0.68</td>
<td>13.6</td>
<td>-0.025</td>
<td>-0.075</td>
</tr>
<tr>
<td></td>
<td>BD</td>
<td>600</td>
<td>0.3</td>
<td>+0.05</td>
<td>+1.02</td>
<td>20.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>DA</td>
<td>400</td>
<td>0.3</td>
<td>+0.1</td>
<td>+2.72</td>
<td>27.2</td>
<td>-0.025</td>
<td>+0.075</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.06</td>
<td>61.2</td>
</tr>
</tbody>
</table>

h<sub>f</sub> calculated for each pipe from the following relation:

\[
\frac{8fLQ^2}{\pi^2gD^4} \leq h_f
\]

\[
\Delta = \frac{-\sum h_f}{2 \sum \frac{h_f}{Q_{initial}}} = \frac{-3.06}{2 \times 61.2} = -0.025 \text{ m}^3/\text{s}
\]

Q<sub>new</sub> = Q<sub>initial</sub> + Δ

Note that, we don’t correct pipe BD because is associated with the two loops, so we calculate the flow in the associated pipe after calculating the correction in each loop.
\[ \Delta = \frac{-\sum h_f}{2 \sum \frac{h_f}{Q_{\text{initial}}}} = \frac{-(1.02)}{2 \times 102} = +0.005 \text{ m}^3/\text{s} \]

Now, to calculate \( Q_{\text{new}} \) for pipe BD, we calculate the associated value for correction:

\[ Q_{\text{new, BD}} = Q_{\text{initial, loop(1)}} + (\Delta_{\text{loop(1)}} - \Delta_{\text{loop(2)}}) \]

\( Q_{\text{new, BD}} = +0.05 + (-0.025 - 0.005) = +0.02 \text{ m}^3/\text{s} \)

Or:

\[ Q_{\text{new, BD}} = Q_{\text{initial, loop(2)}} + (\Delta_{\text{loop(2)}} - \Delta_{\text{loop(1)}}) \]

\( Q_{\text{new, BD}} = -0.05 + (0.005 - (-0.025)) = -0.02 \text{ m}^3/\text{s} \)

Now, we put the corrected flow rate on each pipe of the network:

\[ \sum \]

\[ -1.02 \quad 102 \]

\textbf{Note:} If the sign of the corrected flow rate is the same sign of initial flow rate, the direction of flow will remain unchanged, however if the sign is changed, the flow direction must be reversed.
To calculate the pressure head at point A, we must start from point which have a known head, so we start from the reservoir.

**Important Note:**
The total head at any node in the network is calculated as following:

\[ h_{\text{total}} = h_{\text{pressure}} + h_{\text{elevation}} \rightarrow h_t = \frac{P}{\gamma} + Z \]

By considering a piezometer at each node, such that the water rise on it distance: \( \frac{P}{\gamma} + Z \) and the velocity is zero.

Starts from reservoir at E:

Apply Bernoulli between E and C:

\[
50 + 0 + 0 = \frac{P_C}{\gamma} + 0 + 20 + \frac{8fLQ^2}{\pi^2gD^5}
\]

\[
\Rightarrow \frac{P_C}{\gamma} = 50 - 20 - \frac{8 \times 0.02 \times 200 \times 0.3^2}{\pi^2 \times 9.81 \times 0.4^5} = 27.1\text{m}
\]

Apply Bernoulli between C and B:

\[
20 + 27.1 + 0 = \frac{P_B}{\gamma} + 0 + 25 + \frac{8fLQ^2}{\pi^2gD^5}
\]

\[
\Rightarrow \frac{P_B}{\gamma} = 20 + 27.1 - 25 - \frac{8 \times 0.02 \times 400 \times 0.145^2}{\pi^2 \times 9.81 \times 0.3^5} = 16.38\text{m}
\]

Apply Bernoulli between B and A:

\[
25 + 16.38 + 0 = \frac{P_A}{\gamma} + 0 + 20 + \frac{8fLQ^2}{\pi^2gD^5}
\]

\[
\Rightarrow \frac{P_A}{\gamma} = 25 + 16.38 - 20 - \frac{8 \times 0.02 \times 400 \times 0.075^2}{\pi^2 \times 9.81 \times 0.3^5} = 19.84\text{m} \; \checkmark
\]
10.
For all pipe segments, the initial estimate of the flow in each pipe (m³/s) is shown in the figure below.

![Pipe Network Diagram]

a) Show on the sketch all sources and sinks (in and out flows) of water from the pipe network. Use an arrow to indicate direction (in or out of system) and give the magnitude of the flow.

b) Use Hardy-Cross method to correct the flow rate through each pipe. Do only one iteration (stop after you correct Q), assume \( n = 0.02 \) for all pipes and neglect minor losses.

c) If the pressure head at A = 90m, determine the pressure head at E.

<table>
<thead>
<tr>
<th>Pipe</th>
<th>L(m)</th>
<th>D(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>250</td>
<td>0.5</td>
</tr>
<tr>
<td>AC</td>
<td>350</td>
<td>0.6</td>
</tr>
<tr>
<td>CD</td>
<td>200</td>
<td>0.6</td>
</tr>
<tr>
<td>DB</td>
<td>180</td>
<td>0.2</td>
</tr>
<tr>
<td>DE</td>
<td>380</td>
<td>0.4</td>
</tr>
<tr>
<td>BE</td>
<td>400</td>
<td>0.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Node</th>
<th>Elevation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20</td>
</tr>
<tr>
<td>B</td>
<td>25</td>
</tr>
<tr>
<td>C</td>
<td>28</td>
</tr>
<tr>
<td>D</td>
<td>25</td>
</tr>
<tr>
<td>E</td>
<td>30</td>
</tr>
</tbody>
</table>

Solution
a) By applying continuity equation at each junction, the in and out flow from the system are shown in the following figure:

![Diagram showing flow in and out of the system]

b) We calculate the corrected flow in each pipe, from the following tables:

<table>
<thead>
<tr>
<th>Loop</th>
<th>Pipe</th>
<th>L</th>
<th>D</th>
<th>Q_{initial}</th>
<th>h_f</th>
<th>h_f/Q_{initial}</th>
<th>Δ</th>
<th>Q_{new}</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>AB</td>
<td>250</td>
<td>0.5</td>
<td>+0.5</td>
<td>+3.305</td>
<td>6.61</td>
<td>+0.1028</td>
<td>+0.6028</td>
</tr>
<tr>
<td>I</td>
<td>BD</td>
<td>180</td>
<td>0.2</td>
<td>-0.2</td>
<td>-37.18</td>
<td>185.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>DC</td>
<td>200</td>
<td>0.6</td>
<td>-0.8</td>
<td>-2.72</td>
<td>3.4</td>
<td>+0.1028</td>
<td>-0.6972</td>
</tr>
<tr>
<td>I</td>
<td>CA</td>
<td>350</td>
<td>0.6</td>
<td>-0.8</td>
<td>-4.76</td>
<td>5.95</td>
<td>+0.1028</td>
<td>-0.6972</td>
</tr>
<tr>
<td>I</td>
<td>∑</td>
<td></td>
<td></td>
<td>-41.35</td>
<td>201</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \Delta = \frac{- \sum h_f}{2 \sum \frac{h_f}{Q_{initial}}} = \frac{(-41.35)}{2 \times 201} = +0.1028 \text{ m}^3/\text{s} \]
\[ \Delta = \frac{\sum h_f}{2} = \frac{-623.525}{2} = -0.088 \text{ m}^3/\text{s} \]

\[ Q_{\text{new}, BD} = Q_{\text{initial}, \text{loop}(1)} + (\Delta_{\text{loop}(1)} - \Delta_{\text{loop}(2)}) \]

\[ Q_{\text{new}, BD} = -0.2 + (0.1028 - (-0.088)) = -0.0092 \text{ m}^3/\text{s} \]

Or:

\[ Q_{\text{new}, BD} = Q_{\text{initial}, \text{loop}(2)} + (\Delta_{\text{loop}(2)} - \Delta_{\text{loop}(1)}) \]

\[ Q_{\text{new}, BD} = +0.2 + (-0.088 - 0.1028) = +0.0092 \text{ m}^3/\text{s}. \]

The corrected flows are shown in the following figure:
c)  

@ point A: \( P / \gamma = 90 \) m and \( Z = 20 \) m \( \rightarrow h_{tA} = 90 + 20 = 110 \) m  

Apply Bernoulli between A and B:  

\[
110 = \frac{P_B}{\gamma} + 25 + \frac{8fLQ^2}{\pi^2gD^5}
\]

\[
\Rightarrow \frac{P_B}{\gamma} = 110 - 25 - \frac{8 \times 0.02 \times 250 \times 0.603^2}{\pi^2 \times 9.81 \times 0.5^5} = 80.2 \text{ m}
\]

Apply Bernoulli between B and E:  

\[
80.2 + 25 = \frac{P_E}{\gamma} + 30 + \frac{8fLQ^2}{\pi^2gD^5}
\]

\[
\Rightarrow \frac{P_E}{\gamma} = 105.2 - 30 - \frac{8 \times 0.02 \times 400 \times 0.112^2}{\pi^2 \times 9.81 \times 0.2^5} = 49.29 \text{ m} \checkmark
\]
Chapter (5)
Pumps
1.

The figure below shows a portion of a pump and pipe system. The 30-m long pipeline connecting the reservoir to the pump is 1m in diameter with friction factor $f = 0.02$. For the pump, the required net positive suction head ($NPSH_R$) is 2 m.

If the flow velocity $V$ is 2.5 m/s check the system for cavitation. Take the atmospheric pressure = 100 kPa and the vapor pressure = 2.4 kPa.

Solution

$$(NPSH)_A = \pm h_s - h_{fs} - \sum h_{ms} + \frac{P_{atm}}{\gamma} - \frac{P_{vapor}}{\gamma}$$

$h_s = -(10 - 5) = -5\text{m}$

(−ve sign because the supply tank is below the pump).

$h_{fs} = f \left( \frac{L}{D} \right) \left( \frac{V^2}{2g} \right) = 0.02 \times \left( \frac{30}{1} \right) \left( \frac{2.5^2}{2 \times 9.81} \right) = 0.191\text{m}$

$\sum h_{ms} = (0.5 + 2 \times 2 + 3) \times \frac{V^2}{2g} = 7.5 \times \frac{2.5^2}{2 \times 9.81} = 2.39\text{m}$

$\rightarrow (NPSH)_A = -5 - 0.191 - 2.39 + \frac{100}{9.81} - \frac{2.4}{9.81} = 2.36\text{m}$

$$(NPSH)_A = 2.36 > (NPSH)_R = 2 \rightarrow \text{System is adequate for cavitation} \checkmark.$$
2.
A pump is required to supply water to an elevated tank through a 0.2m diameter pipe which is 300m long and has a friction factor $f$ equal to 0.025. Minor losses causes an additional head loss of $4V^2/2g$ where $V$ is the velocity in the pipe in m/s. The static head between the pump wet well and the elevated tank is 40 m. The relationship between head, $H$, and flow, $Q$, for the pump is given by the following equation:

$$H = 50 - 600Q^2$$

Where $Q$ in m$^3$/s and $H$ in meters.

**a)** under these conditions determine the flow in the pipeline

**b)** if the maximum efficiency of the pump is achieved when the flow is 70 ℓ/s then how could the system be redesigned so that the pump would operate at this efficiency.

**Solution**

Pump Curve Equation is: $H = 50 - 600Q^2$ and since $Q$ in m$^3$/s we calculate the head at the following values of $Q$ as shown in the following table:

<table>
<thead>
<tr>
<th>$Q$ (m$^3$/s)</th>
<th>0</th>
<th>0.05</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$ (m)</td>
<td>50</td>
<td>48.5</td>
<td>44</td>
<td>36.5</td>
<td>26</td>
<td>12.5</td>
</tr>
</tbody>
</table>

Now, we want to find the system head equation:

To find the relation between the required head and flow for the system, the best way is to apply Bernoulli equation on the given system and then find the relation between pump head and flow rate (general case), or by applying the following equation (special case):

$$H = H_{static} + \sum h_L$$

When the pump is used to transfer water from one reservoir to the other.

$$H = H_{static} + \sum h_L$$

$H_{static} = 40$ m (given)

$\sum h_L = h_f + h_m$
\[
\sum h_L = \frac{8fLQ^2}{\pi^2 gD^5} + h_m
\]

\[
h_m = 4 \frac{v^2}{2g} = 4 \times \frac{Q^2}{2gA^2}, \quad \text{but } A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \Rightarrow h_m = 206.7Q^2
\]

\[
\Rightarrow \sum h_L = \frac{8 \times 0.025 \times 300Q^2}{\pi^2 \times 9.81 \times 0.25} + 206.7Q^2 = 2143.26Q^2
\]

So, the following equation is for system curve:

\[
H = 40 + 2143.26Q^2
\]

Now, we calculate the head at the same values of Q (in pump curve) as shown in the following table:

<table>
<thead>
<tr>
<th>Q (m(^3)/s)</th>
<th>0</th>
<th>0.05</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td>40.0</td>
<td>45.4</td>
<td>61.4</td>
<td>88.2</td>
<td>125.7</td>
<td>174.0</td>
</tr>
</tbody>
</table>

Now, we draw the pump curve and the system curve, and the point of intersection between the two curves is the operating point:

As shown in figure above, the pump flow rate is 0.06 m\(^3\)/s (60 ℓ /s) and the pump head is 49m.
The required B, means if we get maximum efficiency if the operating point gives flow rate of \((70 \text{ ℓ/s}) = 0.07 \text{ m}^3/\text{s}\), what should you modify in the system to meet this efficiency?

From the above graph, the corresponding head at \(Q = 0.07\) is \(47.5\text{m}\).

Now, we know the system curve equation:

\[
H = 40 + \sum h_L
\]

At maximum efficiency, \(H = 47.5\) m and \(Q = 0.07\) →

\[47.5 = 40 + \sum h_L \rightarrow \sum h_L = 7.5\text{m}
\]

Now the only factor that we can change is the diameter of the pipe, or changing material of the pipe (change \(f\) value), but assume the same material, so change the diameter of the pipe as following:

\[
\sum h_L = 7.5 = \frac{8 \times 0.025 \times 300 \times 0.07^2}{\pi^2 \times 9.81 \times D^5} + 4 \times \frac{0.07^2}{2 \times 9.81 \times \left(\frac{\pi}{4} \times D^2\right)^2}
\]

\[\rightarrow D = 0.214\text{m} \text{ (to get maximum efficiency)}\]

3.

For the shown figure below, (a) what is the operation point for the system that shown in the figure below, the pump characteristics curve is given below (Assume \(f = 0.014\)), (b) If the efficiency of the system is 80%, what is the power required?, (c) What is the operation point in case of two identical pumps in parallel?
a) The pump curve is given as shown above, now we want to find the system curve. So firstly we find the system curve equation:

By applying Bernoulli equation between the two reservoir, the equation will be:

\[ H = H_{\text{Static}} + \sum h_L \]

\[ H_{\text{Static}} = 3.5 + 15 - 10 = 8.5 \text{ m} \]

\[ \sum h_L = h_f + h_m \]

\[ \sum h_L = \frac{8fLQ^2}{\pi^2 gD^5} + h_m \]

\[ h_m = (0.4 + 0.9 + 1) \frac{v^2}{2g} = 2.3 \times \frac{Q^2}{2gA^2} \]

but \( A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \rightarrow h_m = 118.9Q^2 \)

\[ \rightarrow \sum h_L = \frac{8 \times 0.014 \times (200+200 + 15)Q^2}{\pi^2 \times 9.81 \times 0.2^5} + 118.9Q^2 = 1619Q^2 \]

So, the following equation is for system curve:

\[ H = 8.5 + 1619Q^2 \]
But, in this equation $Q$ is in $m^3/s$ and the horizontal axis in the given graph is in $m^3/min.$, so each value on the graph must be divided by 60 (to transform to $m^3/s$) and then we draw the system curve on the same graph as following:

The value of pump flow (at the operation point) is $3 \text{ m}^3/\text{min} = 0.05 \text{ m}^3/\text{s}$. The value of pump head (at the operation point) is 12.55 m.

b) 
\[ \eta = \frac{\gamma \times Q \times H}{P_{in}} \rightarrow P_{in} = \frac{9810 \times 0.05 \times 12.55}{0.8} = 7694.7 \text{ watt}. \]

c) When we use two pumps in parallel, the system curve will never change, but the pump curve will change (The values of head remains constant but the values of $Q$ multiplied by 2), so we draw the new pump curve by multiplying each value of $Q$ by 2 at each head as following:
So the new value of Q is 4.1 m³/min = 0.0683 m³/s and the new value of H is 16m ✔.
Chapter (6)

Open Channels
Formulas used to describe the flow in open channels

The most common formula is Manning equation:

\[ V = \frac{1}{n} \times R_h^{2/3} \times S^{0.5} \]

- \( n \) = manning coefficient
- \( S \) = The slope of channel bed.

\[ R_h = \text{Hydraulic Radius} = \frac{\text{Wetted Area}}{\text{Wetted Perimeter}} = \frac{A}{P} \]

Most Economical Section of Channels

The most efficient section is satisfied when the flow in the channel is maximum and thereby the minimum wetted perimeter.

I.e. for most economical section:

\[ \frac{dP}{dy} = 0.0 \quad \text{OR} \quad \frac{dP}{dB} = 0.0 \quad \text{such that,} \]

- \( y \) is the depth of water in channel also known by (normal depth)
- \( B \) is the width of the channel.

Energy Principals in Open Channels

\[ E_{\text{specific}} = y + \frac{v^2}{2g} \quad \rightarrow \quad E_s = y + \frac{Q^2}{2A^2 g} \quad \text{(for any cross section)} \]

Special Case for rectangular channel:

Since the width of the channel \( B \) is constant, we can calculate the discharge per unit width; \( q = \frac{Q}{B} \ (m^2/s) \)

\[ \rightarrow E_s = y + \frac{q^2}{2y^2 g} \quad \text{(For rectangular channel)} \]

Sub-critical, critical and supercritical flow:

\[ F_R (\text{Froude Number}) = \frac{V}{\sqrt{gD_h}} \]

\[ D_h = \text{Hydraulic depth of channel} = \frac{\text{Area of flow}}{\text{Water surface depth (T)}} \]
Critical Depth \((y_c)\):

Is the depth of flow of liquid at which the specific energy is minimum \((E_{\text{min}})\) and the flow corresponds to this point is called critical flow \((F_R = 1)\).

So, we can classify the flow also as following:

If \(y > y_c\) → Sub - critical flow \((F_R < 1)\)

If \(y = y_c\) → Critical flow \((F_R = 1)\)

If \(y < y_c\) → Super critical flow \((F_R > 1)\)

For rectangular channel:

\[ F_R = \frac{V}{\sqrt{g y}} \]

\[ y_c^3 = \frac{q^2}{g} \rightarrow y_c = \left(\frac{q^2}{g}\right)^{1/3} \]

The critical velocity is:

\[ V_c = \sqrt{g \times y_c} \]

\[ E_{\text{min}} = E_c = 1.5 y_c \]

Non-Rectangular Shapes (All Shapes):

\[ \frac{Q^2}{g} = \frac{A^3}{T} \quad (\text{To calculate } y_c) \quad , \quad \frac{v_c^2}{2g} = \frac{D_h}{2} \]

\[ E_{\text{min}} = E_c = y_c + 0.5 \left(\frac{A}{T}\right) \]
Hydraulic Jump

\[ y_1 = \text{upstream depth} \quad \text{and} \quad y_2 = \text{downstream depth} \]
\[ y_1 \text{ and } y_2 \text{ are called conjugate depths.} \]
\[ y_1 < y_2 \]

**For rectangular channel:**

\[
\frac{y_2}{y_1} = 0.5 \left[ -1 + \sqrt{1 + 8 \left( \frac{y_c}{y_1} \right)^3} \right] \quad (\text{To calculate } y_2)
\]

\[
\frac{y_1}{y_2} = 0.5 \left[ -1 + \sqrt{1 + 8 \left( \frac{y_c}{y_2} \right)^3} \right] \quad (\text{To calculate } y_1)
\]

**Or in terms of Froude number:**

\[
\frac{y_2}{y_1} = 0.5 \left[ -1 + \sqrt{1 + 8 F_1^2} \right] \quad \rightarrow \quad F_1 = \frac{V_1}{\sqrt{gy_1}} \quad (\text{To calculate } y_2)
\]

\[
\frac{y_1}{y_2} = 0.5 \left[ -1 + \sqrt{1 + 8 F_2^2} \right] \quad \rightarrow \quad F_2 = \frac{V_2}{\sqrt{gy_2}} \quad (\text{To calculate } y_1)
\]

**Head loss** \((H_L) = \Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}\)

**Height of Hydraulic jump** \((h_J) = y_2 - y_1\)

**Length of Hydraulic jump** \((L_J) = 6h_J\)
1.
A trapezoidal channel is to be designed to carry a discharge of 75 m$^3$/s at maximum hydraulic efficiency. The side slopes of the channel are 1V:2H (1 vertical and 2 horizontal) and the Manning’s roughness $n$ is 0.03.

- If the maximum allowable velocity in the channel is 1.75 m/s, what should be the dimensions of the channel (bottom width and height)?
- What should be the longitudinal slope of the channel if the flow is uniform?

**Solution**

*a)*
We draw the following graph (from the given data in the problem):

\[ \sqrt{y^2 + (2y)^2} = \sqrt{5} y \]

\[ A = \frac{Q}{V} = \frac{75}{1.75} = 42.86 \text{ m}^2 \]

At maximum hydraulic efficiency (most economical section) \( \rightarrow \)

\[ \frac{dP}{dy} = 0.0 \quad \text{Or} \quad \frac{dP}{dB} = 0.0 \]

\[ P = B + 2 \times \sqrt{5} y \rightarrow P = B + 4.47y \rightarrow \text{Eq. (1)} \]

\[ A = 42.86 = By + 2 \times \frac{1}{2} \times 2y \times y \rightarrow 42.86 = By + 2y^2 \]

\[ \rightarrow B = \frac{42.86}{y} - 2y \quad \text{(Substitute in Eq. (1))} \rightarrow \]

\[ P = \frac{42.86}{y} - 2y + 4.47y \rightarrow P = \frac{42.86}{y} + 2.47y \]

\[ \frac{dP}{dy} = -\frac{42.86}{y^2} + 2.47 = 0.0 \quad \text{(for most economical)} \]
\[
\frac{42.86}{y^2} = 2.47 \rightarrow y = 4.16 \text{ m} \checkmark .
\]
\[
B = \frac{42.86}{4.16} - 2 \times 4.16 = 1.98 \text{ m} \checkmark .
\]

\text{b) According Manning equation:}

\[
V = \frac{1}{n} \times R_h^{2/3} \times S^{0.5}
\]
\[
V = 1.75 \text{ m/s ,}
\]
\[
n = 0.03 \text{ (given)}
\]
\[
R_h = \frac{A}{P} = \frac{42.86}{1.98 + 4.47 \times 4.16} = 2.083 \text{ m}
\]
\[
\rightarrow 1.75 = \frac{1}{0.03} \times 2.083^{2/3} \times S^{0.5} \rightarrow S = 0.00104 \text{ m/m} \checkmark .
\]

2.

In the figure shown, water flows uniformly at a steady rate of 0.4 m³/s in a very long triangular flume (open channel) that has side slopes 1:1. The flume is on a slope of 0.006, and \(n = 0.012\);

\text{(a) Find normal depth (} y_n \text{).}

\text{(b) Determine whether the flow is subcritical or supercritical.}

\text{(c) Find the specific energy (} E_s \text{) and the critical specific energy (} E_c \text{).}
Solution

a) To find the normal depth, we use Manning equation:
\[ V = \frac{1}{n} \times R_h^{2/3} \times S^{0.5} \rightarrow Q = AV = \frac{A}{n} \times R_h^{2/3} \times S^{0.5} \]
\[ Q = 0.4 \text{ m}^3/\text{s}, \quad n = 0.012, \quad S = 0.006 \text{ (Givens)} \]
\[ A = \text{area of triangle} = \frac{1}{2} \times (2y) \times y = y^2 \]
\[ R_h = \frac{A}{P} = \frac{y^2}{2 \times \sqrt{2} y} = 0.353y \]
now, substitute by all above data in Manning’s equation:
\[ 0.4 = \frac{y^2}{0.012} \times (0.353y)^{2/3} \times 0.006^{0.5} \rightarrow y = 0.457 \text{ m} \checkmark. \]

b) To determine the type of flow, there are two methods:
\[ F_r = \frac{V}{\sqrt{gD_h}} \]
\[ V = \frac{Q}{A} = \frac{0.4}{0.457^2} = 1.915 \text{ m/s} \]
\[ D_h = \frac{A}{T} = \frac{y^2}{2y} = \frac{0.457^2}{2 \times 0.457} = 0.2285 \text{ m} \]
\[ \rightarrow F_r = \frac{1.915}{\sqrt{9.81 \times 0.2285}} = 1.28 > 1 \rightarrow \text{Supercritical} \checkmark. \]

Or, we calculate the critical depth:
\[ Q^2 = \frac{A^3}{g} \rightarrow \frac{0.4^2}{9.81} = \frac{(y_c^2)^3}{2y_c} \rightarrow y_c = 0.504 \]
Since \( y_c = 0.504 > y = 0.457 \rightarrow \text{Supercritical} \checkmark. \]

c) \[ E_S = y + \frac{v^2}{2g} = 0.457 + \frac{1.915^2}{2 \times 9.81} = 0.644 \text{ m} \checkmark. \]
\[ E_c = y_c + \frac{1}{2} \left( \frac{A}{T} \right) = 0.504 + \frac{1}{2} \left( \frac{0.504^2}{2 \times 0.504} \right) = 0.63 \text{ m} \checkmark. \]
\[ E_c = E_{\min} \text{ must be less than } E_S \checkmark. \]
3.
You are asked to design a rectangular channel that has the **minimum wetted perimeter** and that conveys **flow in critical condition**. Find the relationship between the critical depth and the channel width if the flow discharge is constant. Your answer should look something like $y_c = mB$, where $m$ is constant and $B$ is the channel width.

**Solution**

$$P = B + 2y_c \rightarrow \text{Eq. (1)}$$

We want other relation, between $B$ and $y_c$

For rectangular channel:

$$y_c^3 = \frac{q^2}{g}, \quad (q = \frac{Q}{B}) \rightarrow y_c^3 = \frac{Q^2}{B^2g}$$

$$\rightarrow B^2 = \frac{Q^2}{y_c^3g} \rightarrow B = \frac{Q}{y_c^{3/2}\sqrt{g}}$$

$$\rightarrow B = \frac{Q}{\sqrt{g}} y_c^{-3/2} \quad \text{(Substitute in Eq. (1))} \rightarrow$$

$$P = \frac{Q}{\sqrt{g}} y_c^{-3/2} + 2y_c$$

$$\frac{dP}{dy_c} = 0.0 \quad \text{(for most economical)}$$

$$\frac{dP}{dy_c} = -1.5 \frac{Q}{\sqrt{g}} y_c^{-5/2} + 2 = 0.0 \rightarrow 2 = 1.5 \frac{Q}{\sqrt{g}} y_c^{-5/2}$$

But, $Q = AV_c = By_c \times \sqrt{gy_c} = B\sqrt{g} y_c^{3/2}$

$$\rightarrow 2 = 1.5 \frac{B\sqrt{g} y_c^{3/2}}{\sqrt{g}} y_c^{-5/2} \rightarrow y_c^{-2} = 1.5 \frac{B y_c^2}{2} \rightarrow y_c \times y_c^{-2} = 1.5 \frac{B y_c^2}{2}$$

$$\rightarrow y_c = 0.75B \checkmark.$$

We can solve this problem alternatively as following:

$$P = B + 2y_c \rightarrow \frac{dP}{dy_c} = \frac{dB}{dy_c} + 2 \rightarrow \frac{dB}{dy_c} = -2 \rightarrow \text{Eq. (1)}$$

Now, we want to find other relation for $\frac{dB}{dy_c}$
\[ y_c^3 = \frac{q^2}{g} = \frac{Q^2}{B^2g} \rightarrow B^2y_c^3 = \frac{Q^2}{g} \text{ (Now, Derive according to } y_c) \]

\[ (B^2)(3y_c^2) + \left(2B \times \frac{dB}{dy_c}\right) \times (y_c^3) = 0.0 \]

\[ \text{تذكر: مشتقة حاصل ضرب اقترانين:} \]

\[ \text{الم.getUserName()} {\text{الم.getUserName()}} + \text{الم.getUserName()} {\text{الم.getUserName()}} \]

\[ \rightarrow 3B^2y_c^2 + 2By_c^3 \frac{dB}{dy_c} = 0.0 \text{ (Substitute from Eq. (1))} \left(\frac{dB}{dy_c} = -2\right) \rightarrow \]

\[ \rightarrow 3B^2y_c^2 + 2By_c^3 \times -2 = 0.0 \]

\[ \rightarrow 3B^2y_c^2 = 4By_c^3 \rightarrow y_c = 0.75B. \]

4.
A rectangular channel carrying a supercritical stream is to be provided with a hydraulic jump type of energy dissipater. It is required to have an energy loss of 5m in the jump when the inlet Froud number is 8.5.(\(F_1 = 8.5\))

**Determine the conjugate depths.** \((y_1, y_2).\)

**Solution**

\[ \frac{y_2}{y_1} = 0.5 \left[-1 + \sqrt{1 + 8F_1^2}\right] \rightarrow \frac{y_2}{y_1} = 0.5 \left[-1 + \sqrt{1 + 8 \times 8.5^2}\right] \]

\[ \rightarrow y_2 = 11.53y_1 \rightarrow \text{Eq. (1)} \]

\[ \Delta E = 5 = \frac{(y_2 - y_1)^3}{4y_1y_2} \text{ (Substitute from Eq. (1))} \rightarrow \]

\[ 5 = \frac{(11.53y_1 - y_1)^3}{4y_1 \times 11.53y_1} \rightarrow y_1 = 0.1975 \text{ m}. \]

\[ \rightarrow y_2 = 11.53 \times 0.1975 = 2.27 \text{ m}. \]