Lecture Topics

- Doppler CW Radar System,
- FM-CW Radar System,
- Moving Target Indication Radar System, and
- Pulsed Doppler Radar System
Remember that:

An EM wave is a function of both space and time e.g.
The electric field strength \( \mathbf{E}(x, y, z, t) \)

\[
v = \lambda f \quad \lambda = \frac{v}{f} \quad v = 3 \times 10^8 \text{ m/s}
\]

\( v \): is in metre/second
\( f \): is in cycle/second
\( \lambda \): is ?

\[
\lambda = \frac{v}{f} = \frac{\text{metre}}{\text{second}} \times \frac{\text{second}}{\text{cycle}} = \frac{\text{metre}}{\text{cycle}}
\]

An EM wave will go through one cycle in a distance equal to one wavelength
Interpretation

As an example, for a frequency $f = 900 \text{ MHz}$, the wavelength $\lambda = 1/3 \text{ m}$, and the period $T = 1.11 \times 10^{-9} \text{ sec}$.

The wave will travel $1/3 \text{ m}$ per cycle
Phasor Representation

\[ A \angle \theta(t) \]

\[ \theta(t) = \int_{0}^{t} \omega_i(\tau) d\tau + \theta_0 \]
Instantaneous Frequency

Taking the derivatives of both side of the equation:

$$\theta(t) = \int_{0}^{t} \omega_i(\tau) d\tau + \theta_0$$

the result is:

$$\frac{d\theta}{dt} = \omega_i(t)$$

The instantaneous frequency of a sinusoidal signal is given by the time derivative of its phase.
Fixed and Moving Objects (straight line)

R_1 \neq R_2 \neq R_3
Fixed and Moving Objects
(circular path)

\[ R_1 = R_2 = R_3 \]
Doppler CW Radar (Simplified)

CW: Continuous wave
Characteristics of CW Radar

- Transmit unmodulated continuous sinusoidal carrier.
- Echoes will also be unmodulated continuous sinusoidal carrier.
- Time difference between the transmitted and returned echoes cannot be detected.
- Echoed radio energy from a moving target differs in frequency from that transmitted by the radar producing a “beat frequency” that can be detected.
- CW Radar Utilise “Doppler Frequency Shift” for detecting and measuring the radial velocity of moving targets.
Doppler Frequency

The total number of wavelengths of the two-way path between the Radar and Target is given by: \( \frac{2R}{\lambda} \).

Since a wavelength correspond to an angular excursion of \( 2\pi \) radians, the returned echo will have a phase difference given by:

\[
\phi = \left( \frac{2R}{\lambda} \right) 2\pi = \frac{4\pi R}{\lambda} \text{ radians}
\]

Substituting \( \frac{c}{f_t} \) for \( \lambda \)

\[
\phi = \frac{4\pi R f_t}{c} \text{ radians}
\]

Where \( f_t \) is the radar transmitted frequency, and \( c \) the speed of radio waves.
Angular Frequency

For a moving target, $R$ and $\phi$ are functions of time.

$$\phi(t) = 4\pi \frac{R(t) f_t}{c} \quad \text{radians}$$

Differentiating $R$ and $\phi$ with respect to time,

$$\frac{d\phi}{dt} = 4\pi \frac{f_t}{c} \quad \frac{dR}{dt} \quad (a)$$

but,

$$\frac{d\phi}{dt} = \omega_d = 2\pi f_d \quad (b) \quad \text{angular Doppler frequency}$$

and

$$\frac{dR}{dt} = v_r \quad (c) \quad \text{target relative velocity}$$
Equation of Doppler Frequency

Substituting (b) and (c) in (a) and rearranging:

\[ 2\pi f_d = (4\pi f_t / c) v_r \]

\[ f_d = 2 \cdot v_r \cdot f_t / c \]

- \( f_d \): Doppler Frequency
- \( v_r \): Target relative velocity
- \( f_t \): Radar transmit frequency
- \( c \): 300 x 10^6 m/s
Bandwidth Consideration

- A wide bandwidth amplifier is required for the expected range of Doppler frequencies.
- Receiver noise increases with bandwidth, resulting in decrease of receiver sensitivity.
- Use of multiple narrowband filters at the baseband or IF frequencies.
- Use of Phase-Locked-Loop to keep track of the varying Doppler frequency.
IF Doppler Filter Bank

Mixer → IF Amp → Filter No.1 → Det. → Indicator
→ Filter No.2 → Det → Indicator
→ Filter No.3 → Det → Indicator
→ Filter No.4 → Det → Indicator
→ Filter No.n → Det → Indicator
Frequency Response Characteristics

- $f_1$, $f_2$, $f_3$, $f_4$, $f_n$

IF bandwidth
Radial Velocity Sign Detection

Transmit Antenna

$\frac{\pi}{2}$

Channel A

Mix A

Mix B

Channel B

Receive Antenna

$f_t$

$\pm f_d$

$\pm f_d$

CW Trans.

LPF

LPF

Synch Motor Indicator

$\frac{f_t + f_d}{2}$

$\frac{f_t - f_d}{2}$
Spectra of Received Signals

- **No Doppler shift**
  - Frequency: $f_t$
  - Amplitude: $f_t$

- **Approaching target**
  - Frequency: $f_t$, $f_d$
  - Amplitude: $f_t$

- **Receding target**
  - Frequency: $f_t$, $f_d$
  - Amplitude: $f_d$
Signal Representation

Let the transmitted signal be:

\[ E_t = E_0 \cos \omega_0 t \]

For an approaching target, the echo signal will be:

\[ E_r = k_1 E_0 \cos [(\omega_0 + \omega_d)t] \]

For a receding target, the echo signal will be:

\[ E_r = k_1 E_0 \cos [(\omega_0 - \omega_d)t] \]

Note: \( k_1 \) is a scaling factor
Signal Analysis

The output of the Mixer is obtained by “multiplying” the transmitted and echo signals and using the following trigonometric identity.

\[ \cos A \cos B = 0.5 \cos (A-B) + 0.5 \cos (A+B) \]
Mixer Output (the answer)

If the target is approaching, the output of channels A and B:

\[ E_A = k_2 E_o \cos (\omega_d t) \quad \text{Velocity} \]
\[ E_B = k_2 E_o \cos (\omega_d t + \pi/2) \quad \text{Direction} \]

If the target is receding, the output of channels A and B:

\[ E_A = k_2 E_o \cos (\omega_d t) \quad \text{Velocity} \]
\[ E_B = k_2 E_o \cos (\omega_d t - \pi/2) \quad \text{Direction} \]
## Calculations

<table>
<thead>
<tr>
<th>Echo Signal</th>
<th>Local Oscillator</th>
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<tbody>
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<td><strong>For an approaching target,</strong></td>
<td></td>
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Output of A Mixer (approaching target)

\[ E_r = k_1 E_o \cos \left[ (\omega_0 + \omega_d) t \right] \]
\[ \times \quad E_t = E_o \cos \omega_o t \]

Ignoring magnitude for now,

\[ E_A = \cos \left[ (\omega_0 + \omega_d) t \right] \cos \omega_o t \]

\[ E_A = 0.5 \cos \left[ (\omega_0 + \omega_d - \omega_0) t \right] + 0.5 \cos \left[ (\omega_0 + \omega_d + \omega_0) t \right] \]

\[ E_A = 0.5 \cos (\omega_d t) + 0.5 \cos \left[ (2\omega_0 + \omega_d) t \right] \]

The term \( \cos \left[ (2\omega_0 + \omega_d) t \right] \) is double the RF frequency and hence it is filtered out

Then the output of the mixer A is: \( E_A = 0.5 \cos (\omega_d t) \)
Output of B Mixer (approaching target)

\[ E_r = k_1 E_o \cos \left[ (\omega_o + \omega_d) t \right] \]

\[ E_t = E_o \cos(\omega_o t - \pi/2) \]

\[ E_B = \cos \left[ (\omega_o + \omega_d) t \right] \cos(\omega_o t - \pi/2) \]

\[ E_B = 0.5 \cos \left[ (\omega_o + \omega_d - \omega_o) t + \pi/2 \right] + 0.5 \cos \left[ (\omega_o + \omega_d + \omega_o) t - \pi/2 \right] \]

\[ E_B = 0.5 \cos \left[ (\omega_d) t + \pi/2 \right] + 0.5 \cos \left[ (2\omega_o + \omega_d) t - \pi/2 \right] \]

The term \( \cos \left[ (2\omega_o + \omega_d) t - \pi/2 \right] \) is double the RF frequency and hence it is filtered out.

Then the output of the mixer A is: \( E_B = 0.5 \cos \left[ (\omega_d) t + \pi/2 \right] \)
FM - CW Radar Systems
FM- CW Radar Systems

- CW Radar Systems do not give target range information.
- Doppler frequency is zero for stationary targets.
- A version of CW Radar provides range information by incorporating Frequency Modulation (FM) technique.
- FM-CW Radar can employ linear or non-linear frequency modulation.
- Linear Modulation is achieved by Triangular modulating waveform, while Non-Linear Modulation is achieved by Sinusoidal modulating waveform.
Frequency-Time Relationship

The frequency Rate of Change:

\[ f_o = \frac{\Delta f}{\Delta t} \quad \text{Hz / second} \]
Linear Modulation

If the frequency is being increased linearly at the rate \( f_o \) Hz/sec, then during the time \( \Delta t = 2R/c \) the transmitted frequency would have increased by:

\[
\Delta f = \dot{f}_o \Delta t
\]

Substituting \( 2R/c \) for \( \Delta t \) and \( f_b \) for \( \Delta f \)

\[
f_b = \dot{f}_o \cdot 2R/c
\]
Range Calculation

If the transmitted frequency is $f_t$, then the received frequency is given by: $f_t + f_b$

The difference between the transmitted and received frequencies is called the Beat Frequency $f_b$.

The range $R$ is then given by:

$$R = \frac{c}{2} \frac{f_b}{f_o}$$

where $f_o = \frac{\Delta f}{\Delta t}$
Linear Modulation Waveform

\[ \Delta t = \frac{2R}{c} \]

Transmitted: \( f_t \)

Received: \( f_r \)

\( \Delta t = 2R/c \)
Triangular Modulation

Practically, the frequency cannot be increased indefinitely and the target may not be stationary.

The modulating signal waveform used is Triangular, linearly increasing and decreasing (positive and negative slopes).

Let the frequency of the triangular waveform be \( f_m \), and the peak-to-peak frequency deviation \( 2\Delta f \) (FM modulation), then the transmitted frequency is linearly increasing and decreasing by the rate:

\[
\dot{f}_0 = 4f_m \Delta f
\]

\textit{see FM theory}
Frequency Rate of Change

The modulating frequency $f_m = 1 / T$

The frequency rate of change (slope) = $\Delta f / \Delta t$
$\Delta t = T / 4 = 1 / 4f_m$
Stationary Target

The beat frequency is given by:

\[ f_b = \frac{f_o}{2} \frac{R}{c} \]

Substituting \( f_o = 4 f_m \Delta f \)

\[ f_b = 8 f_m \Delta f \frac{R}{c} \]

The range is given by:

\[ R = c \frac{f_b}{(8 f_m \Delta f)} \]
Stationary Target Waveform

\[ \Delta t = \frac{2R}{c} \]

\[ f_b \]
Moving Target

When the target is moving Doppler Effect takes place and the frequency is shifted accordingly.

\[ f_d = 2 \frac{v_r f_t}{c} \]
Moving Target Waveform

\[ f_0 \]

\[ f_b \]
Beat Frequency for Moving Target

For a moving target, the beat frequency will now include an additional component due to Doppler frequency shift. The positive and negative frequency slopes will be:

\[ f_{\text{up}} = f_r - f_d \]

\[ f_{\text{down}} = f_r + f_d \]
Beat Frequency Waveform

\[ f_r - f_d \quad f_r + f_d \]

Frequency

Time
FM-CW Radar Block Diagram

- Transmit Antenna
- FM Transmitter
- Modulator
- Indicator
- Mixer
- Amp
- Limiter
- Frequency Counter
- Receive Antenna
MTI & Pulsed Doppler Radar System
Introduction

So far we examined the functionality of a number of radar systems, Pulsed Radar, CW Radar, and FM-CW Radar Systems.

Pulsed Radar System provides range (distance) information.

Doppler CW Radar System provides the relative velocity of a moving object.

FM-CW Radar System provides range information.

Is it possible to obtain both range and velocity of a moving object using one radar system?
Waveform (Wide Pulse)

Long enough to detect velocity!
Waveform (Narrow Pulse)

Far enough to detect range!
Waveform (Medium Pulse)

Good enough to detect both range and velocity!
MTI Radar (measures range)

The purpose of MTI Radar is to reject signals from fixed or slow-moving unwanted targets and display signals from fast-moving wanted targets.

Examples of slow-moving targets:
- buildings,
- hills,
- trees’
- sea waves, and
- rainfall.

A flying aircraft is an example of fast-moving targets.
Pulsed Doppler Radar
(measures velocity)

- Optimised for speed measurement, the range need not be accurate.

- Pulsed Doppler Radar operates with unambiguous Doppler measurement but with ambiguous range measurement.
Purpose & Characteristics

MTI and Pulsed Doppler Radar systems are used to detect moving targets in severe clutter environment.

PRF : Pulse Repetition Frequency
Purpose and Definition

Both the MTI and Pulsed Doppler Radar systems are based on the same physical principle, but in practice there are generally recognisable differences between them.

- **MTI Radar** operates with ambiguous Doppler Measurement but with unambiguous Range Measurement.
- **Pulsed Doppler Radar** operates with unambiguous Doppler Measurement but with ambiguous Range Measurement.

mainly for range but with some knowledge on speed

mainly for speed but with some knowledge on range
Operational Limits

- Combing the features of Pulsed and CW radar systems to obtain target **Range** and **Speed** information.

- Unambiguous Range Measurement is inversely proportional to the pulse repetition rate.

- Unambiguous Speed Measurement by Doppler frequency shift method requires a “**long enough**” pulse width.

- A compromise must be made on which measurement is to be unambiguous, Rang or Speed.
Coherent MTI Radar System

PRI : Pulse Repetition Interval
Simple CW Radar

Transmit Antenna $\rightarrow$ CW Oscillator $\rightarrow$ Receiver $\rightarrow$ Indicator

$\text{ft}$

$\text{ft} + f_d$

$\text{ft} - f_d$

$\text{fd}$
Pulsed-Doppler Radar

- Transmit Antenna
- Pulse Modulator
  - Pulsed $f_t$
- Power Amplifier
  - $f_t$
- CW Oscillator
  - $f_t$
- Receiver
  - Pulsed $f_t$
  - $f_t + f_d$
  - $f_t - f_d$
- Indicator
  - $f_d$
Waveform Equations

The transmitted carrier = $A_1 \sin (2\pi f_0 t)$

The reference carrier = $A_2 \sin (2\pi f_0 t)$

The returned echo = $A_3 \sin \left[2\pi (f_o +/- f_d) t - 4\pi f_o R / c \right]$

The result of mixing returned echo and reference carrier is:

$$V_{diff} = A_4 \sin \left( 2\pi f_d t - 4\pi f_o R / c \right)$$