Chapter (2)

Eng. Mai Z. Alyazji

October, 2016
### Table 2.1

<table>
<thead>
<tr>
<th>Postulate/ Theorem</th>
<th>Expression 1</th>
<th>Expression 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Postulate 2</td>
<td>( x + 0 = x )</td>
<td>( x \cdot 1 = x )</td>
</tr>
<tr>
<td>Postulate 5</td>
<td>( x + x' = 1 )</td>
<td>( x \cdot x' = 0 )</td>
</tr>
<tr>
<td>Theorem 1</td>
<td>( x + x = x )</td>
<td>( x \cdot x = x )</td>
</tr>
<tr>
<td>Theorem 2</td>
<td>( x + 1 = 1 )</td>
<td>( x \cdot 0 = 0 )</td>
</tr>
<tr>
<td>Theorem 3, involution</td>
<td>( (x')' = x )</td>
<td></td>
</tr>
<tr>
<td>Postulate 3, commutative</td>
<td>( x + y = y + x )</td>
<td>( xy = yx )</td>
</tr>
<tr>
<td>Theorem 4, associative</td>
<td>( x + (y + z) = (x + y) + z )</td>
<td>( x(yz) = (xy)z )</td>
</tr>
<tr>
<td>Postulate 4, distributive</td>
<td>( x(y + z) = xy + xz )</td>
<td>( x + yz = (x + y)(x + z) )</td>
</tr>
<tr>
<td>Theorem 5, De Morgan</td>
<td>( (x + y)' = x'y' )</td>
<td>( (xy)' = x' + y' )</td>
</tr>
<tr>
<td>Theorem 6, absorption</td>
<td>( x + xy = x )</td>
<td>( x(x + y) = x )</td>
</tr>
</tbody>
</table>

### 2.2 Simplify the following Boolean expressions to a minimum number of literals:

\[ \begin{align*}
(a) & \quad xy + xy' \\
(b) & \quad (x + y)(x + y') \\
(f) & \quad a'bc + abc' + abc + a'bc'
\end{align*} \]

**Answer:**

\[ \begin{align*}
(a) & \quad xy + xy' = x(y + y') = x(1) = x \\
b) & \quad (x + y)(x + y') = xx + xy' + xy + yy' = x + x(y + y') = x \\
f) & \quad a'bc + abc' + abc + a'bc' \\
& \quad = b(a' + a) = b
\]

### 2.3 Simplify the following Boolean expressions to a minimum number of literals:

\[ \begin{align*}
(b) & \quad x'yz + xz \\
(d) & \quad xy + x(wz + wz')
\end{align*} \]

**Answer:**

\[ \begin{align*}
b) & \quad x'yz + xz = (x'y + x)z = z(x' + x') = (x + y)
\end{align*} \]
d) \[ xy + x(wz + wz') = x(y + wz + wz') = x(y + w(z + z')) = x(w + y) \]

2.4 Reduce the following Boolean expressions to the indicated number of literals:

(a) \[ A'C' + ABC + AC' \]  to three literals

(b) \[ (x'y' + z)' + z + xy + wz \]  to three literals

(c) \[ A'B(D' + C'D) + B(A + A'CD) \]  to one literal

Answer:

a) \[ A'C' + ABC + AC' = C'(A'+A) + ABC = (C' + AB)(C' + C) = C' + AB \]

b) \[ (x'y' + z)' + z + xy + wz = (x'y')'z' + z + xy + wz \]
\[ = [(x + y)z' + z] + xy + wz = (z + z')(z + x + y) + xy + wz \]
\[ = z + wz + x + xy + y = z(1 + w) + x(1 + y) + y = x + y + z \]

(c) \[ A'B(D' + C'D) + B(A + A'CD) = B(A'D' + A'C'D + A + A'CD) \]
\[ = B(A'D' + A + A'D(C + C')) = B(A + A'(D' + D)) = B(A + A') = B \]

2.5 Draw logic diagrams of the circuits that implement the original and simplified expressions in Problem 2.2.

Answer:

a)
b) Find the complement of $F = wx + yz$; then show that $FF' = 0$ and $F + F' = 1$.

Answer:

$$F' = (wx + yz)' = (wx)'(yz)' = (w' + x')(y' + z')$$

$$FF' = wx(w' + x')(y' + z') + yz(w' + x')(y' + z') = 0$$

$$F + F' = wx + yz + (wx + yz)' = A + A' = 1$$ with $A = wx + yz$

2.9 Given the Boolean functions $F_1$ and $F_2$, show that

(a) The Boolean function $E = F_1 + F_2$ contains the sum of the minterms of $F_1$ and $F_2$.

(b) The Boolean function $G = F_1F_2$ contains only the minterms that are common to $F_1$ and $F_2$.

Answer:

(a) $F_1 + F_2 = \sum m_{1i} + \sum m_{2i} = \sum (m_{1i} + m_{2i})$

(b) $F_1 F_2 = \sum m_i \sum m_j$ where $m_i m_j = 0$ if $i \neq j$ and $m_i m_j = 1$ if $i = j$
2.12 We can perform logical operations on strings of bits by considering each pair of corresponding bits separately (called bitwise operation). Given two eight-bit strings A = 10110001 and B = 10101100, evaluate the eight-bit result after the following logical operations:

(a) AND         (b) OR         (c) XOR         (d) NOT A         (e) NOT B

Answer:

a) 1010 0000
b) 1011 1101
c) 0001 1101
d) 0100 1110
e) 0101 0011

2.13 Draw logic diagrams to implement the following Boolean expressions:

(a) \( F = [(u + x') (y' + z)] \)

Answer:

\[ u \]
\[ x \]
\[ y \]
\[ z \]
2.14 Implement the Boolean function

F = xy + x'y' + y'z

(a) With AND, OR, and inverter gates

(b) With OR and inverter gates

With AND and inverter gates

With NAND and inverter gates

With NOR and inverter gates

Answer:

a)

\[
F = xy + x'y' + y'z
\]

b)

\[
F = x'y + (x + y)' + (y + z)'
\]
c) \[ F = xy + x'y' + y'z \]
\[ = [(xy)'(x'y')'(y'z)']' \]

d) \[ F = xy + x'y' + y'z \]
\[ = [(xy)'(x'y')'(y'z)']' \]

e) \[ F = xy + x'y' + y'z \]
\[ = (x' + y')' + (x + y)' + (y + z')' \]
2.15 Simplify the following Boolean functions T1 and T2 to a minimum number of literals:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>T1</th>
<th>T2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Answer:

\[
T_1 = A'B'C' + A'B'C + A'BC' = A'B'(C' + C)' + A'BC' = A'B' + A'BC' \\
= (A'B' + A'B) (A'B' + C') = A'(B' + B)'(A'B' + C') = A'(A'B' + C') \\
= (A'A'B' + A'C') = (A'B' + A'C') = A'(B' + C') \\
\]

\[
T_2 = (A + B + C)(A + B + C')(A + B' + C) \\
= (A + AB + AC' + AB + B + BC' + AC + BC + CC')(A + B' + C) \\
= (A(1 + B + C' + C) + B(1 + C' + C) + CC')(A + B' + C) \\
= (A + B)(A + B' + C) + AB + AC + AB + BB' + BC = A(1 + B' + C + B) + BC' \\
= A + BC \\
\]

OR

\[
T_2 = T_1' = A'BC + AB'C' + AB'C + ABC' + ABC' \\
= BC(A' + A) + AB'(C' + C) + AB(C' + C) \\
= BC + AB' + AB = BC + A(B' + B) \\
= A + BC
\]
2.16 The logical sum of all minterms of a Boolean function of n variables is 1.

(a) Prove the previous statement for n = 3.

(b) Suggest a procedure for a general proof.

Answer:

a) \( F(A, B, C) = A'B'C' + A'B'C + A'BC' + A'BC + AB'C' + AB'C + ABC' + ABC \\
    = A'(B'C' + B'C + BC' + BC) + A((B'C' + B'C + BC') + BC) \\
    = B'(C' + C) + B(C' + C) = B' + B = 1 \\

b) \( F(x_1, x_2, x_3, \ldots, x_n) = \sum m_i \) has \( 2^n/2 \) minterms with \( x_1 \) and \( 2^n/2 \) minterms with \( x_1' \), which can be factored and removed as in (a). The remaining \( 2^{n-1} \) product terms will have \( 2^{n-1}/2 \) minterms with \( x_2 \) and \( 2^{n-1}/2 \) minterms with \( x_2' \), which and be factored to remove \( x_2 \) and \( x_2' \), continue this process until the last term is left and \( x_n + x_n' = 1 \).

2.17 Obtain the truth table of the following function, and express the function in sum-of-minterms and product-of-maxterms form:

(b) \( (cd + b'c + bd')(b + d) \)

Answer:

\[
(cd + b'c + bd')(b + d) = bcd + b'b'c'0 + bbd' + cdd + b'cd + b'd'd'0 \\
= bcd + bd' + cd + b'cd \\
= bcd + b(c+c')d' + (b+b')cd + b'cd \\
= bcd + bcd' + bc'd' + bcd + b'cd + b'cd \\
\]

<table>
<thead>
<tr>
<th>b</th>
<th>c</th>
<th>d</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\( F = \Pi (0,1,2,5) = \sum (3,4,6,7) \)
2.19 Express the following function as a sum of minterms and as a product of minterms:

\[ F(A, B, C, D) = B'D + A'D + BD \]

Answer:

First look at the number of literals... we have four different literals 
\[ A, B, C, D \]
In the first term \( B'D \), we have \( A, C \) are missing
We get them back as \( (A'+A')B'(C+C)D \), that \( B'D \) is equivalent to
\[ AB'CD + A'B'CD + AB'C'D + A'B'C'D \] and so on...

\[ F = (A+A')B'(C+C)D + A'(B+B')(C+C)D + (A+A')B(C+C)D \]
\[ = AB'CD + A'B'CD + AB'C'D + A'B'C'D + A'BCD + A'BCD + A'BC'D + A'B'C'D + ABCD + A'BC'D \] (omit repeated terms)
\[ = AB'CD + A'B'CD + AB'C'D + A'B'C'D + A'BCD + A'BC'D + ABCD + ABC'D \]
\[ = \sum(11, 3, 9, 1, 7, 5, 15, 13) = \sum(1, 3, 5, 7, 9, 11, 13, 15) \]
\[ = \Pi(0, 2, 4, 6, 8, 10, 12, 14) \]

2.20 Express the complement of the following functions in sum-of-minterms form:

(a) \( F(A, B, C, D) = \sum(2, 4, 7, 10, 12, 14) \)

(b) \( F(x, y, z) = \Pi(3, 5, 7) \)

Answer:

a) \( \sum(0, 1, 3, 5, 6, 8, 9, 11, 13, 15) \)

b) \( = \Pi(3, 5, 7) = \sum(0, 1, 2, 4, 6) \) complement=\( \sum(3, 5, 7) \)

2.21 Convert each of the following to the other canonical form:

(a) \( F(x, y, z) = \sum(1, 3, 5) \)
(b) \( F(A, B, C, D) = \Pi(3, 5, 8, 11) \)

Answer:

a) \( \Pi(0, 2, 4, 6, 7) \)

b) \( \Sigma(0, 1, 2, 4, 6, 7, 9, 10, 12, 13, 14, 15) \)

2.24 Show that the dual of the exclusive-OR is equal to its complement.

Answer:

\( x \oplus y = x'y + xy' \) and \( (x \oplus y)' = (x + y')(x' + y) \)

Dual of \( x'y + xy' = (x' + y)(x + y') = (x \oplus y)' \)

2.29 Determine whether the following Boolean equation is true or false.

\( x'y' + x'z + x'z' = x'z' + y'z' + x'z \)

Answer:

\[
\begin{align*}
\text{False} \\
\downarrow & \downarrow \\
x'y' + x'z + x'z' \neq x'z' + y'z' + x'z \\
\Sigma(0,1,2,3) \neq \Sigma(0,1,2,3,4)
\end{align*}
\]
2.9 Find complement:

(a) \( xy' + x'y = (xy)'(x'y) = (x'+y)(x+y) = x'y' + xy + x'y + xy' \)

(b) \((a+c)(a+b)(a'+b+c') \)
\((a+c)'(a+b)'(a'+b+c)' = ac' + ab + a'b'c'\)

(c) \( z + z'(w' + xy) \)
\( z'(z'(w' + xy)') = z'(w' + xy)' \)
\( = z'(z + (w + xy)')(x + y') \)
\( = z'(z + y) + z'(w + xy)(x + y') = z'(w + z + y' + x' + w'y + z'y') \)

2.11 List Truth table.

(a) \( F = xy + x'y + y'z \)

(b) \( F = bc + a'c' \)
2.23) Draw logic diagram without simplifying.
(a) Bc\(^1\) + AB + Ac\(^1\)D

2.25) By substituting the Boolean expr equivalent of the binary ops as defined in Table 2.3, show the following:

(a) The inhibition operation is neither commutative nor associative.

F = xy\(^1\) = x\(\bar{y}\)
F = x\(\bar{y}\) = x\(\bar{y}\) ≠ not comm.
(x\(\bar{y}\))x\(\bar{z}\) = x\(\bar{y}\)z
x\(\bar{y}\) = x\(\bar{y}\)z

(b) The exclusive-or op. is commutative and associative.

x\(\oplus\)y = xy\(^1\) + xy
(x\(\oplus\)y)\(\oplus\)z = (xy\(^1\) + xy)\(\oplus\)z

x\(\oplus\)z + xy\(^1\) + x\(\bar{y}\)\(\bar{z}\) = x\(\bar{y}\) + y\(\bar{z}\) + x\(\bar{y}\)z

2.26) Show that a positive logic NAND gate is negative logic NOR gate and vice versa.

<table>
<thead>
<tr>
<th>NAND Gate</th>
<th>NAND Pos.</th>
<th>NOR Pos.</th>
<th>NOR Neg.</th>
<th>NAND Neg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
<td>z</td>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>L</td>
<td>L</td>
<td>H</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>H</td>
<td>L</td>
<td>L</td>
<td>H</td>
<td>L</td>
</tr>
<tr>
<td>H</td>
<td>H</td>
<td>L</td>
<td>H</td>
<td>L</td>
</tr>
</tbody>
</table>

\[ x = x y^1 + x y^2 + x y_1 z + x y_1 ^2 \]
2-30 Write Boolean expression for sum of products:

\[ (b + d) (a' + b' + c) \]
\[ a'b + bb + bc + ad + bd + dc \]
\[ = ab(c+c)(d+d') + (a+a')bc(d+d') + a'(b+b')(c+c)d + (a+a')bd \]
\[ + (c+c) + (a+a')(b+b') \]
\[ cd = \frac{abcd + abcd' + abcd + abcd'}{abed + abed + abed + abed + abed + abed} \]
\[ + \frac{abed + abed + abed + abed + abed + abed}{abed + abed + abed + abed + abed + abed} \]

2-31 Write Boolean expression for product of sum form:

\[ F = a'b + a'c' + abc \]
\[ a'b(c+c) + a'(b+b')c' + abc \]
\[ = a'bc + ab'c + a'bc + abc + abc + abc + abc \]
\[ F' = \Sigma (0, 2, 3, 7) = a'bc + ab'c + abc + abc \]

\[ F = (a'bc + ab'c + abc + abc) \]
\[ \xi \]
\[ F = (a + b + c') (a + b + c) (a + b + c') (a + b + c') \]
\[ = \Pi (1, 2, 3, 6) \]

301 2

\[ \xi \]
\[ = a'bc d + a'bc d + a'bc d + a'bc d + a'bc d + a'bc d + a'bc d + a'bc d \]
\[ 000 \]
\[ 001 \]
\[ 010 \]
\[ 011 \]
\[ 100 \]
\[ 101 \]
\[ 110 \]
\[ 111 \]

\[ = \Sigma (0, 3, 4, 5, 6, 7, 9, 13, 14) \]
### Boolean Expression

<table>
<thead>
<tr>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$ab$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

$$F_1 = a'b'c' + a'bc' + ab'c + abc' + abc$$

$$F_2 = a'b'c + ab'c + a'bc$$

$$Z = a'b'c' + ab'c + a'bc$$