Experiment (1): Hydrostatic force on a plane surface

Introduction:
The study of pressure forces acting on plane submerged surfaces is a fundamental topic in the subject of hydrostatic involving assessment of the value of the net thrust and the concept of center of pressure, which are so important in the design of innumerable items of hydraulic equipment and civil engineering projects.

Purpose:
To investigate the pressure acting on a submerged surface and to determine the position of the center of pressure.

Apparatus:
1. Center of pressure apparatus (Figure 1).

![Figure 1: Center of pressure apparatus](image-url)
Figure 2: Schematic diagram of center of pressure apparatus

Theory:

Referring to figure (2) which shows the experimental set-up consider the forces which result in turning moments of the beam and submerged part of the model about the knife edged fulcrum. Liquid pressures on the curved surfaces act at right angles to the curved surfaces, and the design of the model ensures that these forces pass through the line of action of the knife edges and therefore do not exert any turning moment. The hydrostatic pressure on the vertical end surface exerts a force $F$ at the center of pressure which is at depth $H_P$ below the surface. The resulting turning moment about the knife edge from the hydrostatic forces is therefore given by:
\[ F (a + d - y + Hp) \]

Which is resisted by the weight of the mass \( M \) on the balance arm at distance \( L \) from the knife edge:

\[ MgL \]

Now considering the cases of partial immersion and complete immersion separately:

(A) Partial immersion

![Diagram of partial immersion](image.png)

Figure 3: Surface is partially submerged

When the vertical end face of the quadrant is only partially immersed, the geometric properties of the wetted portion of the end face are:

- Area \( A = by \)
- Depth of center \( \bar{H} = y/2 \)
- Second moment of area \( I_o = by^3/12 \)
- Depth of center of pressure \( H_p = \bar{H} + \frac{I_o}{A\bar{H}} = \frac{y}{2} + \frac{by^3/12}{by/2} = \frac{2y}{3} \)

The force acting on the submerged part of the end surface of the model is:

\[ F = \rho g \bar{H} A = \rho g y/2 by = 1/2 \rho g by^2 \]

Taking moments about the knife edge:

\[ MgL = F (a + d - y + H_p) \]
Substituting for $H_p$ and rearranging the above equation shows that the force acting on the wetted end surface can be calculated from the experimental results of $M$ and $y$:

$$F = \frac{MgL}{a + d - y + \frac{2y}{3}} = \frac{MgL}{a + d - \frac{y}{3}}$$

Which can then be compared with the theoretical result:

$$F = \frac{1}{2} \rho g b y^2$$

(B) Complete Immersion

![Figure 4: Surface is fully submerged](image)

When the end surface is fully immersed, the properties of the submerged end face are:

Area $A = bd$

Depth of center of area $\bar{H} = y - d/2$

Second moment of area $I_o = b y^3 / 12$

Depth of center of pressure

$$H_p = \bar{H} + \frac{I_o}{A\bar{H}} = y - \frac{d}{2} + \frac{bd^3 / 12}{bd\bar{H}} = y - \frac{d}{2} + \frac{d^2}{12 \bar{H}}$$

The force acting on the end surface is:

$$F = \rho g \bar{H} A$$

$$= \rho g \left( y - \frac{d}{2} \right) bd$$

Taking moments about the knife edge:
\[ MgL = F (a + d - y + H_p) \]

Substituting for \( H_p \) and rearranging the above equation shows that the force acting on the wetted end surface can be calculated from the experimental results of \( M \) and \( y \):

\[ F = \frac{MgL}{a + d - y + y - \frac{d}{2} + \frac{d^2}{12H}} = \frac{MgL}{a + \frac{d}{2} + \frac{d^2}{12H}} \]

Which can be compared with the theoretical result calculated from:

\[ F = \rho g \left( y - \frac{d}{2} \right) bd \]

**Equipment preparation:**

Position the apparatus on the work surface of the hydraulic bench and adjust the feet to level the base. Attach a length of hose to the drain cock and direct the other end of the hose into the overflow pipe of the volumetric measuring tank. If the quadrant is not assembled to the balance arm then locate the quadrant on the two dowel pins and fasten it to the balance arm by the central screw.

**Procedures:**

1. If necessary measure the dimensions \( a, b \) and \( d \) of the quadrant, and the distance between the pivot and the weight hanger \( L \). Lightly apply wetting agent to reduce surface tension effects.
2. Insert the quadrant into the tank locating the balance arm on the knife edges. Adjust the counter-balance weight until the balance arm is horizontal, as indicated on the datum level indicator.
3. Add all the weights supplied to the weight carrier. Fill the tank with water until the balance beam tips lifting the weights then drain out a small quantity of water to bring the balance arm horizontal, **do not level the balance arm by adjustment of the counter balance weight or the datum setting of the balance arm will be lost**. Record the water level shown on the scale. Fine adjustment of the water level may be achieved by over-filling and slowly draining, using the drain cock.
4. Remove one or more weights from the weight carrier and level the balance arm by draining out more of the water. When the arm is level record the depth of immersion shown on the scale on the quadrant.
5. Repeat reading for reducing masses on the weight carrier.

Data & Results:

$L = 275 \text{ mm}, a = 100 \text{ mm}, d = 100 \text{ mm}, b = 75 \text{ mm}$

(A) Partial Immersion

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<thead>
<tr>
<th>Trials</th>
<th>1</th>
<th>2</th>
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<tbody>
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(B) Complete Immersion

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