Implicit enumeration

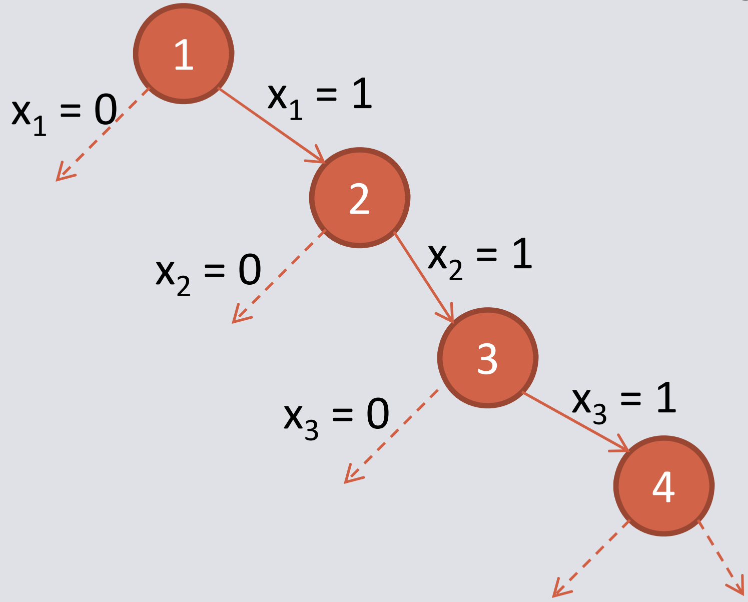
Objectives of the topic:

- Present and apply the implicit enumeration method for the solution of 0-1 integer programming models.
- See the handout.

- The method of implicit enumeration is often used to solve 0-1 models.
- Implicit enumeration is based on the fact that each variable must equal 0 or 1 to simplify both the branching and bounding components of the branch-and-bound process and to determine efficiently when a node is infeasible.
- The tree used in the implicit enumeration method is similar to that of the brand and bound method.
- However, branching in implicit enumeration is specific, since the branching variable has to take either 0 or 1.

Implicit enumeration basics

- At any node of the implicit enumeration tree, values of some variables are specified.
- For example, assume a 0-1 MIP has the variables x_1, x_2, …, x_5, and the solution tree is as illustrated by



- At node 4, the values of x_1, x_2, and x_3 are determined.

- Hence, parts of the solution tree can be eliminated as either non-feasible or do not providing a better solution.
- Elimination of parts of the solution tree can be performed using the following general rules:

1. In any non-terminal node of the tree, select the best values for the undetermined variables. If the chosen solution is feasible, then the solution is the best and further branching is unnecessary.

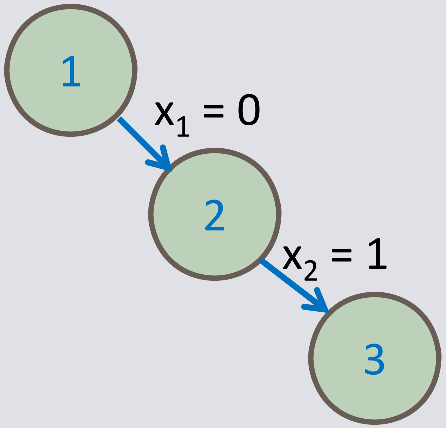
- To illustrate, take the following 0-1 model

$$\max z = 4x_1 + 2x_2 - x_3 + 2x_4$$

$$\text{s.t.} \qquad x_1 + 3x_2 - x_3 - 2x_4 \ge 1$$

$$x = (0,1)$$

- The partially complete solution tree is



- At node 3, $x_1 = 0$ and $x_2 = 1$.
- The best values for the undetermined variables are $x_3 = 0$ and $x_4 = 1$, and they are feasible.
- Hence, no further branching from node 4 is required.

2. If the choice of values for the undetermined variables is not feasible, compare the objective function value with the current best objective function value.

- For example, if the objective function is

$$\max z = 4x_1 + 2x_2 + x_3 - x_4 + 2x_5$$

- Assume the current best integer solution is 6.
- Suppose that a specific node, $x_1 = 0$, $x_2 = 1$, $x_3 = 1$.
- The best values for the undetermined variables are $x_4 = 0$, $x_5 = 1$, $z = 5$.
- Since 5 < 6, this node should be eliminated, regardless whether any feasible solution will come from this node.

3. Determine whether any feasible solution can be generated from the current node.

- Consider the following constraint:

$$-2x_1 + 3x_2 + 2x_3 - 3x_4 - x_5 + 2x_6 \leq -5$$

- Assume at a specific node $x_2 = 1$, $x_3 = 1$, $x_4 = 1$.
- The values for the undetermined variables that can satisfy the constraints are $x_1 = 1$, $x_5 = 1$, and $x_6 = 0$.
- Note that x_1 and x_5 have negative coefficients and x_6 has positive coefficient.
- These values do not satisfy the constraint, hence further branching is not required.

Example

- Use implicit enumeration to solve the 0-1 model

$$\max z = -7x_1 - 3x_2 - 2x_3 - x_4 - 2x_5$$

$$\text{s.t.} \quad -4x_1 - 2x_2 + x_3 - 2x_4 - x_5 \leq -3$$

$$-4x_1 - 2x_2 - 4x_3 + x_4 + 2x_5 \leq -7$$

$$x = (0,1)$$