Engineering Statistics (ENGC 6310) - DR. Samir Safi

Fall 2008

Midterm Exam #1

NAME:____ ID:_____________________

Section A: Multiple-Choice

For each question in this section, circle the correct answer. (Problem is worth 2)
Questions 1-2 refer to the following information:

1- The height of Palestinian men aged 18 to 24 are approximately normally distribution with mean 68 inches and standard deviation 2.5 inches. Half of all young men are shorter than

a) 65.5 inches b) 68 inches c) 70.5 inches

d) Can't tell, because the median height is not given.

2- Only about 5% of young men have heights outside the range

a) 65.5 inches to 70.5 inches b) 63 inches to 73 inches

c) a) 60.5 inches to 75.5 inches d) 58 inches to 78 inches

3. You have measured the systolic blood pressure of a random sample of 25 employees of a company. A 95% confidence interval for the mean systolic blood pressure for the employees is computed to be (122,138). Which of the following statements gives a valid interpretation of this interval?

(a) About 95% of the samples of employees have a systolic blood pressure between 122 and 138.
(b) About 95% of the employees in the company have a systolic blood pressure between 122 and 138.
(c) If the sampling procedure were repeated many times, then approximately 95% of the resulting confidence intervals would contain the mean systolic blood pressure for employees in the company.
(d) If the sampling procedure were repeated many times, then approximately 95% of the sample means would be between 122 and 138.

<u>Questions 4-5 refer to the following information:</u>

4- In a large population of college students, 20 % of the students have experienced feelings of math anxiety. If you take a random sample of 10 students from this population, the probability that exactly 2 students have experienced math anxiety is

a) 0.3020 b) 0.2634 c) 0.2013 d) 0.5 e) 1

5- The standard deviation of the sample proportion of students who have experienced math anxiety is

a) 0.0160 b) 0.1265 c) 0.2530 d) 1 e) 0.2070

6- You want to estimate the mean SAT score for a population of students with a 90% confidence interval. Assume that the population standard deviation is σ = 100. if you want the margin of error of the to be approximately 5, you will need a sample size of

a) 33 b) 1083 c) 11 d) 1476
e) None of the above

<u>Questions 7-8 refer to the following information:</u>

7- A 95% confidence interval for the mean reading achievement score for a population of third grades is (44.2 , 54.2). The margin of error of this interval is

a) 95% b) 5 c) 2.5 d) 54.2
e) The answer cannot be determined from the information given

8- The sample mean is
a) 44.2 b) 54.2 c) 0.95 d) 49.2
e) The answer cannot be determined from the information given

9- Using the same set of data, you compute a 95% confidence interval and a 99% confidence interval. Which of the following statement is correct?

a)	The intervals have the same width	b)	The 99% interval is wider
c)	The 95 % interval is wider	c)	You cannot be determined which interval is wider unless you know n and s

For problems 10-11 use the following information:

Parents in Lake Wobegone think their children are smarter than average. To try and prove this they take a random sample of 2500 children from Lake Wobegone and find their average IQ is 100.8 with a standard deviation of 15. The average IQ outside of Lake Wobegone is 100.

10- The P-value of the test is

a) 0.051 b) 0.012 c) 0.008 d) 0.004

11- A 90% confidence interval for the average IQ of Lake Wobegone children is

a) (100.3,101.3) b) (100,101) c) (99.1,101.9) d) (93.4,108.2)

For problems 12-13, use the following information:

The shelf life of a carbonated beverage is of interest. Ten bottles are randomly selected and tested. The following results are obtained in days:

108	124	124	106	115	138	163	159	134	139

We want to test if the true mean shelf life is greater than 125 days.

12. The alternative hypothesis is

a) $H_a : \overline{X} > 125$ b) $H_o : \mu > 125$ c) $H_a : p > 125$ d) $\mathrm{H_a} : \mu > 125$

13. Which of the following is the test statistic?

a) Z = 0.31 b) T = 0.97 c) Z = 0.9 d) T = 3.07

14. Which of the following is an example of a matched pairs design?
(a) A teacher compares the pre-test and the post-test scores of students.
(b) A teacher compares the scores of students using a computer-based method of instruction with the scores of other students using a traditional method of instruction.
(c) A teacher compares the scores of students in her class on a standardized test with national average score.
(d) A teacher calculates the average of scores of students on a pair of tests and wishes to see if this average is larger than 80%.

15. An airplane is only allowed a gross passenger weight of 8000 kg. If the weights of passengers traveling by air between Toronto and Vancouver have a mean of 78 kg and a standard deviation of 7 kg, the approximate probability that the combined weight of 100 passengers will exceed 8,000 kg is:

a) 0.4978 b) 0.3987 c) 0.0044 d) 0.0022

Section B: Free Response Questions

Question (1): (5 Points)

The weight of the eggs produced by a certain breed of hen is normally distributed with mean 65 g and standard deviation 5 g. If cartons of such eggs can be considered to be simple random sample of size 12 from the population of all eggs, what is the probability that the weight of a carton falls between 750 g and 825 g?

Question #2: (10 Points)

A company wanted to know if attending a course on "how to be a successful salesperson" can increase the average sales of its employees. The company sent six of its salespersons to attend this course. The following table gives the week sales of these salespersons before and after they attended this course.

Before	12	18	25	9	14	16
After	18	24	24	14	19	20

Using the 1% significance level, can you conclude that the mean weekly sales for all salespersons increase as a result of attending this course? Assume that the population of paired difference has a normal distribution.

(a) (2 Points) State H_0 and H_a.

H_o:

H_a:

(b) (4 Points) Calculate the test statistic.

(c) (2 Points) Find the “P-value” or give the rejection region.

(d) (2 Points) State your conclusion.

Question (3): (15 Points)

A political analyst was curious if younger adults were becoming more conservative. He decided to see if the mean age of registered Republicans was lower than that of registered Democrats. He selected an SRS of 128 registered Republicans from a list of registered Republicans and determined the mean age to be $\bar{x}_1 = 39$ years, with a standard deviation $S_1 = 8$ years. He also selected an independent SRS of 200 registered Democrats from a list of registered Democrats and determined the mean age to be $\bar{x}_2 = 40$ years, with a standard deviation $S_1 = 10$ years. Let μ_1 and μ_2 represent the mean ages of the populations of all registered Republicans and Democrats, respectively. Suppose that the distributions of age in the populations of registered Republicans and of registered Democrats have the *same* standard deviation. Assume the pooled two-sample *t*-procedures are safe to use (The two variances are equal).

a. (2 Points) Compute the numerical value of the pooled estimator of σ^2, S_p^2

b. (3 Points) Construct and 90% confidence interval for μ_1-μ_2. Interpret your finding.

c. (2 Points) State H_0 and H_a.

- H_o:
- H_a:

d. (4 Points) Calculate the test statistic.

e. (2 Points) Compute the P-value.

f. (2 Points) State your conclusion at level of significance $\alpha = .05$.

Good Luck

Formula Sheet

Binomial Distribution	
$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}, k=0,1,\ldots,n$	$\mu=np$, $\sigma^2=np(1-p)$

1-Population mean	**Level C confidence interval for μ**	**Hypothesis test H_0: $\mu=\mu_0$**
Large sample **One-sample z test** Use s if σ is unknown	$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$	$z=\frac{\bar{x}-\mu_\circ}{\sigma/\sqrt{n}}$
Small sample and σ unknown **One-sample t test**	$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$ $df=n-1$	$t=\frac{\bar{x}-\mu_\circ}{s/\sqrt{n}}$, $df=n-1$

2-population means	**Level C confidence interval for $\mu_1-\mu_2$**	**Hypothesis test H_0: $\mu_1=\mu_2$**
Large samples **Two-sample z test** Use s_1 and s_2 if σ_1,σ_2 are unknown	$(\bar{x}_1-\bar{x}_2) \pm z^*\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$	$z=\frac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$
Pooled two-sample t test $s_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{(n_1+n_2-2)}$ σ_1,σ_2 unknown & equal	$(\bar{x}_1-\bar{x}_2) \pm t^* s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$ $df=n_1+n_2-2$	$t=\frac{\bar{x}_1-\bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$ $df=n_1+n_2-2$
Two-sample t test σ_1,σ_2 unknown & unequal	$(\bar{x}_1-\bar{x}_2) \pm t^*\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$	$t=\frac{\bar{x}_1-\bar{x}_2}{\sqrt{(s_1^2/n_1)+(s_2^2/n_2)}}$
Matched pairs t-test	$t=\frac{\bar{d}}{S_d/\sqrt{n}}$, $d_i=x_{before}-x_{after}$, $df=n-1$	

Computing P-values	Use z-table for z tests and t-table for t tests Reject $H_\circ$ if P-value$<\alpha$

Sample size for desired margin of error m
One-sample Z interval: $n=\left(\frac{z^*\sigma}{m}\right)^2$

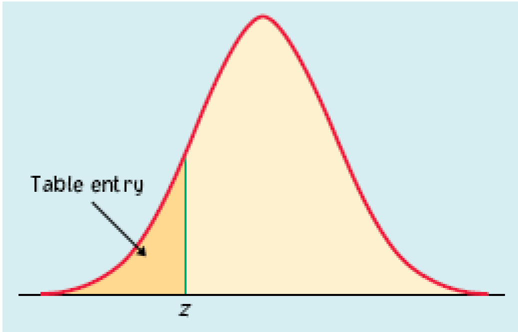

Table entry for z is the area under the standard normal curve left of z.

TABLE A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

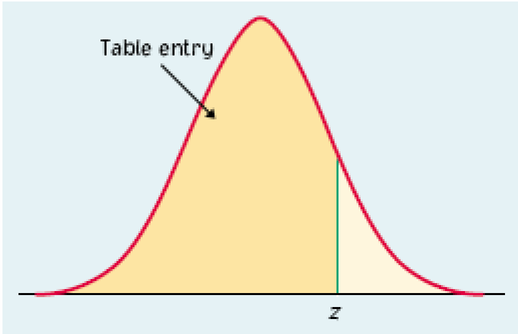

Table entry for z is the area under the standard normal curve to the left of z.

TABLE A Standard normal probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^*.

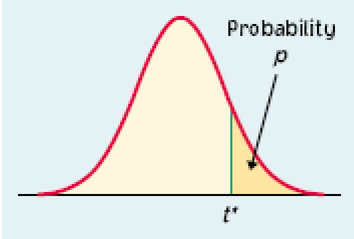

TABLE C *t* **distribution critical values**

	Upper tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											