**Example (8.1):**

Using the *ACI Code* approximate structural analysis, design for a warehouse, a continuous one-way solid slab supported on beams 4.0 m apart as shown in Figure 1. Assume that the beam webs are 30 cm wide. The dead load is 300 kg/m² in addition to own weight of the slab, and the live load is 300 kg/m². Use \( f'_c = 250 \text{ kg/cm}^2 \), \( f_y = 4200 \text{ kg/cm}^2 \).

**Solution:**

1- Select a representative 1 m wide slab strip:

The selected representative strip is shown in Figure 1.

2- Select slab thickness:

The clear span length \( l_n = 4.0 - 0.30 = 3.70 \text{ m} \)

For one-end continuous spans, \( h_{min} = l/24 = 400/24 = 16.67 \text{ cm} \)

Slab thickness is taken as 17 cm.

3- Calculate the factored load \( w_u \) per unit length of the selected strip:

Own weight of slab = 0.17 \times 2.50 = 0.425 \text{ ton/m}^2

\[ w_u = 1.20(0.30+0.425) + 1.60(0.30) = 1.35 \text{ ton/m}^2 \]

For a strip 1 m wide, \( w_u = 1.35 \text{ ton/m} \)

4- Evaluate the maximum factored bending moments in the strip:

The results are shown in the following table. Points at which moments and shear are calculated are shown in Figure 2.
5- Check slab thickness for beam shear:
Effective depth \( d = 17 - 2 - 0.60 = 14.40 \) cm, assuming \( \phi 12 \) mm bars.

\[
V_{u \ max} = \frac{1.15}{2} w_u l_n = \frac{1.15}{2} (1.35)(3.70) = 2.87 \text{ ton}
\]

\[\Phi \ V_c = 0.75(0.53)\sqrt{250} (100)(14.40)/1000 = 9.05 \text{ ton} > 2.87 \text{ tons} \ O.K.\]
i.e. , slab thickness is adequate in terms of resisting beam shear.

6- Design flexural and shrinkage reinforcement:
Steel reinforcement ratios are then calculated, and be checked against minimum and maximum code specified limits, where

\[
\rho = \frac{0.85 f'_c}{f_y} \left[ 1 - \sqrt{\frac{1 - \frac{2.353 \times 10^5 M_u}{\Phi b d^2 f'_c}}{}} \right]
\]

Minimum reinforcement = 0.0018 (100) (17) = 3.06 cm²/m

The reinforcement for positive and negative moments are shown in the following table.

<table>
<thead>
<tr>
<th>Factored moment in t.m</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment coefficient</td>
<td>- 1/24</td>
<td>1/14</td>
<td>- 1/10</td>
<td>-1 /11</td>
<td>1/16</td>
</tr>
<tr>
<td>Factored moment in t.m</td>
<td>- 0.77</td>
<td>1.32</td>
<td>- 1.85</td>
<td>- 1.68</td>
<td>1.16</td>
</tr>
</tbody>
</table>

Calculate the area of shrinkage reinforcement:

Area of shrinkage reinforcement = 0.0018 (100) (17) = 3.06 cm²/m
Select reinforcement bars:
Main and secondary reinforcement bars are also shown in the table
For shrinkage reinforcement use $\phi 10\ mm @ 25\ cm$, or $4\ \phi 10\ mm/m$.

Check bar spacing against code specified values
For main reinforcement, spacing between bars is not to exceed the larger of $3\ (17) = 51\ cm$ and $45\ cm$, which is already satisfied as shown in the table. For shrinkage reinforcement, spacing between bars is not to exceed the larger of $5\ (17) = 85\ cm$ and $45\ cm$, which is also satisfied.

7- Prepare neat sketches showing the reinforcement and slab thickness:
Figure 3 shows reinforcement details and required slab thickness.

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(a)

(b)

Figure 3: (a) Section A-A; (b) reinforcement details
Example (8.2):
Design the slab shown in Example (8.1) using any available structural analysis software.

Solution:
1- Select a representative 1 m wide slab strip:
The selected representative strip is shown in Figure 1.

2- Select slab thickness:
Same as in Example (8.1), the thickness is taken as 17 cm.

3- Calculate the factored load $w_u$ per unit length of the selected strip:
For a strip 1 m wide, $w_u = 1.35 \text{ton/m}$

4- Evaluate the maximum factored shear forces and bending moments in the strip:
Shear force and bending moment diagrams are shown in Figure 4.

5- Check slab thickness for beam shear:
Effective depth $d = 17 - 2 - 0.60 = 14.40 \text{cm}$, assuming $\phi 12 \text{mm}$ bars.

$$V_{u_{\text{max}}} = 3.27 \text{ton}$$

$$\Phi \ V_c = 0.75(0.53)\sqrt{250 (100)(14.40)/1000} = 9.05 \text{ton} > 3.27 \text{tons O.K.}$$

i.e., slab thickness is adequate in terms of resisting beam shear.

6- Design flexural and shrinkage reinforcement:
Steel reinforcement ratios are calculated and checked against minimum and maximum code specified limits.

For $M_u = -2.28 \, t.m$

$$\rho = \frac{0.85 \times 250}{4200} \left[ 1 - \sqrt{1 - \frac{2.353 \times 10^5 (2.28)}{0.9 \times 100 \times (14.4)^2 \times 250}} \right] = 0.0030$$

$$A_s = 0.0030 \times (100) \times (14.4) = 4.32 \text{ cm}^2 \text{ /m}, \text{ use } \phi 10 \text{ mm @ 15 cm}.$$  

For $M_u = -1.87 \, t.m$

$$\rho = \frac{0.85 \times 250}{4200} \left[ 1 - \sqrt{1 - \frac{2.353 \times 10^5 (1.87)}{0.9 \times 100 \times (14.4)^2 \times 250}} \right] = 0.0024$$

$$A_s = 0.0024 \times (100) \times (14.4) = 3.46 \text{ cm}^2 \text{ /m}, \text{ use } \phi 10 \text{ mm @ 20 cm}.$$  

For $M_u = -1.66 \, t.m$

$$\rho = \frac{0.85 \times 250}{4200} \left[ 1 - \sqrt{1 - \frac{2.353 \times 10^5 (1.66)}{0.9 \times 100 \times (14.4)^2 \times 250}} \right] = 0.0022$$

$$A_s = 0.0022 \times (100) \times (14.4) = 3.17 \text{ cm}^2 \text{ /m}, \text{ use } \phi 10 \text{ mm @ 20 cm}.$$  

For $M_u = 1.68 \, t.m$

$$\rho = \frac{0.85 \times 250}{4200} \left[ 1 - \sqrt{1 - \frac{2.353 \times 10^5 (1.68)}{0.9 \times 100 \times (14.4)^2 \times 250}} \right] = 0.0022$$

$$A_s = 0.0022 \times (100) \times (14.4) = 3.17 \text{ cm}^2 \text{ /m}, \text{ use } \phi 10 \text{ mm @ 20 cm}.$$  

For $M_u = 0.93 \, t.m$

$$\rho = \frac{0.85 \times 250}{4200} \left[ 1 - \sqrt{1 - \frac{2.353 \times 10^5 (0.93)}{0.9 \times 100 \times (14.4)^2 \times 250}} \right] = 0.0012$$

$$A_s = 0.0012 \times (100) \times (14.4) = 1.728 \text{ cm}^2 < A_{s,\text{min}}$$

$$A_s = 0.0018 \times (100) \times (17.0) = 3.06 \text{ cm}^2 \text{ /m}, \text{ use } \phi 10 \text{ mm @ 25 cm}.$$  

For $M_u = 0.74 \, t.m$

$$A_s < A_{s,\text{min}}$$
Calculate the area of shrinkage reinforcement:
Area of shrinkage reinforcement = 0.0018 \times 100 \times 17 = 3.06 \text{ cm}^2/\text{m}, \text{ use } \phi 10 \text{ mm } @ 25 \text{ cm}.

Select reinforcement bars:
It is already done in step 6.

Check bar spacing:
Same as in Example (8.1).

7- Prepare neat sketches showing the reinforcement and slab thickness:
Figure 5 shows details of the required reinforcement.

Example (8.3):
Design a one-way ribbed slab to cover a 4 m \times 10 m panel, shown in Figure 6. The covering materials weigh 150 kg/\text{m}^2, concrete hollow blocks are 40 cm \times 25 cm \times 20 cm in dimension, each 20 kg in weight, equivalent partition load is equal to 75 kg/\text{m}^2, and the live load is 200 kg/\text{m}^2. Use \( f_c' = 250 \text{ kg/} \text{cm}^2 \), \( f_y = 4200 \text{ kg/} \text{cm}^2 \), and \( \gamma_{\text{plaster}} = 2100 \text{ kg/} \text{m}^3 \).

Solution:
1. **The direction of ribs is chosen:**
Ribs are arranged in the short direction as shown in Figure 6.

2. **The overall slab thickness h is determined:**
Minimum slab thickness, \( h_{\text{min}} = \frac{400}{16} = 25 \text{ cm} \)
Topping slab thickness = 25 – 20 = 5 cm
Let width of web be equal to 10 cm
*Design of topping slab:* \( w_u = 1.20 \left( 0.05 \times 2.5 + 0.075 + 0.15 \right) + 1.60 \left( 0.20 \right) = 0.74 \text{ ton} / \text{m}^2 \)
For a strip 1 m wide, \( w_u = 0.74 \text{ ton} / \text{m} \)
\[ M_u = w_u \cdot l^2 / 12 = 0.74 \left( 0.40 \right)^2 / 12 = 0.011 \text{ m} \]
\[ t = \sqrt{\frac{3 \cdot M_u}{\Phi b \cdot f_c}} = \sqrt{\frac{3 \left( 0.011 \right) \left( 10 \right)^3}{0.9 \left( 100 \right) \sqrt{250}}} = 1.52 \text{ cm} < 5 \text{ cm} \]
Area of shrinkage reinforcement
\[ A_s = 0.0018 \left( 100 \right) \left( 5 \right) = 0.90 \text{ cm}^2 / \text{m} \]
Use 4 ø 6 mm / m in both directions.

3. **The factored load on each of the ribs is to be computed:**
Factored load per rib is shown below, supported by Figure 7.
Figure 7: Factored load calculation

Total volume (hatched) = 0.5 \times 0.25 \times 0.25 = 0.03125 \, m^3

Volume of one hollow block = 0.4 \times 0.20 \times 0.25 = 0.02 \, m^3

Net concrete volume = 0.03125 - 0.02 = 0.01125 \, m^3

Weight of concrete = 0.01125 \times 2.5 = 0.028125 \, ton

Weight of concrete / m^2 = 0.028125/(0.5) (0.25) = 0.225 \, ton/m^2

Weight of hollow blocks / m^2 = 20/(0.5) (0.25) (1000) = 0.16 \, ton/m^2

Weight of plastering coat / m^2 = 0.02 (2100/1000) = 0.042 \, ton/m^2

Total dead load = 0.225 + 0.16 + 0.042 + 0.075 + 0.15 = 0.652 \, ton/m^2

\[ w_u = 1.20(0.652) + 1.60(0.20) = 1.10 \, \text{ton/m}^2 \]

\[ w_u / m = 1.10 \, \text{t/m} \]

\[ w_u / \text{rib} = 1.10 (50/100) = 0.55 \, \text{t/m} \]

4. **Critical shear forces and bending moments are determined:**

   Maximum factored shear force = 0.55 (4/2) = 1.10 \, ton

   Maximum factored bending moment = 0.55 (4) (4)/8 = 1.10 \, ton \cdot m

5. **Check rib strength for beam shear:**

   Effective depth \( d = 25 - 2 - 0.60 - 0.6 = 21.8 \, cm \), assuming \( \phi 12 \, mm \) reinforcing bars and \( \phi 6 \, mm \) stirrups.

   \[ 1.1 \Phi \, V_c = 1.1 (0.75) (0.53) \sqrt{250 \, (10) \, (21.8) / 1000} = 1.51 \, \text{ton} > 1.10 \, \text{t} \]

   Though shear reinforcement is not required, 4\( \phi 6 \, mm \) U-stirrups per meter run are to be used to carry the bottom flexural reinforcement.

6. **Design flexural reinforcement:**

   Since the maximum factored moment creates compression in the flange, the section of maximum positive moment is to be designed as a T-section, shown in Figure 8.

   Assuming that \( a < 5 \, cm \), compressive force in concrete is
\[ C = 0.85 \left( f'_c \right) (b_c) (a) \]

\[ C = 0.85 \left( 250 \right) \left( 50 \right) (a) /1000 = 10.625 \ a \ ton \]

\[ M_u = \Phi M = \Phi (C) (d - a / 2) \]

\[ 1.10 = 0.9 \left( 10.625 a \right) \left( 21.8 - a / 2 \right) /100 \]

\[ a^2 - 43.6 \ a + 23.06 = 0 \]

Figure 8: Section size at maximum positive moment

Solving this equation gives \( a = 0.54 \ cm \), which means that the assumption made before is valid and the other root is too large to be considered.

From equilibrium, \( C = T \)

Area of flexural reinforcement

\[ A_s = \frac{10.625 \left( 0.54 \right) \left( 1000 \right)}{4200} = 1.37 \ cm^2 \]

Use \( 2 \phi 10 \ mm \), one is straight and the other is bent-up in each rib at its bottom side.

7. Neat sketches showing arrangement of ribs and details of the reinforcement are to be prepared:

Figure 9 shows required rib reinforcement and concrete dimensions.
Figure 9: (a) Reinforcement on plan; (b) Reinforcement details at section A-A

Loads on main beam:

The load on the main beams which run perpendicular to the ribs include:

a. Load from the slab $= \frac{4}{2}(1.10) = 2.20 \text{ t/m}$

b. Weight of partition loads applied directly on the beam, if any.

c. Part of the beam own weight is not included in slab design calculations (projection above or below the slab).

Notice that loads in (b) and (c) need to be factorized by multiplying each of them by a factor of 1.2.
**Example (8.4):**

Design the ribbed slab shown in Figure 10. The covering materials weigh 200 kg/m$^2$, concrete hollow blocks are 40 cm $\times$ 25 cm $\times$ 17 cm in dimension, each 17 kg in weight, and the live load is 350 kg/m$^2$. Use $f' = 300$ kg/cm$^2$, $f_y = 4200$ kg/cm$^2$, and $\gamma_{plaster} = 2100$ kg/m$^3$.

![Figure 10: Plan of ribbed slab](image)

**Solution:**

1- The direction of ribs is chosen:
Ribs are arranged in the short direction as shown in Figure 10.

The overall slab thickness $h$ is determined:

$h_{\min} = 400/18.5 = 21.62$ cm, for one end continuous panels

$h_{\min} = 150/8 = 18.75$ cm, for cantilevered panels

Use an overall slab thickness of 22.0 cm.

Topping slab thickness = 22 − 17 = 5 cm

Let width of web be equal to 10 cm

Area of shrinkage reinforcement $A_s = 0.0018 (100)(5) = 0.9$ cm$^2$/m
Use 4 φ 6 mm/ m in both directions.

2- The factored load on each of the ribs is to be computed:
Factored load per rib is shown below supported by Figure 11.

![Figure 11: Factored load calculation](image)

Total volume (hatched) = 0.5 × 0.25 × 0.22 = 0.0275 m³
Volume of one hollow block = 0.4 × 0.17 × 0.25 = 0.017 m³
Net concrete volume = 0.0275 - 0.017 = 0.0105 m³
Weight of concrete = 0.0105 × 2.5 = 0.02625 ton
Weight of concrete / m² = 0.02625 / (0.5) (0.25) = 0.21 ton/ m²
Weight of hollow blocks / m² = 17/(0.5) (0.25) (1000) = 0.136 ton/ m²
Weight of plastering coat / m² = 0.02 (2100/1000) = 0.042 ton/ m²
Total dead load = 0.21 + 0.136 + 0.042 + 0.2 = 0.588 ton/ m²

\[ w_u = 1.20(0.588) + 1.60(0.35) = 1.27 \text{ ton/ m}^2 \]

For a strip 1 m wide, \( w_u = 1.27 \text{ ton/ m} \)
\[ w_u / \text{ rib} = 1.27(50/100) = 0.64 \text{ t/ m} \]

3- Critical shear forces and bending moments are determined:
Analyzing the strip shown in Figure 12 using the BENARI structural software, one gets the shown shear force and bending moment diagrams.
4- Check rib width for beam shear:
Effective depth \(d = 22 - 2 - 0.60 - 0.6 = 18.80\) cm, assuming \(\phi 12\) mm reinforcing bars and \(\phi 6\) mm stirrups.

\[1.1 \Phi V_c = 1.1 (0.75)(0.53)\sqrt{300 (10)(18.8)/1000} = 1.42\) ton\]

Since critical shear section can be taken at distance \(d\) from faces of beam, the rib shear resistance will be considered adequate and the assumed web width will be kept unchanged.

Though shear reinforcement is not required, \(4 \phi 6\) mm U-stirrups per meter run are to be used to carry the bottom flexural reinforcement.

5- Design flexural reinforcement:

a. Positive moment reinforcement:

Since factored positive moments create compression in the flange, the sections at their corresponding locations are to be designed as T-sections.

Since the positive moment values are relatively small, the largest of the three values will be considered here and same reinforcement will be used at the two other locations.
Assuming that \( a < 5 \, \text{cm} \), compressive force in concrete, shown in Figure 13, is given by

\[
C = 0.85 \left( f'_c \right) \left( b' \right) (a)
\]

\[
C = 0.85 \times 300 \times 50 \times (a) / 1000 = 12.75 \, \text{a ton}
\]

\[
M_u = \Phi \, M_n = \Phi (C) (d - a / 2)
\]

\[
0.80 = 0.9 \times (12.75 \times a)(18.8 - a / 2) / 100
\]

\[
a^2 - 37.6 \, a + 13.94 = 0
\]

Solving this equation gives \( a = 0.375 \, \text{cm} \), which means that the assumption made before is valid and the other root is too large to be considered.

From equilibrium, \( C = T \)

Area of flexural reinforcement

\[
A_s = \frac{12.75 \times (0.375) \times (1000)}{4200} = 1.14 \, \text{cm}^2
\]

Use 2\( \phi \, 10 \, \text{mm} \), one is straight and the other is bent-up in each rib at its bottom side.

b. Negative moment reinforcement:

Since factored negative moments create compression in the web, the sections at their corresponding locations are to be designed as rectangular sections.

For \( M_u = -1.07 \, \text{t.m} \)

\[
\rho = \frac{0.85(300)}{4200} \left[ 1 - \sqrt{1 - \frac{2.353 \times 10^2 (1.07)}{0.9(10)(18.8)^2(300)}} \right] = 0.0086
\]

\[
A_s = 0.0086 \times (10)(18.8) = 1.62 \, \text{cm}^2, \text{ use}
\]

1\( \phi \, 10 \, \text{mm} + 1\( \phi \, 12 \, \text{mm} \) per rib.

For \( M_u = -0.83 \, \text{t.m} \)
\[
\rho = \frac{0.85(300)}{4200} \left[1 - \sqrt{1 - \frac{2.353 \times 10^5 (0.83)}{0.9 (10)(18.8)^2 (300)}}\right] = 0.0066
\]

\[A_s = 0.0066 (10)(18.8) = 1.24 \text{ cm}^2, \text{ use } 2 \phi 10 \text{ mm per rib.}\]

For \(M_u = -0.72 \text{ t.m}\)

\[
\rho = \frac{0.85(300)}{4200} \left[1 - \sqrt{1 - \frac{2.353 \times 10^5 (0.72)}{0.9 (10)(18.8)^2 (300)}}\right] = 0.0057
\]

\[A_s = 0.0057 (10)(18.8) = 1.07 \text{ cm}^2, \text{ use } 2 \phi 10 \text{ mm per rib.}\]

c. Neat sketches showing arrangement of ribs and details of the reinforcement are to be prepared:

Figure 14 shows required rib reinforcement and concrete dimensions.
Figure 14: (a) Slab reinforcement; (b) section A-A