11.1 Introduction

Footings are structural elements that transmit column or wall loads to the underlying soil below the structure. Footings are designed to transmit these loads to the soil without exceeding its safe bearing capacity, to prevent excessive settlement of the structure to a tolerable limit, to minimize differential settlement, and to prevent sliding and overturning. The settlement depends upon the intensity of the load, type of soil, and foundation level. Where possibility of differential settlement occurs, the different footings should be designed in such away to settle independently of each other.

Foundation design involves a soil study to establish the most appropriate type of foundation and a structural design to determine footing dimensions and required amount of reinforcement.

Because compressive strength of the soil is generally much weaker than that of the concrete, the contact area between the soil and the footing is much larger than that of the columns and walls.

11.2 Footing Types

The type of footing chosen for a particular structure is affected by the following:

1. The bearing capacity of the underlying soil.
2. The magnitude of the column loads.
3. The position of the water table.
4. The depth of foundations of adjacent buildings.

Footings may be classified as deep or shallow. If depth of the footing is equal to or greater than its width, it is called deep footing, otherwise it is called shallow footing. Shallow footings comprise the following types:
1- Isolated Footings:

An isolated footing is used to support the load on a single column. It is usually either square or rectangular in plan. It represents the simplest, most economical type and most widely used footing. Whenever possible, square footings are provided so as to reduce the bending moments and shearing forces at their critical sections. Isolated footings are used in case of light column loads, when columns are not closely spaced, and in case of good homogeneous soil. Under the effect of upward soil pressure, the footing bends in a dish shaped form. An isolated footing must, therefore, be provided by two sets of reinforcement bars placed on top of the other near the bottom of the footing. In case of property line restrictions, footings may be designed for eccentric loading or combined footing is used as an alternative to isolated footing. Figure 11.1 shows square and rectangular isolated footings.

11.3 Depth of Footing

The depth to which foundations shall be carried is to satisfy the following:

a. Ensuring adequate bearing capacity.
b. In the case of clay soils, footings are to penetrate below the zone where shrinkage and swelling due to seasonal weather changes are likely to cause appreciable movement.

c. The footing should be located sufficiently below maximum scouring depth.

d. The footing should be located away from top soils containing organic materials.

e. The footing should be located away from unconsolidated materials such as garbage.

All footings shall extend to a depth of at least 0.50 meter below natural ground level. On rock or such other weather-resistant natural ground, removal of the top soil may be all that is required. In such cases, the surface shall be cleaned, so as to provide a suitable bearing. Usually footings are located at depths of 1.5 to 2.0 meters below natural ground level.

11.4 Pressure Distribution Below Footings

The distribution of soil pressure under a footing is a function of the type of soil, the relative rigidity of the soil and the footing, and the depth of foundation at level of contact between footing and soil. A concrete footing on sand will have a pressure distribution similar to Figure 11.2.a. When a rigid footing is resting on sandy soil, the sand near the edges of the footing tends to displace laterally when the footing is loaded. This tends to decrease in soil pressure near the edges, whereas soil away from the edges of footing is relatively confined. On the other hand, the pressure distribution under a footing on clay is similar to Figure 11.2.b. As the footing is loaded, the soil under the footing deflects in a bowl-shaped depression, relieving the pressure under the middle of the footing. For design purposes, it is common to assume the soil pressures are linearly distributed. The pressure distribution will be uniform if the centroid of the footing coincides with the resultant of the applied loads, as shown in Figure 11.2.

![Figure 11.2: Pressure distribution under footing; (a) footing on sand; (b) footing on clay; (c) equivalent uniform distribution](image-url)
11.4.1 Ultimate Bearing Capacity of Soil

The maximum intensity of loading at the base of a foundation which causes shear failure of soil is called *ultimate bearing capacity of soil*, denoted by \( q_u \).

11.4.2 Allowable Bearing capacity of Soil

The intensity of loading that the soil carries without causing shear failure and without causing excessive settlement is called *allowable bearing capacity of soil*, denoted by \( q_a \). It should be noted that \( q_a \) is a service load stress. The allowable bearing capacity of soil is obtained by dividing the ultimate bearing capacity of soil by a factor of safety on the order of 2.50 to 3.0.

The allowable soil pressure for soil may be either gross or net pressure permitted on the soil directly under the base of the footing. The gross pressure represents the total stress in the soil created by all the loads above the base of the footing. These loads include: (a) column service loads; (b) the weight of the footing; and (c) the weight of the soil on the top of the footing, or

\[
q_{\text{gross}} = q_{\text{soil}} + q_{\text{footing}} + q_{\text{column}} \tag{11.1}
\]

For moment and shear calculations, the upward and downward pressures of the footing mass and the soil mass get cancelled. Thus, a net soil pressure is used instead of the gross pressure value, or

\[
q_{\text{net}} = q_{\text{gross}} - q_{\text{footing}} - q_{\text{soil}} \tag{11.2}
\]

Figure 11.3 shows schematic representation of allowable gross and net soil pressures.
11.5 Concentrically loaded Footings

If the resultant of the loads acting at the base of the footing coincides with the centroid of the footing area, the footing is concentrically loaded and a uniform distribution of soil pressure is assumed in design, as shown in Figure 11.4. The magnitude of the pressure intensity is given by

\[ q = \frac{P}{A} \]  

(11.3)

where \( A \) is the bearing area of the footing, and \( P \) is the applied load.
11.6 Design of Isolated Footings

Design of isolated rectangular footings is detailed in the following steps.

1- Select a trial footing depth.

According to ACI Code 15.7, depth of footing above reinforcement is not to be less than 15 cm for footings on soil. Noting that 7.5 cm of clear concrete cover is required if concrete is cast against soil, a practical minimum depth is taken as 25 cm.

2- Establish the required base area of the footing.

The allowable net soil pressure is

\[ q_{all\ (net)} = q_{all\ (gross)} - \gamma_c (h_c) - \gamma_s (d_f - h_c) \]

where \( h_c \) is assumed footing depth, \( d_f \) is distance from ground surface to the contact surface between footing base and soil, \( \gamma_c \) is weight density of concrete, and \( \gamma_s \) is weight density of soil on top of footing.

Based on ACI Code 15.2.2, base area of footing is determined from unfactored forces and moments transmitted by footing to soil and the allowable soil pressure evaluated through principles of soil mechanics. The required base area of the footing is obtained by dividing the column service loads by the allowable net soil pressure of the soil, or

\[ A_{req} = \frac{P_D + P_L}{q_{all\ (net)}} \]  \hspace{1cm} (11.4)

where \( P_D \) and \( P_L \) are column service dead and live loads respectively.

Select appropriate \( L \), and \( B \) values, if possible, use a square footing to achieve greatest economy.

3- Evaluate the net factored soil pressure.

Evaluate the net factored soil pressure by dividing the factored column loads by the chosen footing area, or

\[ q_{all\ (net)} = \frac{1.2 P_D + 1.6 P_L}{L \times B} \]  \hspace{1cm} (11.5)

4- Check footing thickness for punching shear.

Since large soil pressures are present under footings, high shear stresses are produced and since shear reinforcement is not normally used, shear rather than moment commonly
determines the minimum required depth of footing. The depth of the footing must be set so that the shear capacity of the concrete equals or exceeds the critical shear forces produced by factored loads.

As discussed in Chapter 4, the critical section for punching shear is located at distance $d/2$ from column faces and usually takes the shape of the column. Footing thickness is adequate for resisting punching shear once $V_u \leq \Phi V_c$.

The critical punching shear force can be evaluated using one of the two following methods:

$V_u = q_u \left( (L) B - (C_1 + d)(C_2 + d) \right)$  \hspace{1cm} (11.6.a)

$V_u = (1.2 P_D + 1.6 P_L) - q_u \left( (C_1 + d)(C_2 + d) \right)$  \hspace{1cm} (11.6.b)

where $C_1$ and $C_2$ are column cross sectional dimensions, shown in Figure 1.5.

Figure 11.5.a: Critical sections for punching and beam shears (square footings)
Figure 11.5.b: Critical sections for punching and beam shears (rectangular footings)

Punching shear force resisted by concrete $V_c$ is given as the smallest of:

$$V_c = 0.53 \sqrt{f'_c \left( l + \frac{2}{\beta} \right)} \lambda b_0 d$$  \hspace{1cm} (11.7)

$$V_c = \lambda \sqrt{f'_c} b_0 d$$  \hspace{1cm} (11.8)

$$V_c = 0.27 \left( \frac{\alpha_s d}{b_0} + 2 \right) \lambda \sqrt{f'_c} b_0 d$$  \hspace{1cm} (11.9)

When $\beta = 2$, equations (11.7) and (11.8) give the same value, if $\beta > 2$ Eq. (11.7) gives smaller value than that evaluated using Eq. (11.8).

Since there are two layers of reinforcement, an average value of $d$ may be used. The average effective depth is given as

$$d_{avg} = h_c - 7.5 \text{cm} - d_b$$, where $d_b$ is bar diameter.

Increase footing thickness if additional shear strength is required.

5- **Check footing thickness for beam shear in each direction.**

If $V_{u} \leq \Phi V_c$, thickness will be adequate for resisting beam shear without using shear reinforcement. The critical section for beam shear is located at distance $d$ from column faces.
a- In the short direction:
The factored shear force is given by
\[ V_u = q_u \text{ (net)} B \left( \frac{L-C_2}{2} \right) - d \] (11.10)

The factored shearing force resisted by concrete is given as
\[ V_c = 0.53 \sqrt{f'_c} B d \] (11.11)

b- In the long direction:
The factored shear force is given by
\[ V_u = q_u \text{ (net)} L \left( \frac{B-C_1}{2} \right) - d \] (11.12)

The factored shearing force resisted by concrete is given as
\[ V_c = 0.53 \sqrt{f'_c} L d \] (11.13)

Increase footing thickness if necessary until the condition \( V_u \leq \Phi V_C \) is satisfied.

6- Compute the area of flexural reinforcement in each direction.
The critical section for bending is located at face of column, or wall, for footings supporting a concrete column or wall, as specified by ACI Code 15.4.2. Figure 11.6 shows critical sections for flexure for footings supporting concrete columns, masonry walls, and columns with steel base plates.

a- Reinforcement in the long direction:
\[ M_u = q_u \text{ (net)} \frac{B}{2} \left( \frac{L-C_2}{2} \right)^2 \] (11.14)

b- Reinforcement in the short direction:
\[ M_u = q_u \text{ (net)} \frac{L}{2} \left( \frac{B-C_1}{2} \right)^2 \] (11.15)
The reinforcement ratio is calculated based on rectangular section design, where the minimum reinforcement ratio $\rho_{\text{min}}$ is not to be less than 0.0018.
According to *ACI Code 15.4.3*, for square footings, the reinforcement is identical in both directions as shown in Figure 11.6.a, neglecting the slight difference in effective depth values in the two directions. For rectangular footings, *ACI Code 15.4.4* specifies that the reinforcement in the long direction is uniformly distributed while portion of the total reinforcement in the short direction, $\gamma_s A_s$, is to be distributed uniformly over a band width, centered on centerline of column, equal to the length of the short side of footing. Remainder of reinforcement required in short direction, $(1-\gamma_s)A_s$ is to be distributed uniformly outside center band width of footing as shown in Figure 11.6.b.

where $A_s$ is the total reinforcement required in the short direction, $\beta$ equals the ratio of the long side to the short side of the footing and $\gamma_s$ is given as

$$\gamma_s = \frac{2}{1+\beta} \quad (11.16)$$

7- **Check for bearing strength of column and footing concrete.**

All forces applied at the base of a column or wall must be transferred to the footing by bearing on concrete and/or by reinforcement. Tensile forces must be resisted entirely by the reinforcement. Bearing on concrete for column and footing must not exceed the concrete bearing strength.

The joint could fail by crushing of the concrete at the bottom of the column where the column bars are no longer effective or by crushing the concrete in the footing under the column.
For a supported column, the bearing capacity $\Phi P_n$ is

$$\Phi P_n = \Phi \left(0.85 f'_c A_i\right)$$

(11.17)

where

- $f'_c =$ compressive strength of the column concrete
- $A_i =$ column cross-sectional area
- $\Phi =$ strength reduction factor for bearing = 0.65

For a supporting footing,

$$\Phi P_n = \Phi \left(0.85 f'_c A_i\right) \frac{A_2}{A_1} \leq 2.0 \Phi \left(0.85 f'_c A_i\right)$$

(1.18)

where

- $f'_c =$ compressive strength of the footing concrete
- $A_2 =$ area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the footing and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal.

When bearing strength is exceeded, reinforcement in the form of dowel bars must be provided to transfer the excess load. A minimum area of reinforcement must be provided across the interface of column or wall and footing, even where concrete bearing strength is not exceeded.

For columns, minimum dowel reinforcement is given by ACI Code 15.8.2.1 as

$$A_{i, \text{min}} = 0.005 A_y$$

(11.19)

where $A_y =$ column gross cross-sectional area

Required dowel reinforcement is given by

$$A_{i, \text{req}} = \frac{(P_u - \Phi P_n)}{\Phi f_y}$$

(11.20)

8- Check for anchorage of the reinforcement.

Both flexural and dowel reinforcement lengths are checked for anchorage to prevent bond failure of the dowels in the footing and to prevent failure of the lap splices between the dowels and the column bars, as shown in Figure 11.7.
Prepare neat design drawings showing footing dimensions and provided reinforcement.

Example (11.1):

Design an isolated square footing to support an interior column 40 cm × 40 cm in cross section and carries a dead load of 80 tons and a live load of 60 tons.

Use $f'_c = 250 \text{ kg/cm}^2$, $f_y = 4200 \text{ kg/cm}^2$, $q_{all \ (gross)} = 2.0 \text{ kg/cm}^2$, $\gamma_{soil} = 1.7 \text{ t/m}^3$, and $D_f = 1.25 \text{ m}$. 

Figure 11.8.a: Footing dimensions
Solution:

1- Select a trial footing depth:
Assume that the footing is 50 cm thick.

2- Establish the required base area of the footing:

\[ q_{all} (net) = 20 - 0.75 (1.7) - 0.5 (2.5) = 17.475 \text{ t/m}^2 \]

\[ A_{req} = \frac{P}{q_{all} (net)} = \frac{80 + 60}{17.475} = 8.011 \text{ m}^2 \]

For a square footing, \( B = \sqrt{8.011} = 2.83 \text{ m} \)

Use 285 cm \( \times \) 285 cm \( \times \) 50 cm footing, as shown in Figure 11.8.a.

3- Evaluate the net factored soil pressure:

\[ P_u = 1.20 (80) + 1.60 (60) = 192 \text{ tons} \]

\[ q_{u} (net) = \frac{P_u}{B^2} = \frac{192}{(2.85)^2} = 23.64 \text{ t/m}^2 \]

4- Check footing thickness for punching shear:
Average effective depth \( d = 50 - 7.5 - 1.6 = 40.9 \text{ cm} \)

The factored shear force

\[ V_u = (23.64)(2.85) - (0.809)(0.809) = 176.54 \text{ tons} \]

\[ b_o = 4 (40 + 40.9) = 323.6 \text{ cm} \]

\( \Phi V_c \) is the smallest of:

\[ \Phi V_c = 0.53 \Phi \sqrt{f_c'} \left( 1 + \frac{2}{\beta} \right) \lambda b_o d \]

\[ = 0.53 \left( 0.75 \right) \sqrt{250} \left( 1 + \frac{2}{1} \right) \left( 323.6 \right) \left( 40.9 \right) / 1000 = 249.55 \text{ tons} \]

\[ \Phi V_c = \Phi \lambda \sqrt{f_c'} b_o d \]

\[ = 0.75 \sqrt{250} \left( 323.6 \right) \left( 40.9 \right) = 156.95 \text{ tons} \]

\[ \Phi V_c = 0.27 \Phi \left( \frac{\alpha_s}{\beta} + 2 \right) \lambda \sqrt{f_c'} b_o d \]
\[
\Phi V_c = 0.27 \left(\frac{40 \times (40.9)}{323.6} + 2\right) \sqrt{250 \times 323.6 \times (40.9) / 1000} = 299 \text{ tons}
\]

\[
\Phi V_c = 156.95 \text{ tons} < 176.54 \text{ tons}
\]

Increase footing thickness to 55 cm, and repeat punching shear check. 

Average effective depth \( d = 55 - 7.5 - 1.6 = 45.9 \text{ cm} \)

The factored shear force 
\[
V_u = (23.64) [(2.85)(2.85) - (0.859)(0.859)] = 174.57 \text{ tons}
\]

\[
b_0 = 4(40 + 45.9) = 343.6 \text{ cm}
\]

\[
\Phi V_c = \Phi \lambda \sqrt{f'_c b_0 d}
\]

\[
= 0.75 \sqrt{250 \times 343.6 \times (45.9)} = 187.02 \text{ tons}
\]

\[
\Phi V_c = 187.02 \text{ tons} > 174.57 \text{ tons}
\]

i.e. footing thickness is adequate for resisting punching shear.

5- Check footing thickness for beam shear in each direction:

![Critical sections for beam shear and moment](image-url)
Maximum factored shear force \( V_u \) is located at distance \( d \) from faces of column, as shown in Figure 11.8.b, or:

\[
V_u = (23.64)(2.85)(0.766) = 51.61 \text{ tons} < 82.22 \text{ tons}
\]

6- **Compute the area of flexural reinforcement in each direction:**

The critical section for moment is located at column faces, as shown in Figure 11.8.b, or:

\[
M_u = (23.64)(2.85) \frac{(1.225)^2}{2} = 50.55 \text{ t.m}
\]

\[
\rho = \frac{0.85(250)}{4200} \left[ 1 - \sqrt{\frac{2.353(10)^5 (50.55)}{(0.9)(285)(45.9)^2 (250)}} \right] = 0.00228
\]

\[
A_s = 0.00228(285)(45.9) = 29.82 \text{ cm}^2, \text{ use } 15 \phi 16 \text{ mm in both directions.}
\]

![Figure 11.8.c: Critical section for bearing](image)

7- **Check for bearing strength of column and footing concrete:**

For column,

\[
\Phi P_n = 0.65(0.85)(250)(40)(40)/1000 = 221 \text{ tons} > 192 \text{ tons}
\]

i.e. use minimum dowel reinforcement, \( A_s = 0.005(40)(40) = 8.0 \text{ cm}^2 \)

Use 4 \( \phi 16 \text{ mm} \) for dowel reinforcing bars.

For footing,

\[
\Phi P_n = 0.65(0.85)(250)(40) \sqrt{\frac{(260)^2}{(40)^2}} / 1000 = 1436.5 \text{ tons} > 442 \text{ tons}
\]
i.e. $\Phi P_n = 442$ tons $> 192$ tons

8- Check for anchorage of the reinforcement:

**Bottom reinforcement** ($\phi 16$ mm):

$\psi_t = \psi_e = \lambda = 1$ and $\psi_s = 0.8$

$c_b$ is the smaller of:

$7.5 + 0.8 = 8.3 \text{ cm}$, or $\frac{285 - 15 - 1.6}{14(2)} = 9.58 \text{ cm}$, i.e., $c_b = 8.3 \text{ cm}$

$$\frac{c_b + K_{\text{tr}}}{d_b} = \frac{8.3 + 0}{1.6} = 5.1875 > 2.5 \text{ , taken as 2.5}$$

$$l_d = d_b \left( \frac{f_y \psi_t \psi_e \psi_s}{3.5 \lambda \left( \frac{c_b + K_{\text{tr}}}{d_b} \right) \sqrt{f'_c}} \right) = 1.6 \left( \frac{4200 (0.8)}{3.5 (2.5) \sqrt{250}} \right) = 38.86 \text{ cm}$$

Available length $= 122.5 - 7.5 = 115.0 \text{ cm} > 38.86 \text{ cm}$

**a- Dowel reinforcement** ($\phi 16$ mm):

$$\frac{0.075 f_y}{\lambda \sqrt{f'_c}} d_b \geq 0.0044 f_y d_b$$

$$l_d = \frac{0.075 (1.6) (4200)}{\sqrt{250}} = 31.88 \text{ cm}$$

d $= 0.0044 (1.6) (4200) = 29.57 \text{ cm} > 20 \text{ cm}$

Available length $= 55 - 7.5 - 1.6 - 1.6 = 42.7 \text{ cm} > 31.88 \text{ cm}$

**b- Column reinforcement splices:**

$$l_{sp} = 0.0073 (1.6) (4200) = 49.06 \text{ cm}$$

taken as 50 cm.

9- Prepare neat design drawings showing footing dimensions and provided reinforcement:

Design drawings are shown in Figure 11.8.d.
Example (11.2):

Design an isolated rectangular footing to support an interior column 40 cm × 40 cm in cross section and carries a dead load of 80 tons and a live load of 60 tons. Use $f'_c = 250\text{ kg/cm}^2$, $f_s = 4200\text{ kg/cm}^2$, $q_{all}\text{ (gross)} = 2.0\text{ kg/cm}^2$, $\gamma_{soil} = 1.7\text{ t/m}^3$, and $D_f = 1.25\text{ m}$.

Space limitations are such that one lateral dimension cannot exceed 2.5m.

Solution:
1- Select a trial footing depth:
Assume that the footing is 55 cm thick.

2- Establish the required base area of the footing:

\[ q_{all\ (net)} = 20 - 0.7(1.7) - 0.55(2.5) = 17.435 \, t/m^2 \]

\[ A_{req} = \frac{P}{q_{all\ (net)}} = \frac{80 + 60}{17.435} = 8.03 \, m^2 \]

Let \( B = 2.5 \, m, \quad L = \frac{8.03}{2.5} = 3.21 \, m \)

Use 325 cm × 250 cm × 55 cm footing.

3- Evaluate the net factored soil pressure:

\[ P_u = 1.20(80) + 1.60(60) = 192 \, tons \]

\[ q_{u\ (net)} = \frac{P_u}{L \times B} = \frac{192}{3.25 \times 2.5} = 23.63 \, t/m^2 \]

4- Check footing thickness for punching shear:

The critical section for punching shear is shown in Figure 11.9.a.

Average effective depth \( d = 55 - 7.5 - 1.6 = 45.9 \, cm \)
The factored shear force
\[ V_u = (23.63)(3.25)(2.5) - 0.859(0.859) = 174.56 \text{ tons} \]
\[ b_o = 4 \times (40 + 45.9) = 343.6 \text{ cm} \]
\[ \Phi V_c \text{ is the smallest of:} \]
\[ \Phi V_c = 0.53 \Phi \sqrt{f'\text{c}} \left( I + \frac{2}{\beta} \right) \lambda b_o d \]
\[ = 0.53 \times \frac{0.75}{\sqrt{250}} \left( 1 + \frac{2}{I} \right) (1)(343.6)(45.9)/1000 = 297.37 \text{ tons} \]
\[ \Phi V_c = \Phi \lambda \sqrt{f'\text{c}} b_o d \]
\[ = 0.75 \sqrt{250} \times (343.6)(45.9) = 187.02 \text{ tons} \]
\[ \Phi V_c = 0.27 \Phi \left( \frac{\alpha_s}{b_o} + 2 \right) \lambda \sqrt{f'\text{c}} b_o d \]
\[ = 0.27 \times \frac{0.75}{343.6} \left( 40 \frac{(45.9)}{343.6} + 2 \right) \sqrt{250} \times (343.6)(45.9)/1000 = 370.82 \text{ tons} \]
\[ \Phi V_c = 187.02 \text{ tons} > 174.56 \text{ tons} \]
i.e. footing thickness is adequate for resisting punching shear.

5- Check footing thickness for beam shear in each direction:

a- In the short direction:
\[ \Phi V_c = 0.75 \times (0.53) \sqrt{250} \times (250)(45.9)/1000 = 72.12 \text{ tons} \]
Maximum factored shear force \( V_u \) is located at distance \( d \) from faces of column,
\[ V_u = (23.63)(2.5)(0.966) = 57.07 \text{ tons} < 72.12 \text{ tons} \]

b- In the long direction:
\[ \Phi V_c = 0.75 \times (0.53) \sqrt{250} \times (325)(45.9)/1000 = 93.76 \text{ tons} \]
Maximum factored shear force \( V_u \) is located at distance \( d \) from faces of column,
\[ V_u = (23.63)(3.25)(0.591) = 45.39 \text{ tons} < 93.76 \text{ tons} \]
6- Compute the area of flexural reinforcement in each direction:

a- Reinforcement in long direction:

The critical section for bending is shown in Figure 11.9.d.

\[
M_u = (23.63)(2.5) \frac{(1.425)^2}{2} = 59.98 \text{ t.m}
\]

\[
\rho = \frac{0.85(250)}{4200} \left[ 1 - \sqrt{1 - \frac{2.353(10)^5(59.98)}{(0.9)(250)(45.9)^2(250)}} \right] = 0.00311
\]

\[A_s = 0.00311(250)(45.9) = 35.69 \text{ cm}^2,\] use 15 \( \phi 18 \) mm in the long direction.

b- Reinforcement in short direction:
The critical section for bending is shown in Figure 11.9.e.

\[ M_u = (23.63)(3.25)\left(\frac{1.05}{2}\right) = 42.33 \text{ t.m} \]

\[ \rho = \frac{0.85(250)}{4200} \left[ 1 - \sqrt{1 - \frac{2.353(10)^5(42.33)}{(0.9)(325)(45.9)^2(250)}} \right] = 0.00166 \]

\[ A_{s, min} = 0.0018(325)(55) = 32.17 \text{ cm}^2 \]

Central band reinforcement = \( \gamma_s A_s = \left( \frac{2}{1 + 325/250} \right)(32.17) = 27.97 \text{ cm}^2 \)

Use 14 φ 16 mm in the central band.

For each of the side bands, \( A_s = \left( \frac{32.17 - 27.97}{2} \right) = 2.10 \text{ cm}^2 \)

Use 2 φ 16 mm in each of the two side bands.

7- Check for bearing strength of column and footing concrete:

For column,

\[ \Phi P_n = 0.65(0.85)(250)(40)(40)/1000 = 221 \text{ tons} > 192 \text{ tons} \]

i.e. use minimum dowel reinforcement, \( A_s = 0.005(40)(40) = 8.0 \text{ cm}^2 \).

Use 4 φ 16 mm for dowel reinforcing bars.

For footing,
\( \Phi P_n = 0.65 \left( 0.85 \right) \left( 250 \right) \left( 40 \right) \left( 40 \right) \sqrt{\left( \frac{260}{40} \right)^2 / 1000} = 1436.5 \text{ tons} > 476 \text{ tons} \)

i.e. \( \Phi P_n = 476 \text{ tons} > 192 \text{ tons} \)

8- Check for anchorage of the reinforcement:

\textbf{a- Reinforcement in long direction (}\( \phi 18 \text{ mm})\): \[
\psi_f = \psi_s = \lambda = 1 \text{ and } \psi_s = 0.8 \\
\]

\( c_b \) is the smaller of:

\[
7.5 + 0.9 = 8.4 \text{ cm, or } \frac{325 - 15 - 1.8}{15(2)} = 10.27 \text{ cm, i.e., } c_b = 8.40 \text{ cm} \\
\]

\[
\frac{c_b + K_{tr}}{d_b} = \frac{8.40 + 0}{1.8} = 4.67 > 2.5, \text{ take it equal to 2.5} \\
\]

\[
l_d = d_b \left( \frac{f_y \psi_f \psi_s \psi_s}{3.5 \lambda \left( \frac{c_b + K_{tr}}{d_b} \right) \sqrt{f'_c}} \right) = 1.8 \left( \frac{4200 (0.8)}{3.5 (2.5) \sqrt{250}} \right) = 43.72 \text{ cm} \\
\]

Available length = 142.5 – 7.5 = 135.0 cm > 43.72 cm

\textbf{b- Reinforcement in short direction (}\( \phi 16 \text{ mm})\): \[
\psi_f = \psi_s = \lambda = 1 \text{ and } \psi_s = 0.8 \\
\]

\( c_b \) is the smaller of:

\[
7.5 + 0.8 = 8.3 \text{ cm, or } \frac{250 - 15 - 1.6}{14(2)} = 8.34 \text{ cm, i.e., } c_b = 8.3 \text{ cm} \\
\]

\[
\frac{c_b + K_{tr}}{d_b} = \frac{8.30 + 0}{1.6} = 5.19 > 2.5, \text{ take it equal to 2.5} \\
\]

\[
l_d = d_b \left( \frac{f_y \psi_f \psi_s \psi_s}{3.5 \lambda \left( \frac{c_b + K_{tr}}{d_b} \right) \sqrt{f'_c}} \right) = 1.6 \left( \frac{4200 (0.8)}{3.5 (2.5) \sqrt{250}} \right) = 38.86 \text{ cm} \\
\]

Available length = 105.0 – 7.5 = 97.5 cm > 38.86 cm
c- Column reinforcement splices:

\[ l_{sp} = 0.0073(1.6)(4200) = 49.06 \text{ cm} \], taken as 50 cm.

9- Prepare neat design drawings showing footing dimensions and provided reinforcement:

Design drawings are shown in Figure 11.9.f.

![Design drawings](image)

**Figure 11.9.f: Design drawings**

### 11.7 Problems

**P11.1** Design a circular footing to support a column 40 cm in diameter, and carries a service dead load of 60 tons, and a service live load of 20 tons.

Use \( f'_c = 300 \text{ kg} / \text{cm}^2 \), \( f_y = 4200 \text{ kg} / \text{cm}^2 \), \( q_{all} (\text{gross}) = 1.7 \text{ kg} / \text{cm}^2 \), \( \gamma_{\text{sol}} = 1.7 \text{ t} / \text{m}^3 \), and \( D_f = 1.5 \text{ m} \).

**P11.2** Design an isolated footing to support a column 25 cm × 60 cm in cross section. The column carries a service dead load of 60 tons, and a service live load of 30 tons, in addition to a service dead load moment of 8 t.m, and a service live load moment of 4 t.m.
Use $f_c' = 300 \text{ kg} / \text{cm}^2$, $f_y = 4200 \text{ kg} / \text{cm}^2$, and $q_{alt \text{ (net)}} = 1.7 \text{ kg} / \text{cm}^2$. 