Direct Design Method

The direct design method consists of certain steps for distributing moments to slab and beam sections to satisfy safety requirements and most serviceability requirements simultaneously. Three basic steps are involved:

a. Determination of the total factored static moment;
b. Distribution of the total factored static moment to negative and positive sections;
c. Distribution of negative and positive factored moments to the column and middle strips and to the beams if any.

Limitations

The direct design method was developed from theoretical procedures for determination of moments in slabs, requirements for simple design, construction procedures, and performance of existing slabs. Therefore, the slab system, to be designed using the direct design method, should conform to the following limitations as given by ACI Code 13.6.1:

1. There must be three or more spans in each direction.
2. Slab panels must be rectangular with a ratio of longer to shorter span, center-to-center of supports, not greater than 2.0.
3. Successive span lengths, center-to-center of supports, in each direction must not differ by more than one-third of the longer span.
4. Columns must not be offset more than 10% of the span in the direction of offset from either axis between centerlines of successive columns.
5. Loads must be due to gravity only and uniformly distributed over the entire panel. The live load must not exceed 2 times the dead load.
6. For a panel with beams between supports on all sides, the relative stiffness of beams in two perpendicular directions \( \frac{\alpha_{f1} l_2^2}{\alpha_{f2} l_1^2} \) is not to be less than 0.20 and not greater than 5.0.
where

\[ \alpha_f = \text{ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels, if any, on each side of the beam} \]

\[ \alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} \]

\( E_{cb} \) = modulus of elasticity of beam concrete

\( E_{cs} \) = modulus of elasticity of slab concrete

\( I_b \) = moment of inertia about centroidal axis of gross section of beam

\( I_s \) = moment of inertia about centroidal axis of gross section of slab

\( \alpha_{f1} = \alpha_f \) in direction of \( l_1 = \frac{\alpha' f_1 + \alpha' f_3}{2} \), as shown in Figure 10

\( \alpha_{f2} = \alpha_f \) in direction of \( l_2 = \frac{\alpha' f_2 + \alpha' f_4}{2} \), as shown in Figure 10

Figure 10: Panel bounded by four beams

Figure 11 shows dimensions of interior and edge beams that need to be considered in relative stiffness calculations.
Design Procedure

1. Determination of the total factored static moment:

Total factored static moment for a span is determined in a strip bounded laterally by centerline of panel on each side of centerline of supports, as shown in Figure 12.

Figure 11: Effective beam section; (a) interior beam; (b) exterior beam
Figure 12: (a) Column strip for \( l_2 \leq l_1 \); (b) Column strip for \( l_2 > l_1 \)

Absolute sum of positive and average negative factored moments in each direction is not to be less than

\[
M_0 = q_u l_2 l_n^2 / 8 \tag{14}
\]

Where \( l_n \) is clear span in the direction moments are being considered, measured face-to-face of supports and not be less than 0.65 \( l_1 \), and \( q_u \) is factored load per unit area of slab, where the transverse span of panels on either side of the centerline of supports varies, \( l_2 \) in Eq. (14) is taken as the average of adjacent transverse spans. Besides, when the span adjacent and parallel to an edge is being considered, the distance from edge to panel centerline is to be substituted for \( l_2 \) in Eq. (14).

2. Distribution of the total factored static moment to negative and positive moments:

According to ACI Code 13.6.3, in an interior span, total factored static moment is distributed in such a way that the negative factored moment at
face of support is taken as $0.65 M_o$, and positive factored moment at midspan is taken as $0.35 M_o$.

In end spans, total factored static moment $M_o$ is distributed as shown in Table 6 and Figure 13.

**Table 6: Distribution of total static moment in end spans**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exterior edge unrestrained</td>
<td>0.75</td>
<td>0.70</td>
<td>0.70</td>
<td>0.65</td>
</tr>
<tr>
<td>Beams between all supports</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
<td>0.65</td>
</tr>
<tr>
<td>No beams between interior supports</td>
<td>0.63</td>
<td>0.57</td>
<td>0.52</td>
<td>0.50</td>
</tr>
<tr>
<td>Without edge beam</td>
<td>0.52</td>
<td>0.50</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>With edge beam</td>
<td>0.35</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exterior edge fully restrained</td>
<td>0.00</td>
<td>0.16</td>
<td>0.26</td>
<td>0.30</td>
</tr>
<tr>
<td>Interior negative factored moment</td>
<td></td>
<td>0.00</td>
<td>0.16</td>
<td>0.26</td>
</tr>
<tr>
<td>Positive factored moment</td>
<td></td>
<td>0.63</td>
<td>0.57</td>
<td>0.52</td>
</tr>
<tr>
<td>Exterior negative factored moment</td>
<td></td>
<td>0.00</td>
<td>0.16</td>
<td>0.26</td>
</tr>
</tbody>
</table>

**Figure 13:** (a) Exterior edge fully restrained; (b) exterior edge unrestrained; (c) beams between all supports; (d) No beams between interior supports and without edge beam; (e) No beams between interior supports and with edge beam
3. Distribution of the positive and negative factored moments to the column and middle strips:

Definitions:

Column strip:

Column strip is a design strip with a width on each side of a column centerline equal to 0.25 \( l \) or 0.25 \( l_2 \), whichever is less, as shown in Figure 12. Column strip includes beams, if any.

Middle strip:

Middle strip is a design strip bounded by two column strips, shown in Figure 12.

a. Factored moments in column strips:

According to ACI Code 13.6.4, column strip moments, as percentages of total factored positive and negative moments, are given in Table 7.

In Table 7, \( \beta_t \) is defined as ratio of torsional stiffness of edge beam section to flexural stiffness of a width of slab equal to span length of beam, center-to-center of supports = \( E_e \beta C / 2 E_{cs} I_s \), where \( C \) is cross-sectional constant to define torsional properties given by Eq. (15)

\[
C = \sum \left( 1 - 0.63 \frac{x}{y} \right) \frac{x^3 y}{3} \tag{15}
\]

where \( x \) is the shorter overall dimension of rectangular part of cross section and \( y \) is the longer overall dimension of rectangular part of cross section. The cross section is to be divided into separate rectangular parts and carrying out the summation given in Eq. (15) in such away to give the largest value of \( C \), as shown in Figure 14.
Table 7: Column strip factored moments

<table>
<thead>
<tr>
<th></th>
<th>$l_2/l_1$</th>
<th>0.50</th>
<th>1.0</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exterior negative</td>
<td>$\alpha_l l_2/l_1 = 0$</td>
<td>$\beta_l = 0$</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>factored moment</td>
<td>$\alpha_l l_2/l_1 \geq 1$</td>
<td>$\beta_l = 0$</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>$\beta_l \geq 2.5$</td>
<td></td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>$\beta_l \geq 2.5$</td>
<td></td>
<td>90</td>
<td>75</td>
</tr>
<tr>
<td>Positive factored</td>
<td>$\alpha_l l_2/l_1 = 0$</td>
<td></td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>moment</td>
<td>$\alpha_l l_2/l_1 \geq 1$</td>
<td></td>
<td>90</td>
<td>75</td>
</tr>
<tr>
<td>Interior negative</td>
<td>$\alpha_l l_2/l_1 = 0$</td>
<td></td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>factored moment</td>
<td>$\alpha_l l_2/l_1 \geq 1$</td>
<td></td>
<td>90</td>
<td>75</td>
</tr>
</tbody>
</table>

Figure 14: Torsional cross sectional dimensions for $\beta_t$ calculations

b. Factored moments in beams caused by slab loads:
As specified by *ACI Code 13.6.5*, beams between supports are to be designed to resist 85% of column strip moments if $\alpha_f l_2/l_1 \geq 1$. For values of $\alpha_f l_2/l_1$ between 1.0 and zero, proportion of column strip moments resisted by beams is obtained by linear interpolation between 85 and zero. In addition to these moments, beams are to be proportioned to resist moments caused by factored loads applied directly on these beams.

c. Factored shears in beams caused by slab loads:
As specified by ACI Code 13.6.8, beams with $\alpha_f l_2 / l_1 \geq 1$ are to be designed for shear caused by factored loads on tributary areas which are bounded by 45 degree lines drawn from the corners of the panels and the centerlines of the adjacent panels parallel to the long sides, as shown in Figure 4. For values of $\alpha_f l_2 / l_1$ less than 1.0, linear interpolation assuming beams carry no load at $\alpha_f = 0$, is permitted to be used. In addition to these shears, beams are to be proportioned to resist shears caused by factored loads applied directly on these beams.

**Example (4):**
Design the two-way flat plate with no edge (spandrel) beams, shown in Figure 15.a, given the following: interior columns are 40 cm × 40 cm, exterior columns are 30 cm × 30 cm, covering materials weigh 229 kg/m$^2$, equivalent partition load 50 kg/m$^2$ and the live load is 300 kg/m$^2$. Use $f'_c = 280$ kg/cm$^2$ and $f_y = 4200$ kg/cm$^2$. 
Figure 15.a: Two-way flat plate
Solution:

1- Evaluate slab thickness:

\[ l_{n1} = 700 - 20 - 15 = 665 \text{ cm} \]

\[ l_{n2} = 700 - 20 - 20 = 660 \text{ cm} \]

\[ l_{n3} = 700 - 15 - 15 = 670 \text{ cm} \]

For flat plates with no edge beams, minimum slab thickness \( h_{\text{min}} = 670/30 = 22.33 \text{ cm} \), taken as 22 cm.

2- Check limitations for slab analysis by the direct design method:

The first five conditions are satisfied, while the sixth condition does not apply due to the nonexistence of beams.

3- Calculate the factored load on the slab:

\[ q_u = 1.2 \left[ 0.22 \times (2.5) + 0.229 + 0.05 \right] + 1.6 \times (0.30) = 1.475 \text{ t/m}^2 \]

4- Check slab thickness for shear:

\[ d_{\text{avg}} = 22 - 2 - 2 = 18 \text{ cm} \], assuming \( \phi \) 20 mm reinforcing bars.

a. Interior columns, shown in Figure 15.b:

![Figure 15.b: Beam shear and punching shear](image)
Punching shear:

\[ b_o = 4 (40 + 18) = 232 \text{ cm} \]

\[ V_u = 1.475 \left[ 7 (6) - (0.58)^2 \right] = 61.45 \text{ ton} \]

\[ \Phi V_c = 1.06 \left( 0.75 \sqrt{280} (232)(18) / 1000 \right) > 55.55 \text{ ton} > V_u \]

Beam Shear:

Section 1-1:

\[ V_u = 1.475 (6) \left( \frac{7.0 - 0.40}{2} - 0.18 \right) = 27.61 \text{ ton} \]

\[ \Phi V_c = 0.53 \left( 0.75 \sqrt{280} (600)(18) / 1000 \right) > 71.84 \text{ ton} > V_u \]

Section 2-2:

\[ V_u = 1.475 (7) \left( \frac{6.0 - 0.40}{2} - 0.18 \right) = 27.05 \text{ ton} \]

\[ \Phi V_c = 0.53 \left( 0.75 \sqrt{280} (700)(18) / 1000 \right) = 83.81 \text{ ton} \quad \text{O.K.} \]

b. Corner columns, shown in Figure 15.c:

![Figure 15.c: Beam shear and punching shear](image-url)
Punching shear:

\[ b_o = 2 \left(30 + 18 / 2 \right) = 78 \text{ cm} \]

\[ V_u = 1.475 \left[ 3.65 \left(3.15 \right) - \left(0.39\right)^2 \right] = 16.73 \text{ ton} \]

\[ \Phi V_c = 1.06 \left(0.75\right) \sqrt{280} \left(78\right) \left(18\right) / 1000 = 18.68 \text{ ton} > V_u \]

Beam Shear:

Section 1-1:

\[ V_u = 1.475 \left[3.15 \left(3.65 - 0.18 - 0.30\right)\right] = 14.73 \text{ ton} \]

\[ \Phi V_c = 0.53 \left(0.75\right) \sqrt{280} \left(315\right) \left(18\right) / 1000 = 37.71 \text{ ton} > V_u \]

Section 2-2:

\[ V_u = 1.475 \left[3.65 \left(3.15 - 0.18 - 0.30\right)\right] = 14.37 \text{ ton} \]

\[ \Phi V_c = 0.53 \left(0.75\right) \sqrt{280} \left(365\right) \left(18\right) / 1000 = 43.70 \text{ ton} > V_u \]

c. Edge columns, shown in Figure 15.d and Figure 15.e:

Figure 15.d: Beam shear and punching shear
In Figure 15.d,

**Punching shear:**

\[ b_o = 2 \left(30 + 18 / 2\right) + 30 + 18 = 126 \text{ cm} \]

\[ V_u = 1.475 \left[3.15 \left(7.0 - (0.39)(0.48)\right)\right] = 32.25 \text{ ton} \]

\[ \Phi V_c = 1.06 \left(0.85\right)\sqrt{280 \left(126\right)\left(18\right)} / 1000 = 30.17 \text{ ton} > V_u \]

**Beam Shear:**

Section 1-1:

\[ V_u = 1.475 \left(3.15\right) \left[\left(7.0 - 0.30\right) / 2\right] - 0.18 \right] = 14.73 \text{ ton} \]

\[ \Phi V_c = 0.53 \left(0.75\right)\sqrt{280 \left(315\right)\left(18\right)} / 1000 = 37.71 \text{ ton} > V_u \]

Section 2-2:

\[ V_u = 1.475 \left[7.0 \left(3.15 - 0.18 - 0.30\right)\right] = 27.57 \text{ ton} \]

\[ \Phi V_c = 0.53 \left(0.75\right)\sqrt{280 \left(700\right)\left(18\right)} / 1000 = 83.81 \text{ ton} > V_u \]

In Figure 15.e.

**Punching shear**

\[ b_o = 2 \left(30 + 18 / 2\right) + 30 + 18 = 126 \text{ cm} \]

\[ V_u = 1.475 \left[3.65(6) - (0.39)(0.48)\right] = 32.03 \text{ ton} \]
\[ \Phi V_c = 1.06(0.75)\sqrt{280}(126)(18)/1000 = 30.17 \text{ton} > V_u \]

**Beam shear**

Section 1-1:

\[ V_u = 1.475(6)[3.65 - 0.30 - 0.18] = 28.05 \text{ ton} \]

\[ \Phi V_c = 0.53(0.75)\sqrt{280}(600)(18)/1000 = 71.84 \text{ton} > V_u \]

Section 2-2:

\[ V_u = 1.475(3.65)\left[\left(\frac{6.0 - 0.3}{2}\right) - 0.18\right] = 14.37 \text{ ton} \]

\[ \Phi V_c = 0.53(0.75)\sqrt{280}(365)(18)/1000 = 43.70 \text{ton} > V_u \]

5- Calculate the factored static moment:

![Diagram](image)

**Figure 15.f: Column and middle strips in the short direction**
Strip in the shorter direction:
Width of intermediate strip = 700 cm and width of column strip is the smaller of \((l_1/2)\) and \((l_2/2)\), taken as \((600/2) = 300\) cm, as shown in Figure 15.f.

Total factored static moment:
Clear span for exterior panels = 600 – 15 – 20 = 565 cm
Clear span for interior panels = 600 – 20 – 20 = 560 cm
The larger of the two values will be used in moment calculations.

\[ M_o = 1.475 \left(7\right)\left(5.65\right)^2 / 8 = 41.20\ t.m \]

6- Distribute the total factored static moment into positive and negative moments:
The results are shown in the following table.

7- Distribute the positive and negative moments to the column and middle strips:
The results are shown in the same table.

Negative and positive factored moments:

<table>
<thead>
<tr>
<th>Slab moment ((t.m))</th>
<th>End spans</th>
<th>Interior spans</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Extreme</td>
<td>Positive</td>
</tr>
<tr>
<td>Total moments ((M_T))</td>
<td>0.26(M_o)</td>
<td>0.52(M_o)</td>
</tr>
<tr>
<td></td>
<td>10.71</td>
<td>21.42</td>
</tr>
<tr>
<td>Column strip moments</td>
<td>1.0(M_T)</td>
<td>0.60(M_T)</td>
</tr>
<tr>
<td></td>
<td>10.71</td>
<td>12.85</td>
</tr>
<tr>
<td>Middle strip moments</td>
<td>0.00</td>
<td>8.57</td>
</tr>
</tbody>
</table>

Bending moment diagrams are shown in Figure 15.g and Figure 15.h.
Figure 15: (continued); (g) Bending moment diagram for column strip; (h) bending moment diagram for middle strip

8- Design the reinforcement:

Column strip reinforcement:

Design sections at maximum positive and negative moments as rectangular sections where, \( d = 18 \text{ cm} \), and \( b = 300 \text{ cm} \), \( f' = 280 \text{ kg/cm}^2 \), and \( f_y = 4200 \text{ kg/cm}^2 \).

<table>
<thead>
<tr>
<th>( M, \text{ t.m} )</th>
<th>( \rho )</th>
<th>( A_s )</th>
<th>Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Straight</td>
</tr>
<tr>
<td>10.71</td>
<td>0.0030</td>
<td>16.20</td>
<td>6 ( \phi 12 \text{ mm} )</td>
</tr>
<tr>
<td>12.85</td>
<td>0.0036</td>
<td>19.44</td>
<td>9 ( \phi 12 \text{ mm} )</td>
</tr>
<tr>
<td>21.63</td>
<td>0.0062</td>
<td>33.48</td>
<td>15 ( \phi 12 \text{ mm} )</td>
</tr>
<tr>
<td>8.65</td>
<td>0.0024</td>
<td>12.96</td>
<td>6 ( \phi 12 \text{ mm} )</td>
</tr>
</tbody>
</table>

Half middle strip reinforcement:
Design sections at maximum positive and negative moments as rectangular sections where, \( d = 18 \text{ cm} \), and \( b = 400 \text{ cm} \), \( f'_c = 280 \text{ kg/cm}^2 \), and \( f_y = 4200 \text{ kg/cm}^2 \).

<table>
<thead>
<tr>
<th>( M, t.m )</th>
<th>( \rho )</th>
<th>( A_y )</th>
<th>Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 8.57 )</td>
<td>0.0018</td>
<td>15.84</td>
<td>7 ( \phi 12 \text{ mm} ) ( \quad ) 8 ( \phi 12 \text{ mm} )</td>
</tr>
<tr>
<td>( 7.21 )</td>
<td>0.0018</td>
<td>15.84</td>
<td>--- ( \quad ) 8 ( \phi 12 ) + 8 ( \phi 12 )</td>
</tr>
<tr>
<td>( 5.77 )</td>
<td>0.0018</td>
<td>15.84</td>
<td>7 ( \phi 12 \text{ mm} ) ( \quad ) 8 ( \phi 12 \text{ mm} )</td>
</tr>
</tbody>
</table>

9- Prepare design drawings:

Figure 15.i shows reinforcement detail.

Continue from step 4 to calculate for the Strip in the longer direction

5- Calculate the factored static moment:

Strip in the longer direction:

Width of intermediate strip = 600 cm and width of column strip is the smaller of \( (l_1/2) \) and \( (l_2/2) \), taken as \( (600/2) = 300 \text{ cm} \).

Total factored static moment:

Clear span for exterior panels = 700 – 15 – 20 = 665 cm

Clear span for interior panels = 700 – 20 – 20 = 660 cm

The larger of the two values will be used in moment calculations.

\[ M_o = 1.475 (6)(6.65)^2 / 8 = 48.92 \text{ t.m} \]
Figure 15.i: Reinforcement layout
6- Distribute the total factored static moment into positive and negative moments:

The results are shown in the following table.

7- Distribute the positive and negative moments to the column and middle strips:

The results are shown in the following table, also given in Figure 15.j.

<table>
<thead>
<tr>
<th>Slab moment (t.m)</th>
<th>End spans</th>
<th>Interior spans</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exterior negative</td>
<td>Positive</td>
</tr>
<tr>
<td>Total moments ($M_T$)</td>
<td>$0.26M_o$ = 12.72</td>
<td>$0.52M_o$ = 25.44</td>
</tr>
<tr>
<td>Column strip moments</td>
<td>$1.0M_T$ = 12.72</td>
<td>$0.60M_T$ = 15.26</td>
</tr>
<tr>
<td>Middle strip moments</td>
<td>0.00</td>
<td>10.18</td>
</tr>
</tbody>
</table>

Figure 15.j: Column and middle strips in the long direction

Negative and positive factored moments:
Bending moment diagrams are shown in Figure 15.k and Figure 15.l.

Figure 15: (continued); (k) Bending moment diagram for column strip; (l) bending moment diagrams for middle strip

8- Design the reinforcement:

Column strip reinforcement:

Design sections at maximum positive and negative moments as rectangular sections where, \( d = 18 \text{ cm} \), and \( b = 300 \text{ cm} \), \( f_c' = 280 \text{ kg/cm}^2 \), and \( f_y = 4200 \text{ kg/cm}^2 \).

<table>
<thead>
<tr>
<th>( M, t.m )</th>
<th>( \rho )</th>
<th>( A_y )</th>
<th>Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.72</td>
<td>0.00357</td>
<td>19.28</td>
<td>8 ( \phi 12 \text{ mm} )</td>
</tr>
<tr>
<td>15.26</td>
<td>0.00431</td>
<td>23.27</td>
<td>11 ( \phi 12 \text{ mm} )</td>
</tr>
<tr>
<td>25.68</td>
<td>0.00747</td>
<td>40.34</td>
<td>19 ( \phi 12 \text{ mm} )</td>
</tr>
<tr>
<td>23.85</td>
<td>0.00690</td>
<td>37.26</td>
<td>19 ( \phi 12 \text{ mm} )</td>
</tr>
<tr>
<td>10.27</td>
<td>0.00286</td>
<td>15.46</td>
<td>7 ( \phi 12 \text{ mm} )</td>
</tr>
</tbody>
</table>

Half middle strip reinforcement:
Design sections at maximum positive and negative moments as rectangular sections where, \( d = 18 \ cm \), and \( b = 300 \ cm \), \( f'_c = 280 \text{ kg/cm}^2 \), and \( f_y = 4200 \text{ kg/cm}^2 \).

<table>
<thead>
<tr>
<th>( M, t.m )</th>
<th>( \rho )</th>
<th>( A_d )</th>
<th>Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Straight</td>
</tr>
<tr>
<td>10.18</td>
<td>0.0028</td>
<td>15.12</td>
<td>7 ( \phi ) 12 ( mm )</td>
</tr>
<tr>
<td>8.56</td>
<td>0.0024</td>
<td>12.96</td>
<td>-----</td>
</tr>
<tr>
<td>7.95</td>
<td>0.0022</td>
<td>11.88</td>
<td>-----</td>
</tr>
<tr>
<td>6.85</td>
<td>0.0019</td>
<td>11.88</td>
<td>5 ( \phi ) 12 ( mm )</td>
</tr>
</tbody>
</table>

9- **Prepare design drawings:**

Figure 15.m shows reinforcement detail.
Figure 15.m: Reinforcement details
**Example (5):**

For the two-way solid slab with beams on all column lines, shown in Figure 16.a, evaluate the moments acting on any of the internal beams, using the direct design method. All columns are 30 cm × 30 cm in cross section, all beams are 30 cm × 60 cm in cross section, slab thickness is equal to 14 cm, covering materials weigh 183 kg/m² and the live load is 400 kg/m². Use $f'_c = 280$ kg/cm² and $f_y = 4200$ kg/cm².

![Figure 16.a: Two-way solid slab with beams](image-url)
Solution:

1- Evaluate the constants $\alpha$ and $\beta_i$:

Beam sectional properties are shown in Figure 16.b.

Calculation of relative beam stiffness $\alpha$:

a. Internal beams:

Using Figure 5, relative stiffness of beam is calculated as follows:

\[
\alpha = \left( \frac{b}{l_2} \right) \left( \frac{a}{h} \right)^3 f = \left( \frac{30}{600} \right)(4.285)^3 (1.775) = 6.982
\]

Calculation of torsional constant $\beta_i$ (Figure 16.c):

\[
C = \sum \left( 1 - 0.63 \frac{x}{y} \right) \left( \frac{x^3 y}{3} \right)
\]
Figure 16.c: Beam sectional properties for relative stiffness calculations

\[ C_A = \left( 1 - 0.63 \left( \frac{14}{76} \right) \right) \left( 14 \right)^3 \left( \frac{76}{3} \right) + \left( 1 - 0.63 \left( \frac{30}{46} \right) \right) \left( 30 \right)^3 \left( \frac{46}{3} \right) \]
\[ = 305347.3 \text{ cm}^4 \]

\[ C_B = \left( 1 - 0.63 \left( \frac{30}{60} \right) \right) \left( 30 \right)^3 \left( \frac{60}{3} \right) + \left( 1 - 0.63 \left( \frac{14}{46} \right) \right) \left( 14 \right)^3 \left( \frac{46}{3} \right) \]
\[ = 403907.3 \text{ cm}^4 \]

\[ C_{\text{max}} = C_B = 403907.3 \text{ cm}^4 \]

\[ \beta_i = \frac{403907.3}{2 \left( 600 \right) \left( 14 \right)^3 / 12} = 1.472 \]

\[ q_u = 1.2 \left( 0.14 \left( 2.5 \right) + 0.183 \right) + 1.6 \left( 0.40 \right) = 1.28 \text{ t/m}^2 \]

2- Determine the total factored static moment (internal strip):

Total factored static moment:

\[ M_0 = q_u l_2 l_{ln}^2 / 8 = 1.28 \left( 6.0 \right) \left( 5.7 \right)^2 / 8 = 31.19 \text{ t.m} \]

Negative and positive factored moments:

\[ \alpha_{f I} l_2 / l_1 = 6.982 > 1.0 \]

For \( \beta_i = 0.0 \), exterior negative moment is 100% of total strip moment

For \( \beta_i \geq 2.5 \), exterior negative moment is 75% of total strip moment

By linear interpolation, for \( \beta_i = 1.472 \), exterior negative moment is 85.28% of total strip moment.
3- Evaluate the moments in one of the internal beams:

The results are shown in the following table:

<table>
<thead>
<tr>
<th>Bending moment (t.m)</th>
<th>End spans</th>
<th>Interior spans</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exterior negative</td>
<td>Positive</td>
</tr>
<tr>
<td>Total moments ( M_T )</td>
<td>( 0.16 M_o = 4.990 )</td>
<td>( 0.57 M_o = 17.778 )</td>
</tr>
<tr>
<td>Column strip mom. ( M_c )</td>
<td>( 0.85 M_T = 4.241 )</td>
<td>( 0.75 M_T = 13.333 )</td>
</tr>
<tr>
<td>Beam moments ( M_c )</td>
<td>( 0.85 M_c = 3.60 )</td>
<td>( 0.85 M_c = 11.33 )</td>
</tr>
</tbody>
</table>

Problems

P.1 Design the slab shown in Figure P.1, using the approximate coefficient method (solid). All columns are 40 cm × 40 cm in cross section, all beams are 40 cm × 60 cm in cross section, covering materials weigh 200 kg/m², equivalent partition load is equivalent to 75 kg/m² and the live load is 300 kg/m². Use \( f'_c = 350 \text{ kg/cm}^2 \) and \( f_y = 4200 \text{ kg/cm}^2 \).
P.2 Design a ribbed slab for the plan shown in Figure P.1 using the relevant approximate method.

P.3 Design the slab shown in P.1 using the direct design method.

P.4 Design the slab shown in P.1 using the direct design method with edge beams only. Design one of these beams.

P.5 Design the slab shown in P.1 using the direct design method, with interior beams only (no edge beams). Also, design one of these beams.