EELE 3332 – Electromagnetic II
Microstrip Transmission Lines

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Transmission lines

- Coaxial cable
- Two-wire transmission line
- Parallel Plate Waveguide
- Microstrip Line
- Strip Line
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Microstrip line is one of the most popular types of planar transmission lines because of ease of fabrication and integration with other passive and active microwave devices.

The geometry is shown, it consists of a conductor of width $W$ printed on a grounded dielectric substrate of thickness $h$ and relative permittivity $\varepsilon_r$.
Microstrip
Microstrip

Magnetic field

Electric field
Since some of the field lines are in the dielectric region and some are in air, (the dielectric region does not fill the air region above the strip \((y>h))\), an effective dielectric constant \(\varepsilon_e\) is considered in the analysis.

The effective dielectric constant satisfies the relation \(1 < \varepsilon_e < \varepsilon_r\) and it is dependent on the substrate thickness \(h\), and conductor width \(W\).

\[
\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \frac{1}{\sqrt{1+12h/W}}
\]

\(\varepsilon_e\) can be interpreted as the dielectric constant of a homogeneous medium that replaces the air and dielectric regions of the microstrip, as shown.
Given the dimensions of the microstrip line, the characteristic impedance can be calculated as

\[
\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12h/W}}
\]

The velocity is calculated as

\[
u = \frac{c}{\sqrt{\varepsilon_e}}
\]

The propagation constant is

\[
\beta = \frac{\omega}{u} = \sqrt{\varepsilon_e} \frac{\omega}{c}
\]

\[
Z_0 = \begin{cases} 
\frac{60}{\sqrt{\varepsilon_e}} \ln \left( \frac{8h}{W} + \frac{W}{4h} \right) & \text{for } W/h \leq 1 \\
\frac{120\pi}{\sqrt{\varepsilon_e} \left[ W/h + 1.393 + 0.667 \ln \left( W/h + 1.444 \right) \right]} & \text{for } W/h \geq 1
\end{cases}
\]
For a given characteristic impedance $Z_0$ and dielectric constant $\varepsilon_r$, the $W / h$ ratio can be found as:

$$
\frac{W}{h} = \left\{ \begin{array}{ll}
\frac{8e^A}{e^{2A} - 2} & \text{for } W / h < 2 \\
2 \left[ B - 1 - \ln(2B - 1) + \frac{\varepsilon_r - 1}{2\varepsilon_r} \left( \ln(B - 1) + 0.39 - \frac{0.61}{\varepsilon_r} \right) \right] & \text{for } W / h > 2
\end{array} \right.
$$

where

$$
A = \frac{Z_0}{60} \sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1} \left( 0.23 + \frac{0.11}{\varepsilon_r} \right)
$$

$$
B = \frac{377\pi}{2Z_0\sqrt{\varepsilon_r}}
$$
Attenuation can be caused by dielectric loss and conductor loss.

If $\alpha_d$ is attenuation constant due to dielectric loss, and $\alpha_c$ is the attenuation due to conductor loss, the total attenuation constant is:

$$\alpha = \alpha_d + \alpha_c$$

Considering the microstrip as a quasi-TEM line, the attenuation due to dielectric loss can be determined as:

$$\alpha_d = \frac{k_0 \varepsilon_r (\varepsilon_e - 1) \tan \delta}{2 \sqrt{\varepsilon_e (\varepsilon_r - 1)}} \text{Np/m} \quad , \quad k_0 = \frac{2\pi f}{c}$$

where $\tan \delta$ is the loss tangent of the dielectric.

The attenuation due to conductor loss is given approximately by:

$$\alpha_c = \frac{R_s}{Z_0 W} \quad \text{Np/m}$$

where $R_s = \sqrt{\omega \mu_0 / 2\sigma}$ is the surface resistivity of the conductor.
Example: Calculate the width and length of a microstrip line for a 50 Ω characteristic impedance and a 90° phase shift at 2.5 GHz. The substrate thickness is h=0.127 cm, with \( \varepsilon_r = 2.20 \).

Solution: We first find \( W/h \) for \( Z_0 = 50 \, \Omega \), and initially guess that \( W/h > 2 \).

\[
B = \frac{377\pi}{2Z_0\sqrt{\varepsilon_r}} = 7.985
\]

\[
W/h = \frac{2}{\pi} \left[ B - 1 - \ln(2B - 1) + \frac{\varepsilon_r - 1}{2\varepsilon_r} \left( \ln(B - 1) + 0.39 - \frac{0.61}{\varepsilon_r} \right) \right] = 3.081
\]

So \( W/h > 2 \); otherwise we would use the expression for \( W/h < 2 \).

\[ \Rightarrow \text{So, } W = 3.081h = 0.391 \, \text{cm} \]

The effective dielectric constant is

\[
\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12h/W}} = 1.87
\]

The length \( l \) for a 90° phase shift is found as

\[
\phi = 90° = \beta l = \sqrt{\varepsilon_e} \frac{\omega}{c} l \Rightarrow l = \frac{(90°)(\pi/180)(3 \times 10^8)}{(2\pi f)\sqrt{\varepsilon_e}} \Rightarrow l = 2.19 \, \text{cm}
\]
The geometry is shown, it consists of a thin conducting strip of width \( W \) centered between two wide conducting ground planes of separation \( h \), and the entire region between the ground planes is filled with a dielectric.

In practice, stripline is usually constructed by etching the centre conductor on a grounded substrate of thickness \( h/2 \), and then covering with another grounded substrate of the same thickness.

Notice that all fields exist within the dielectric substrate.
The velocity is: \[ u = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\varepsilon_r}} \]

The propagation constant is \[ \beta = \frac{\omega}{u} = \sqrt{\varepsilon_r} \frac{\omega}{c} \]

The characteristic impedance is: \[ Z_0 = \frac{30\pi}{\sqrt{\varepsilon_r}} \frac{h}{W_e + 0.441h} \]

where \( W_e \) is the effective width of the centre conductor given by

\[
W_e = \begin{cases} 
0 & \text{for } \frac{W}{h} > 0.35 \\
\frac{W}{h} - (0.35 - \frac{W}{h})^2 & \text{for } \frac{W}{h} < 0.35
\end{cases}
\]

These formulas assume a zero strip thickness, and are quoted as being accurate to about 1% of the exact results.

* Notice that \( Z_0 \) decreases as the strip width \( W \) increases.
Given the characteristic impedance $Z_0$, and height $h$ and permittivity $\varepsilon_r$, the strip width can be found by:

$$W = \begin{cases} 
   x & \text{for } \sqrt{\varepsilon_r Z_0} < 120 \\
   0.85 - \sqrt{0.6 - x} & \text{for } \sqrt{\varepsilon_r Z_0} > 120
\end{cases}$$

where

$$x = \frac{30\pi}{\sqrt{\varepsilon_r Z_0}} - 0.441$$
The attenuation due to dielectric loss is given by:

\[ \alpha_d = \frac{k \tan \delta}{2} \quad \text{Np/m (general for TEM waves)}, \quad k = \frac{\omega}{u} = \frac{\omega \sqrt{\varepsilon_r}}{c} \]

The attenuation due to conductor loss is approximately:

\[ \alpha_c = \begin{cases} 
\frac{2.7 \times 10^{-3} R_s \varepsilon_r Z_0}{30\pi (h - t)} A & \text{for } \sqrt{\varepsilon_r} Z_0 < 120 \\
0.16R_s\frac{B}{Z_0 h} & \text{for } \sqrt{\varepsilon_r} Z_0 > 120 
\end{cases} \quad \text{Np/m} \]

with \( A = 1 + \frac{2W}{h - t} + \frac{1}{\pi} \frac{h + t}{h - t} \ln \left( \frac{2h - t}{t} \right) \)

\[ B = 1 + \frac{h}{(0.5W + 0.7t)} \left( 0.5 + \frac{0.414t}{W} + \frac{1}{2\pi} \ln \frac{4\pi W}{t} \right) \]

where \( t \) is the thickness of the strip.
Microstrip applications
Low Pass Filters
Bandpass Filters
Microstrip Antennas
Example: Calculate the width and length of a microstrip line for a 50 Ω characteristic impedance and a 90° phase shift at 2.5 GHz. The substrate thickness is h=0.127cm, with $\varepsilon_r=2.20$. Use **txline** software to design.