Michelson Interferometer

Objective

Determination of the wave length of the light of the helium-neon laser by means of Michelson interferometer subsection Principle and Task Light is made to produce interference in the Michelson arrangement by the use of two mirrors. The wave length is determined by displacing one mirror using the micrometer screw.

Theory

The phenomenon of interference of light has proved the validity of the wave theory of light. Interference is produced due to the superposition of two waves. It is not possible to show interference due to two independent sources of light because a large number of difficulties are involved. The two independent sources may emit light waves of largely different amplitude and wavelength, and the phase difference between the two may change with time.

Coherent sources

Two sources are said to be coherent if they emit light waves of the same frequency, nearly the same amplitude and are always in phase with each other. It means that the two sources must emit radiation of the same color (wavelength). In actual it is not possible to have two independent sources which are coherent. But for experimental purpose, two virtual sources formed from a single source can act as coherent sources. Interference of light takes place between the waves by two methods:

1. From the real source and a virtual source.
2. From two virtual sources formed due to a single source.

In all such cases, the two sources will act, as if they are perfectly similar in all respects.

Composition of two simple harmonic motions in a straight line

Let two waves of the same frequency $\omega$ but of different amplitudes and different phase are represented by the equations

$$y_1 = a_1 \sin(\omega t - \alpha_1)$$

and

$$y_2 = a_2 \sin(\omega t - \alpha_2)$$

If these two waves interfere, the resulting wave

$$y = y_1 + y_2$$
can be described as
\[ y = A \sin(\omega t - \alpha) \] (4)
with the amplitude
\[ A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \delta, \] (5)
where
\[ \delta = \alpha_1 - \alpha_2 \] (6)
is the phase difference between the two waves.
For \( a_1 = a_2 = a \)
\[ A^2 = 4a^2 \cos^2 \frac{\delta}{2}, \] (7)
The intensity distribution according to Eq. (7) is:
\[ I \sim A^2 = 4a^2 \cos^2 \frac{\delta}{2} \] (8)

**Special cases**

1. Maxima occur if the phase difference \( \delta \) is an even of \( \pi \)
\[ \delta = 2n\pi, \quad n = 0, 1, 2, 3, \ldots. \] (9)

2. Minima occur if the phase difference \( \delta \) is an odd of \( \pi \)
\[ \delta = (2n + 1)\pi, \quad n = 0, 1, 2, 3, \ldots. \] (10)

**Michelson interferometer**

Michelson interferometer consists of two highly polished mirrors \( M_1 \) and \( M_2 \) and two plane glass plates \( A \) and \( C \) parallel to each other. The rear side of the glass plate \( A \) is half silvered so that light coming from the source \( S \) is equally reflected and transmitted by it. Light from a monochromatic source \( S \) after passing through the lens \( L \), falls on the plate \( A \). The lens \( L \) makes the beam parallel. The plate \( A \) is inclined at an angle of 45°. One-half of the energy of the incident beam is reflected by the plate \( A \) towards the mirror \( M_1 \) and the other half is transmitted towards the mirror \( M_2 \). These two beams (reflected and transmitted) are reflected back by the mirrors \( M_1 \) and \( M_2 \). These two beams return to the plate \( A \). The beam reflected back by \( M_1 \) is transmitted through the glass plate \( A \) and the beam reflected back by \( M_2 \) is reflected by glass plate \( A \) towards the eye as shown in Fig. 1.1. The beam going towards the mirror \( M_1 \) and reflected back, has to pass twice through the glass plate \( A \). Therefore, to compensate for the path, the plate \( C \) is used between the mirror \( M_2 \) and \( A \). The light beam going towards the mirror \( M_2 \) and reflected back towards \( A \) also passes twice through the compensating plate \( C \). Therefore, the paths of the two rays in glass are the same. The mirror \( M_2 \) can be moved with the help of the micrometer handle \( H \). The distance through which the mirror \( M_2 \) is moved can be read on the scale of the micrometer. The planes of the mirrors \( M_1 \) and \( M_2 \) can be
Figure 1: Michelson interferometer.

made perfectly perpendicular with the help of the fine screws attached to them.

- **Remark**
The compensating plate $C$ is a necessity for white light fringes but can be dispensed with, while using monochromatic light.

If the mirrors $M_1$ and $M_2$ are perfectly perpendicular, the observer’s eye will see the images of the mirrors $M_1$ and $M_2$ through $A$. There will be an air film between the two images and distance can be varied with the help of the micrometer handle $H$. The fringes will be perfectly circular. If the two images of $M_1$ and $M_2$ are inclined (the mirrors $M_1$ and $M_2$ not perfectly perpendicular) the enclosed air film will be wedge-shaped and straight line fringes will be observed. When the mirror $M_2$ is moved away or towards the glass plate $A$ with the help of handle $H$, the fringes cross the center of the field of view of the observer’s eye. If $M_2$ is moved through a distance $\lambda/2$, one fringe will cross the field of view and will move to the position previously occupied by the next fringe.
Circular fringes are produced in Michelson interferometer if the two mirrors $M_1$ and $M_2$ are perpendicular to each other, that is, if the mirror $M_1$ is parallel to the virtual mirror $M_2'$ which is the image of $M_2$ (Fig. 1.2).

The source is an extended one and $S_1$ and $S_2$ are the virtual images of the source due to $M_1$ and $M_2'$. If the distance $M_1M_2'$ is $d$, the distance between $S_1$ and $S_2$ is $2d$. The path difference between the two beams will be

$$\Gamma = 2d \cos \theta.$$  \hspace{1cm} (11)

The relation between the phase difference $\delta$ and the path difference $\Gamma$ is

$$\delta = \frac{2\pi}{\lambda} \Gamma.$$  \hspace{1cm} (12)

Substituting from Eq. (9) and Eq. (11) into Eq. (12), maxima thus occur if

$$2d \cos \theta = n\lambda, \hspace{0.5cm} n = 0, 1, 2, 3, \cdots.$$  \hspace{1cm} (13)
If the position of the movable mirror $M_2$ is changed so that $d$ for example decreases then, according to Eq. (13), the diameter of the ring will also decrease since $n$ is fixed for this ring:

$$d \downarrow \Rightarrow \cos \theta \uparrow \Rightarrow \theta \downarrow \Rightarrow \text{radius of the } n^{th} \text{ ring} \downarrow.$$  

For the first bright ring ($n = 1$), if this ring is disappeared ($\theta = 0$) then, from Eq. (13)

$$d = \frac{\lambda}{2}. \quad (14)$$

One ring thus disappears each time $d$ is reduced by $\lambda/2$. If the movable mirror is displaced a distance $d'$ ($d'$ is the micrometer reading), a number $n$ of fringes is crossed the field of view. If

$$\frac{\lambda}{2} \sim 1$$

$$d' \sim n,$$

then

$$d' = n \frac{\lambda}{2} \quad (15)$$

or

$$2d' = n\lambda. \quad (16)$$

**Equipment**

Support base, helium-neon lazer, Michelson interferometer, screen.

**Setup and Procedure**

The experimental set up is as shown in Fig.1.3. In order to obtain the largest possible number of interference fringes, the two mirrors of the interferometer are first of all adjusted.

**Adjustment of the interferometer**

1. The laser beam is adjusted to strike the half-silvered mirror at an angle of 45°.
2. The laser beam is split into two beams.
3. The resulting two beams are reflected by the mirrors and impinge on the screen.
4. Two points are obtained on the screen.
5. By means of the two adjusting screws, both points of light are made to coincide.
Figure 3: Experimental set-up for measuring wavelengths with the Michelson interferometer.

6. The points of light are enlarged and interference patterns are observed on the screen (bands, circles).

7. By careful readjustment, an interference image of concentric circles can be obtained.

**Measuring the wavelength of the laser light**

8. The micrometer screw is turned to any initial position at which the center of the circles is dark.

9. The initial reading of the micrometer is read.

10. The micrometer screws is now further turned in the same direction and 40 light-dark periods are counted.

11. The distance travelled by the mirror must be read on the micrometer screw and divided by ten (when the micrometer screw moves 10 \textit{mm} the mirror is displaced 1 \textit{mm}).

12. If the central point of the circles moves outside the light spot area, a readjustment has to be performed in order to obtain concentric fringes.
13. Repeat paragraphs 9-12 for 60, 80, 100, 120, 140, 160 and 180 circles.

48. Record your measurements in Table 1.1.

**Measurements and Calculations**

Table 1: The results of a typical measurements for the determination of the wavelength of the light of the helium-neon laser by means of Michelson interferometer.

<table>
<thead>
<tr>
<th>( n )</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
<th>180</th>
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<tr>
<td>( d ) (mm)</td>
<td></td>
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- Plot \( n \) as x-axis versus \( d \) as y-axis.
- Find the slope, then calculate the wavelength \( \lambda \) from Eq. (15) where

\[
\lambda = 2 \text{slope}.
\]
Figure 4: Displacement of the mirror $d$ as a function of number of fringes $n$ that crossed the field of view in Michelson interferometer.