Chapter 1

1.3 Two solid cylindrical rods $AB$ and $BC$ are welded together at $B$ and loaded as shown. Determine the magnitude of the force $P$ for which the tensile stress in rod $AB$ has the same magnitude as the compressive stress in rod $BC$.

\[
\sigma_{AB} = \frac{P}{\pi (2)^2}
\]

\[
\sigma_{BC} = \frac{60 - P}{\pi (3)^2}
\]

\[
\sigma_{AB} = \sigma_{BC}
\]

\[
\frac{P}{\pi (4)} = \frac{60 - P}{\pi (9)}
\]

# $P = 18.46$ kips
1.7 Each of the four vertical links has an \(8 \times 36\)-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (a) points \(B\) and \(D\), (b) points \(C\) and \(E\).

\[\Sigma F_y = 0 \rightarrow F_{BD} = 32.5 \text{ kN} \ (\uparrow)\]

\[\Sigma F_y = 0 \rightarrow F_{CE} = 12.5 \text{ kN} \ (\uparrow)\]

(a) at \(B\) and \(D\)

\[G_{BD} = \frac{32.5 \times 10^3}{2 \times (36 - 16) \times 8 \times 10^{-6}} = 101.56 \text{ MPa} \]

(b) at \(C\) and \(E\)

\[G_{CE} = \frac{12.5 \times 10^3}{2 \times 36 \times 8 \times 10^{-6}} = 21.7 \text{ MPa} \]

The calculations involve the application of force equilibrium to determine the normal stress in the links. The normal stress is calculated using the formula for stress, 

\[\sigma = \frac{F}{A}\]

where \(F\) is the force and \(A\) is the area of the cross-section. The calculations show the forces acting on the links and the resulting stress values.

In the case of (a), the force is 32.5 kN acting upwards, and in the case of (b), the force is 12.5 kN also acting upwards.
1.17 A load $P$ is applied to a steel rod supported as shown by an aluminum plate into which a 0.6-in.-diameter hole has been drilled. Knowing that the shearing stress must not exceed 18 ksi in the steel rod and 10 ksi in the aluminum plate, determine the largest load $P$ that can be applied to the rod.

$$
\tau_{\text{steel}} = \frac{P}{A} = 18 \times 10^3
$$

$$
18 \times 10^3 = \frac{P}{\pi (0.6)(0.4)} \Rightarrow P = 13.57 \text{ Kips}
$$

$$
\tau_{\text{alum}} = \frac{P}{A}
$$

$$
10 \times 10^3 = \frac{P}{\pi (1.6)(0.25)} \Rightarrow P = 12.56 \text{ Kips}
$$

$P = 12.56 \text{ Kips}$
1.20 The axial force in the column supporting the timber beam shown is $P = 20$ kips. Determine the smallest allowable length $L$ of the bearing plate if the bearing stress in the timber is not to exceed 400 psi.

$$
\sigma_b = \frac{P}{A}
$$

$$
\psi_{400} = \frac{20 \times 10^3}{6 \times L}
$$

$L = 8.33$ in
1.22 A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (a) the maximum bearing stress on the concrete footing, (b) the size of the footing for which the average bearing stress in the soil is 145 kPa.

(a)

\[ \sigma_b = \frac{P}{A} \]

\[ \sigma_b = \frac{40 \times 10^3}{0.12 \times 0.1} \]

\[ \sigma_b = 3.33 \text{ MPa} \]

(b)

\[ \sigma_b = \frac{P}{A} \]

\[ 145 \times 10^3 = \frac{40 \times 10^3}{b^2} \]

\[ b = 0.525 \text{ m} \]
1.26 Link $AB$, of width $b = 50$ mm and thickness $t = 6$ mm, is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is $-140$ MPa, and that the average shearing stress in each of the two pins is $80$ MPa, determine (a) the diameter $d$ of the pins, (b) the average bearing stress in the link.

(a)

\[ \sigma = \frac{P}{A} \]

\[ -140 \times 10^6 = \frac{-P}{b \times t} \]

\[ -140 \times 10^6 = \frac{-P}{0.05 \times 0.006} \]

\[ P = 42 \text{ KN} \]

\[ \tau_{\text{pin}} = \frac{P}{A} \]

\[ 80 \times 10^6 = \frac{42 \times 10^3}{\frac{\pi}{4} d^2} \]

\[ d = 0.0258 \text{ m} \]

(b)

\[ \sigma_b = \frac{P}{A} \]

\[ = \frac{42 \times 10^3}{0.0258 \times 0.006} \]

\[ \sigma_b = 271.3 \text{ MPa} \]
1.29 The 1.4-kip load $P$ is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

$$\sin \theta_0 = \frac{S}{x} \quad \Rightarrow \quad x = \frac{S}{\sin \theta_0}$$

$$A = 3 \times x = \frac{3 \times S}{\sin \theta_0}$$

$$G = \frac{P}{A}$$

$$G = \frac{1.4 \sin \theta_0}{15} = 0.07 \text{ KSI}$$

$$\tau = \frac{P \cos \theta_0}{15} = 0.04 \text{ KSI}$$
1.32 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 620 kPa, determine (a) the largest load $P$ that can be safely applied, (b) the corresponding tensile stress in the splice.

(a)

\[
\sin 45 = \frac{75}{x}
\]

\[
x = \frac{75}{\sin 45}
\]

\[
A = 150 \times \frac{75}{\sin 45}
\]

\[
T = 620 \times 10^3 = \frac{P \cos 45}{150 \times 75 / \sin 45}
\]

\[
P = 13.95 \text{ kN}
\]

(b)

\[
6 = \frac{P \sin 45}{150 \times 75 / \sin 45}
\]

\[
6 = 620 \text{ kPa}
\]
1.36 A centric load $P$ is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 18 MPa, determine (a) the magnitude of $P$, (b) the orientation of the surface on which the maximum shearing stress occurs, (c) the normal stress exerted on that surface, (d) the maximum value of the normal stress in the block.

(a) $$
\tau_{\text{max}} = 18 \text{ MPa} \quad \Rightarrow \quad \Theta = 45^\circ
$$

$$
\tau = \frac{P \sin \Theta \cos \Theta}{A_0}
$$

$$
18 \times 10^6 = \frac{P \sin 45^\circ \cos 45^\circ}{0.14 \times 0.14}
$$

$$
P = 706 \text{ KN}
$$

(b) \text{ at } \Theta = 45^\circ

(c) $$
\sigma = \frac{P \cos^2 \Theta}{A_0}
$$

$$
\sigma = \frac{706 \times 10^3 \times \cos^2 45^\circ}{0.14^2} = 18 \text{ MPa}
$$

(d) $$
\sigma_{\text{max}} \rightarrow \Theta = 0^\circ \rightarrow 
\sigma = \frac{P}{A_0}
$$

$$
\sigma = \frac{706 \times 10^3}{0.14^2} = 36 \text{ MPa}
$$
1.43 Two wooden members shown, which support a 3.6-kip load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 360 psi and the clearance between the members is $\frac{1}{4}$ in. Determine the required length $L$ of each splice if a factor of safety of 2.75 is to be achieved.

$$ F_s = \frac{G_u}{G_{all}} $$

$$ G_{all} = \frac{360}{2.75} = 130.91 \text{ psi} $$

$$ \tau = \frac{P}{A} $$

$$ 130.91 = \frac{3.6 \times 10^3}{2 \times [5 \times x]} $$

$$ x = 2.75 \text{ in.} $$

$$ L = 2x + \frac{h}{4} $$

$$ L = 5.75 \text{ in.} $$
1.45 A load \( P \) is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that \( b = 40 \) mm, \( c = 55 \) mm, and \( d = 12 \) mm, determine the load \( P \) if an overall factor of safety of 3.2 is desired.

\[ \text{Based on Double shear in pin} \]
\[ \text{Tall} = \frac{145 \times 10^6}{3.2} = 45.3 \text{ MPa} \]
\[ \text{Tall} = \frac{P}{A} \]
\[ 45.3 \times 10^6 = \frac{P}{2 \times \frac{\pi}{4} \times d^2} \]
\[ P = 10.25 \text{ KN} \]

\[ \text{Based on Double shear in wood} \]
\[ \text{Tall} = \frac{7.5}{3.2} = 2.34 \text{ MPa} \]
\[ \text{Tall} = 2.34 \times 10^6 = \frac{P}{2 \times [0.04 \times 0.055]} \]
\[ P = 10.3 \text{ KN} \]

\[ \text{Based on normal stress on wood} \]
\[ \text{Tall} = \frac{60}{3.2} = 18.75 \text{ MPa} \]
\[ \text{Tall} = 18.75 \times 10^6 = \frac{P}{0.04 [0.04 - 0.012]} \]
\[ P = 21 \text{ KN} \]
1.55 In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load \( P \) if an overall factor of safety of 3.0 is desired.

\[ \tau_{all,\ connections} = \frac{100}{3} \text{ Pa} \]
\[ \tau_{all,\ links} = \frac{250}{3} \text{ Pa} \]

\[ \sigma_T = 0 \rightarrow \]

\[ F_{BD} (200) = P (320) \]
\[ F_{BD} = 1.9 P \]
\[ A_T = 0.9 P \]

* check normal stress in link BD

\[ C = \frac{F_{BD}}{2(8)(20)(10^{-6})} \]
\[ \frac{250 \times 10^6}{3} = \frac{1.9 P}{2(8)(20)(10^{-6})} \]

\[ P = 14.03 \text{ kN} \]

* check shear stress in pin B

\[ \tau_{a} = \frac{0.9 P}{2 \times \frac{8}{3} \times (0.008)^2} \]

\[ P = 3.72 \text{ kN} \]

* use \( P = 3.72 \text{ kN} \).