3.3 Knowing that \( d = 1.2 \text{ in.} \), determine the torque \( T \) that causes a maximum shearing stress of 7.5 ksi in the hollow shaft shown.

\[
T_{\text{max}} = \frac{T_c}{J}
\]

\[
7.5 \times 10^3 = \frac{T \times 0.8}{\frac{\pi}{2} [0.8^4 - 0.6^4]}
\]

\[
T = 4.12 \text{ K.in}
\]
3.5 A torque $T = 3 \text{ kN} \cdot \text{m}$ is applied to the solid bronze cylinder shown. Determine (a) the maximum shearing stress, (b) the shearing stress at point $D$, which lies on a 15-mm-radius circle drawn on the end of the cylinder, (c) the percent of the torque carried by the portion of the cylinder within the 15-mm radius.

(a) \[ T_{\text{max}} = \frac{T}{J} \frac{C}{J} = \frac{3 \times 10 \times 0.03}{\frac{\pi}{2} (0.03)^4} \] \[ T_{\text{max}} = 70.7 \text{ MPa} \]

(b) \[ T_D = \frac{T}{J} \frac{C}{J} = \frac{3 \times 10 \times 0.015}{\frac{\pi}{2} (0.03)^4} \] \[ T_D = 35.4 \text{ MPa} \]

(c) \[ T = \frac{T_C}{J} = \frac{T}{C} \frac{J}{C} \] \[ T_D = \frac{\frac{\pi}{2} (0.015)^4 \times 35.4 \times 10^6}{0.015} = 187.5 \text{ Nm} \] \[ \text{percent} = \frac{0.1875}{3} \times 100 = 6.25\% \]
3.11 Knowing that each of the shafts $AB$, $BC$, and $CD$ consists of a solid circular rod, determine $(a)$ the shaft in which the maximum shearing stress occurs, $(b)$ the magnitude of that stress.

\[ \tau = \frac{48 \times 7.5 \times 10^{-3}}{\frac{\pi}{2} (7.5)^4 \times 10^{-12}} \]

\[ = 72.43 \text{ MPa} \]

\[ \text{for shaft } AB \rightarrow \]

\[ \tau = \frac{96 \times 9 \times 10^{-3}}{\frac{\pi}{2} (9)^4 \times 10^{-12}} \]

\[ = 83.83 \text{ MPa} \]

\[ \text{for shaft } BC \rightarrow \]

\[ \tau = \frac{156 \times 10.5 \times 10^{-3}}{\frac{\pi}{2} (10.5)^4 \times 10^{-12}} \]

\[ = 85.79 \text{ MPa} \]

\[ \text{for shaft } CD \rightarrow \]

\[ \text{Max } \tau \rightarrow \text{ shaft } CD \rightarrow 85.79 \text{ MPa} \]
3.21 A torque of magnitude $T = 1000 \text{ N} \cdot \text{m}$ is applied at $D$ as shown. Knowing that the diameter of shaft $AB$ is 56 mm and that the diameter of shaft $CD$ is 42 mm, determine the maximum shearing stress in (a) shaft $AB$, (b) shaft $CD$.

\[ \tau_{AB} = \frac{T_{AB} C}{J} = \frac{2500 \times 0.028}{\frac{\pi}{2} (0.028)^4} \]

\[ \tau_{AB} = 72.5 \text{ MPa} \]

\[ \tau_{CD} = \frac{T_{CD} C}{J} = \frac{1000 \times 0.021}{\frac{\pi}{2} (0.021)^4} \]

\[ \tau_{CD} = 68.7 \text{ MPa} \]
3.37 The aluminum rod $BC$ ($G = 26$ GPa) is bonded to the brass rod $AB$ ($G = 39$ GPa). Knowing that each rod is solid and has a diameter of 12 mm, determine the angle of twist $(a)$ at $B$, $(b)$ at $C$.

$$\phi = \frac{TL}{JG}$$

$$\phi_B = \phi_{B/A}$$

$$\phi_{B/A} = \frac{100 \times 0.2}{\frac{\pi}{2}(0.006)^4 \times 39 \times 10^9}$$

$$= 0.2519 \text{ rad} \times \frac{180}{\pi}$$

$$\phi_B = 14.43^\circ$$

$$\phi_C = \phi_{C/B} + \phi_{B/A}$$

$$\phi_{C/B} = \frac{100 \times 0.3}{\frac{\pi}{2}(0.006)^4 \times 26 \times 10^9}$$

$$= 0.5668 \text{ rad} \times \frac{180}{\pi}$$

$$\phi_{C/B} = 32.47^\circ$$

Now:

$$\phi_C = 14.43 + 32.47$$

$$\phi_C = 46.9^\circ$$
3.45 The design of the gear-and-shaft system shown requires that steel shafts of the same diameter be used for both $AB$ and $CD$. It is further required that $\tau_{\text{max}} \leq 60$ MPa and that the angle $\phi_D$ through which end $D$ of shaft $CD$ rotates not exceed $1.5^\circ$. Knowing that $G = 77$ GPa, determine the required diameter of the shafts.

\[
\frac{T_{CD}}{T_{AB}} = \frac{40}{100}
\]

\[
T_{AB} = 2500 \text{ N.m}
\]

* Based on stress →

\[
\tau_{\text{max}} = \frac{T_C}{J}
\]

\[
60 \times 10^6 = \frac{2500 \times c}{\frac{\pi}{2} c^4} \quad \Rightarrow \quad c = 29.8 \text{ mm}
\]

* Based on rotation angle →

\[
\phi_D = \phi_{D/c} + \phi_C
\]

\[
\phi_{C/D} = \frac{1000 \times 0.6}{\frac{\pi}{2} c^4}
\]

\[
\frac{\phi_C}{\phi_B} = \frac{r_B}{r_C}
\]

\[
\phi_B = \frac{\phi_B/A}{\frac{2500 \times 0.4}{\frac{\pi}{2} c^4}}
\]

\[
\phi_D = 1.5 \times \frac{\pi}{180} = 26.18 \times 10^{-3} \text{ rad}
\]

\[
\phi_C = \phi_B \times \frac{100}{40}
\]

Now →

\[
\phi_D = \frac{1000 \times 0.6}{\frac{\pi}{2} c^4} + \frac{2500 \times 0.4}{\frac{\pi}{2} c^4} \times \frac{100}{40}
\]

\[
C = 31.46 \text{ mm}
\]

Choose $C = 31.46 \text{ mm}$
3.51 A torque of magnitude $T = 4 \, \text{kN} \cdot \text{m}$ is applied at end $A$ of the composite shaft shown. Knowing that the modulus of rigidity is $77 \, \text{GPa}$ for the steel and $27 \, \text{GPa}$ for the aluminum, determine (a) the maximum shearing stress in the steel core, (b) the maximum shearing stress in the aluminum jacket, (c) the angle of twist at $A$.

$$T = T_s + T_A$$

$$\gamma = T_s + T_A \rightarrow 0$$

$\phi_s = \phi_A$

$$\frac{T_s \times 25}{77 \times 10^9 \times \frac{\pi}{2} \times 0.027} = \frac{T_A \times 25}{27 \times 10^9 \times \frac{\pi}{2} [0.036^4 - 0.027^4]}$$

$$T_s = 1.32 \, T_A \rightarrow 2$$

Now $\gamma = 1.32 \, T_A + T_A$

$$T_A = 1.724 \, \text{kN} \cdot \text{m}$$

$$T_s = 2.276 \, \text{kN} \cdot \text{m}$$

$$T_s = \frac{2.276 \times 10^3 \times 0.027}{\frac{\pi}{2} (0.027)^4} = 73.6 \, \text{GPa}$$

$$T_A = \frac{1.724 \times 10^3 \times 0.036}{\frac{\pi}{2} (0.036^4 - 0.027^4)} = 34.7 \, \text{GPa}$$

$$\phi_A = TL \cdot \frac{J}{G}$$

$$= \frac{2.276 \times 10^3 \times 25}{77 \times 10^9 \times \frac{\pi}{2} \times 0.027^4}$$

$$= 88.5 \times 10^{-3} \, \text{rad}$$

$$= 5.07^\circ$$
3.74 The two solid shafts and gears shown are used to transmit 16 hp from the motor at A operating at a speed of 1260 rpm to a machine tool at D. Knowing that the maximum allowable shearing stress is 8 ksi, determine the required diameter \( a \) of shaft AB, \( b \) of shaft CD.

\[
P = 2\pi \int T_{AB}
\]

\[
16 \times 6600 = 2\pi \times \frac{1260}{60} \times T_{AB}
\]

\[
T_{AB} = 800.32
\]

\[
\frac{T_{AB}}{T_{CD}} = \frac{r_{B}}{r_{C}}
\]

\[
T_{CD} = \frac{800.32 \times 5}{3} = 1333.87
\]

\[
T_{AB} = \frac{T_{AB} \times C_{AB}}{\frac{\pi}{2} \cdot C_{AB}^4}
\]

\[
8 \times 10 = \frac{800.32 \times C_{AB}}{\frac{\pi}{2} \cdot C_{AB}^4} \rightarrow C_{AB} = 0.399 \text{ in} \rightarrow d_{AB} = 0.799 \text{ in}
\]

\[
T_{CD} = \frac{T_{CD} \cdot C_{CD}}{\frac{\pi}{2} \cdot C_{CD}^4}
\]

\[
8 \times 10 = \frac{1333.87 \times C_{CD}}{\frac{\pi}{2} \cdot C_{CD}^4} \rightarrow C_{CD} = 0.473 \text{ in} \rightarrow d_{CD} = 0.947 \text{ in}
\]
3.83 A 1.6-m-long tubular steel shaft (G = 77.2 GPa) of 42-mm outer
diameter $d_1$ and 30-mm inner diameter $d_2$ is to transmit 120 kW
between a turbine and a generator. Knowing that the allowable shearing stress is 65 MPa and that the angle of twist must not exceed 3°,
determine the minimum frequency at which the shaft can rotate.

\[
P = \frac{2\pi f T}{2\pi T} \quad \Rightarrow \quad f = \frac{P}{2\pi T}
\]

* Based on shear stress →

\[
T = \frac{T_C}{J}
\]

\[
65 \times 10^6 = \frac{T \times 0.021}{\frac{\pi}{2} \left(0.021^4 - 0.015^4\right)}
\]

\[
T = \text{N.m}
\]

* Based on angle of twist →

\[
G = \frac{TL}{JG}
\]

\[
3 \times \frac{\pi}{180} = \frac{T \times 1.6}{77.2 \times 10^9 \times \frac{\pi}{2} \times \left(0.021^4 - 0.015^4\right)}
\]

\[
T = 570.878 \text{ N.m}
\]

→ To determine min. $f$ →

\[
f = \frac{120 \times 10^3}{2\pi \times 570.878} = 33.45 \text{ Hz}
\]
3.87 The stepped shaft shown must transmit 45 kW. Knowing that the allowable shearing stress in the shaft is 40 MPa and that the radius of the fillet is \( r = 6 \text{ mm} \), determine the smallest permissible speed of the shaft.

\[
T = K \frac{T_C}{J}
\]

\[
D = 60 \quad \frac{D}{d} = \frac{60}{30} = 2
\]

\[
d = 30 \quad \frac{V}{d} = \frac{6}{30} = 0.2
\]

From graph \( K = 1.26 \)

\[
40 \times 10^6 = 1.26 \times \frac{T \times 0.015}{\frac{\pi}{2} \times 0.015^4}
\]

\[
T = 168.3 \text{ N.m}
\]

\[
f = \frac{P}{2\pi T}
\]

\[
f = \frac{45 \times 10^3}{2\pi \times 168.3} = 42.55 \text{ Hz}
\]
3.125 Determine the largest allowable square cross section of a steel shaft of length 20 ft if the maximum shearing stress is not to exceed 10 ksi when the shaft is twisted through one complete revolution. Use $G = 11.2 \times 10^6$ psi.

\[
\text{square} \quad a = b \quad \Rightarrow \quad \frac{a}{b} = 1
\]

\[T_{\text{max}} = \frac{T}{C_1 a b^2}
\]

\[10 \times 10 = \frac{T}{0.208 a^3} \Rightarrow 1
\]

\[\varphi = 2\pi = \frac{TL}{C_2 a b^3 G}
\]

\[2\pi = \frac{T \times 20 \times 12}{0.1406 a^4 \times 11.2 \times 10^6} \Rightarrow 2
\]

Solving 1 and 2 \[T = 0.267 \text{ Ib in} \]

\[a = b = 0.0505 \text{ in} \]

#
The torque $T$ causes a rotation of 0.6° at end $B$ of the aluminum bar shown. Knowing that $b = 15$ mm and $G = 26$ GPa, determine the maximum shearing stress in the bar.

$$\phi = \frac{T L}{C_2 a b^3 G}$$

$$0.6 \times \frac{\pi}{180} = \frac{T \times 0.75}{0.029 \times 0.03 \times 0.015^3 \times 26 \times 10^9}$$

$$T = 8.417 \text{ N}\cdot\text{m}$$

$$\tau_{\text{max}} = \frac{T}{C_1 a b^2} = \frac{8.417}{0.029 \times 0.03 \times 0.015^2}$$

$$\tau_{\text{max}} = 5.07 \text{ GPa}$$

<table>
<thead>
<tr>
<th>$a/b$</th>
<th>$c_1$</th>
<th>$c_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.205</td>
<td>0.1406</td>
</tr>
<tr>
<td>1.2</td>
<td>0.219</td>
<td>0.1661</td>
</tr>
<tr>
<td>1.5</td>
<td>0.231</td>
<td>0.1958</td>
</tr>
<tr>
<td>2.0</td>
<td>0.246</td>
<td>0.229</td>
</tr>
<tr>
<td>2.5</td>
<td>0.258</td>
<td>0.249</td>
</tr>
<tr>
<td>3.0</td>
<td>0.267</td>
<td>0.263</td>
</tr>
<tr>
<td>4.0</td>
<td>0.282</td>
<td>0.281</td>
</tr>
<tr>
<td>5.0</td>
<td>0.291</td>
<td>0.291</td>
</tr>
<tr>
<td>10.0</td>
<td>0.312</td>
<td>0.312</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.333</td>
<td>0.333</td>
</tr>
</tbody>
</table>
3.140 A torque $T = 5 \text{ kN} \cdot \text{m}$ is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points $a$ and $b$.

\[
\tau = \frac{T}{2\pi a t}
\]

\[
A = \left(75 - 6\right) \left(125 - 10\right) \times 10^{-6}
\]

\[
A = 7.935 \times 10^{-3} \text{ m}^2
\]

\[
\tau_a = \frac{5 \times 10^3}{2 \times 7.935 \times 10^{-3} \times 0.006}
\]

\[
\tau_a = 52.5 \text{ MPa}
\]

\[
\tau_b = \frac{5 \times 10^3}{2 \times 7.935 \times 10^{-3} \times 0.01}
\]

\[
\tau_b = 31.5 \text{ MPa}
\]
3.141 A 90-N \cdot m torque is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points \( a \) and \( b \).

\[ \tau = \frac{T}{2At} \]

\[ A = 13 \times 52 + 13 \times 39 + \frac{\pi}{4} (39)^2 \]

\[ = 2.37 \times 10^{-3} \text{ m}^2 \]

\[ \tau_a = \frac{90}{2 \times 2.37 \times 10^{-3} \times 0.004} \]

\[ \tau_a = 4.74 \times 10^6 \text{ Pa} \]

\[ \tau_b = \frac{90}{2 \times 2.37 \times 10^{-3} \times 0.002} \]

\[ \tau_b = 9.49 \text{ MPa} \]