Part 2: Foundation Analysis

Chapter 4: Shallow Foundations: Ultimate Bearing Capacity

Introduction
This chapter aims at grasping:

- Fundamental concepts for ultimate bearing capacity of shallow foundations.
- Effect of water table and soil compressibility on ultimate bearing capacity.
- Bearing capacity of shallow foundations subjected to vertical eccentric loading and eccentrically inclined loading.

Generally speaking, shallow foundations must have two main satisfactorily characteristics:

(1) They have to be safe against overall shear failure in the soil that supports them.
(2) They cannot undergo excessive displacement, or settlement.
(The term excessive is relative, because the degree of settlement allowed for a structure depends on several considerations.)

Ultimate bearing capacity, \( q_u \)
The load per unit area of the foundation at which shear failure in soil occurs, or,

The ultimate pressure per unit area of the foundation that can be supported by the soil in excess of the pressure caused by the surrounding soil at the foundation level.

Nature of bearing capacity failure in soil
Bearing capacity failure occurs when the shear stresses in a soil exceeds its shear strength. Also called shear failure. There are three different natures of soil shear failure:

(a) general shear failure,
(b) local shear failure, and

(c) punching shear failure.
Terzaghi’s Bearing Capacity Theory

The first comprehensive theory for the evaluation of the ultimate bearing capacity of rough shallow foundations was presented by Terzaghi.

Terzaghi’s theory has been based on three assumptions:

- A foundation is shallow if its \( D_f \leq B \).
  - later investigators have suggested that foundations are shallow if its \( D_f \leq (3 - 4)B \).
  - a continuous, or strip, foundation is a one whose width-to-length ratio approaches zero, \( B/L \approx 0 \).
- The effect of the soil above the bottom of a foundation may be assumed to be replaced by an equivalent surcharge \( (q = \gamma \times D_f) \). Therefore, the shearing resistance of that soil along the failure surfaces is neglected.
- The equation is derived for general shear of a soil failure surfaces.

![Diagram of Terzaghi's Bearing Capacity Theory](image)

For continuous foundations:
\[
q_u = c'N_c + qN_q + 0.5\gamma BN_y
\]

For square foundations:
\[
q_u = 1.3\ c'N_c + qN_q + 0.4\gamma BN_y
\]

For circular foundations:
\[
q_u = 1.3\ c'N_c + qN_q + 0.3\gamma BN_y
\]

NB: \( N_c, N_q, N_y \) are bearing capacity factors that represent, respectively, the contributions of cohesion, surcharge, and unit weight of soil to the ultimate load-bearing capacity. These factors are functions of soil friction angle, \( \phi' \), and can be obtained from Table 4.1 demonstrated in page 164 of the Textbook.
NB: if you were given a soil friction angle of local shear failure, you would, at first, transfer it to its corresponding value of that soil general shear failure by means of the following equation:

$$\phi'_{\text{general}} = \tan^{-1}\left(\frac{2}{3}\tan(\phi'_{\text{local}})\right)$$

**Factor of Safety, FS**

It is a term used to describe the load carrying capacity of a system beyond the expected or actual loads, and can be defined as the ratio of the maximum stress that a foundation can withstand to the maximum stress estimated for it.

**Allowable bearing capacity, $q_{\text{all}}$**

It is the maximum bearing stress that can be applied to the foundation such that it is safe against instability due to shear failure and the maximum tolerable settlement is not exceeded.

$$q_{\text{all(gross)}} = q_{\text{all}} = \frac{q_u}{FS}$$

$$q_{\text{all(net)}} = \frac{q_{u(net)}}{FS}$$

$$q_{u(net)} = q_u - q$$

$$q = \gamma_c H_c - \gamma_s H_s$$

$$q \cong \gamma D_f$$

$$q_{u(net)} = q_u - q$$

$$q_{u(net)} = q_u - \gamma_c H_c - \gamma_s H_s$$

$$q_{u(net)} = q_u - \gamma D_f$$

$$q_{\text{all(net)}} = \frac{q_u - q}{FS}$$

$$q_{\text{all(net)}} = \frac{q_u - \gamma_c H_c - \gamma_s H_s}{FS}$$

$$q_{\text{all(net)}} = \frac{q_u - \gamma D_f}{FS}$$
Modification of Load-Bearing Capacity Equations for Water Table

If the water table is close to the foundation, some modifications of the load-bearing capacity equations will be necessary. Therefore, there are three cases:

**Case I:** \(0 \leq D_1 \leq D_f\)

For the second term of the equation:

\[
q = D_1 \gamma + D_2 (\gamma_{sat} - \gamma_w) = D_1 \gamma + D_2 \gamma'
\]

\(q = \text{effective surcharge of soil}\)

For the third term of the equation:

\[
\gamma' = \gamma_{sat} - \gamma_w
\]

\(\gamma' = \text{effective unit weight of soil}\)

**Case II:** \(0 \leq d < B\)

For the second term of the equation:

\[
q = \gamma D_f
\]

\(q = \text{effective surcharge of soil}\)

no change has been offered

For the third term of the equation:

\[
\bar{\gamma} = \gamma' + \frac{d}{B} (\gamma - \gamma')
\]
\[ \gamma' = \gamma_{sat} - \gamma_w \]
\[ \gamma' = \text{effective unit weight of soil} \]

**Case III: \( d \geq B \)**

There is no effect on the ultimate bearing capacity, and thus no term needs a modification.

**NB:** the previous modifications are based on the assumption that there is no seepage force in the soil.

**Meyerhof’s Equation: The General Bearing Capacity Equation**

There are two shortcomings that Terzaghi’s equations has been wrapped up:

- They do not address the case of rectangular foundations, when \( 0 < B/L < 1 \).
- They do not take into account the shearing resistance along the failure surface in soil above the bottom of the foundation.
- The load on the foundation may be inclined.

In order to account for all shortcomings mentioned above, Meyerhof suggested an equation that is commonly known as the general bearing capacity equation which is shown under:

\[ q_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_yF_{ys}F_{yd}F_{yi} \]

**NB:** \( N_c, N_q, N_y \) are bearing capacity factors and can be obtained from Table 4.2 in page 169 of the Textbook, whereas,

- \( F_{cs}, F_{qs}, F_{ys} \) are shape factors,
- \( F_{cd}, F_{qd}, F_{yd} \) are depth factors, and
- \( F_{ci}, F_{qi}, F_{yi} \) are load inclination factors and these factors can be obtained by using the equations listed in Table 4.3 in page 170 of the Textbook, and those are as given as under:
Shape

\[ F_{cs} = 1 + \left( \frac{B}{L} \right) \left( \frac{N_q}{N_c} \right) \]

\[ F_{q_s} = 1 + \left( \frac{B}{L} \right) \tan \phi' \]

\[ F_{\gamma_s} = 1 - 0.4 \left( \frac{B}{L} \right) \]

Depth

\[ \frac{D_f}{B} \leq 1 \]

For \( \phi = 0 \):

\[ F_{cd} = 1 + 0.4 \left( \frac{D_f}{B} \right) \]

\[ F_{q_d} = 1 \]

\[ F_{\gamma_d} = 1 \]

For \( \phi' > 0 \):

\[ F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi'} \]

\[ F_{qd} = 1 + 2 \tan \phi' \left( 1 - \sin \phi' \right)^2 \left( \frac{D_f}{B} \right) \]

\[ F_{\gamma_d} = 1 \]

\[ \frac{D_f}{B} > 1 \]

For \( \phi = 0 \):

\[ F_{cd} = 1 + 0.4 \tan^{-1} \left( \frac{D_f}{B} \right) \]

\[ F_{qd} = 1 \]

\[ F_{\gamma_d} = 1 \]

For \( \phi' > 0 \):

\[ F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi'} \]

\[ F_{qd} = 1 + 2 \tan \phi' \left( 1 - \sin \phi' \right)^2 \tan^{-1} \left( \frac{D_f}{B} \right) \]

\[ F_{\gamma_d} = 1 \]
Eccentrically Loaded Foundations

Foundations may be subjected to moments in addition to the vertical load. In such cases, the distribution of pressure by the foundation on the soil is not uniform, due to that some eccentricity has been offered which can be determined as under:

\[ e = \frac{Overall \ Moment}{Vertical \ Loads} = \frac{M}{Q} \]

The nominal distribution of pressure is:

\[ q = \frac{Q}{BL} \pm \frac{Mc}{l} \]

The maximum pressure, \( q_{\text{max}} \), and the minimum pressure, \( q_{\text{min}} \), can be determined by the formulas expressed in the following three cases:

**If \( e < B/6 \):**

\[ q_{\text{max}} = \frac{Q}{BL} \left( 1 + \frac{6e}{B} \right) \]
\[ q_{\text{min}} = \frac{Q}{BL} \left( 1 - \frac{6e}{B} \right) \]

**If \( e = B/6 \):**

\[ q_{\text{max}} = \frac{Q}{BL} \left( 1 + \frac{6e}{B} \right) \]
\[ q_{\text{min}} = 0.0 \]

**If \( e > B/6 \):**

\[ q_{\text{max}} = \frac{4Q}{3L(B - 2e)} \]
Ultimate Bearing Capacity under Eccentric Loading: One-Way Eccentricity

Effective Area Method (Meyerhof)
The following is a step-by-step procedure for determining the ultimate load that the soil can support and the factor of safety against bearing capacity failure:

**Step 1.** Determine the effective dimensions of the foundation:
- \( B' = \text{effective width} = \min \{ (B - 2e), L \} \)
- \( L' = \text{effective length} = \max \{ (B - 2e), L \} \)
- \( A' = \text{effective area} = B' \times L' \)

**Step 2.** Use an adopted equation for the ultimate bearing capacity:
- \( q_u = c' N_c + q N_q + 0.5 \gamma B N_y \) (continuous foundation)
- \( q_u = 1.3 \ c' N_c + q N_q + 0.4 \gamma B N_y \) (square foundation)
- \( q_u = 1.3 \ c' N_c + q N_q + 0.3 \gamma B N_y \) (circular foundation)
- \( q_u = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + 0.5 B \gamma N_y F_{ys} F_{yd} F_{yi} \) (general)
NB: use $B'$ in the equations of the shape factors ($F_{cs}, F_{qs}, F_{ys}$).

use $B$ in the equations of the depth factors ($F_{cd}, F_{qd}, F_{yd}$).

use $B$ in the equations of the water table correction.

**Step 3.** The total ultimate load that the foundation can sustain is:

$$Q_u = q'_u \times A' = q'_u \times B' \times L'$$

**Step 4.** The factor of safety against bearing capacity failure is:

$$FS = \frac{Q_u}{Q_{all}}$$

NB: the factor of safety against $q_{max}$ is $FS = \frac{q'_u}{q_{all}}$ and the following criteria must be satisfactory \{\(q_{max} \leq q_{all}, q_{min} \geq 0.0\}\).

**Problems**