Chapter 8: Mat Foundations

Introduction
Under normal conditions, square and rectangular footings are economical for supporting columns and walls. However, under certain circumstances, it may be desirable to construct a footing that supports a line of two or more columns. These footings are referred to as combined footings. When more than one line of columns is supported by a concrete slab, it is called a mat foundation.

Combined footings can be classified generally into four categories:

1. Rectangular combined footing
2. Trapezoidal combined footing
3. Strap footing

Mat foundations are generally used with soil that has a low bearing capacity. This chapter has been demonstrated in two distinct parts, which are as given as under:

Part A: Geotechnical design, and

Part B: Structural design.
Part A: Geotechnical Design

The geotechnical design of foundations will be illustrated for isolated and combined footings in the next following sections.

A. Isolated Footing

Most economical type.
It can be rectangular, circular or square.
It is preferred that the shape of the footing matches the shape of column above.

\[ A = B \times L \]

B. Combined Footing

1) Rectangular combined footing
In such a case, two or more columns can be supported on a single rectangular foundation. If the net allowable soil pressure is known, the size of the foundation \((B \times L)\) can be determined in the following manner:
a. No property line (i.e. extension is permitted from both sides)

\[
A = \frac{Q_1 + Q_2}{q_{net(\text{all})}}
\]

\[
X = \frac{Q_2 L_3}{Q_1 + Q_2}
\]

\[
L = 2(L_2 + X)
\]

\[
L_1 = L - L_2 - L_3
\]

\[
B = \frac{A}{L}
\]
b. One property line (i.e. extension is permitted from one side only)

\[
\begin{align*}
A &= \frac{Q_1 + Q_2}{q_{\text{net(all)}}} \\
X &= \frac{Q_2L_1}{Q_1 + Q_2} \\
L &= 2 \left( X + \frac{C_1}{2} \right) \\
L_2 &= L - L_1 - \frac{C_1}{2} \\
B &= \frac{A}{L}
\end{align*}
\]
c. Two property line (i.e. extension is obstructed from both sides)

\[ X = \frac{Q_2 L_1}{Q_1 + Q_2} \]

\[ e = \frac{L}{2} - X - \frac{C_1}{2} \]

If \( e > \frac{L}{6} \), use the following equation:

\[ q_{\text{max}} = q_{\text{all(\text{net})}} = \frac{4(Q_1 + Q_2)}{3B(L - 2e)} \]

\( B = \text{been found} \)

Otherwise, use the following equation:

\[ q_{\text{max}} = q_{\text{all(\text{net})}} = \frac{R}{B \times L} \left( 1 + \frac{6e}{L} \right) \]

\( B = \text{been found} \)
(2) Trapezoidal combined footing
Trapezoidal combined footing is sometimes used as an isolated spread foundation of columns carrying large loads where space is tight. The size of the foundation that will uniformly distribute pressure on the soil can be obtained in the following manner:

\[ A = \frac{Q_1 + Q_2}{q_{net(all)}} \]

\[ X = \frac{Q_2 L_3}{Q_1 + Q_2} \]

\[ A = \frac{B_1 + B_2}{2} L \quad (eqn. 1) \]
Solving equations (1) and (2):

\[ B_1 = \text{been found} \]
\[ B_2 = \text{been found} \]

b. One property line (i.e. extension is permitted from one side only)
Solving equations (1) and (2):

\[ B_1 = \text{been found} \]
\[ B_2 = \text{been found} \]

c. Two property line (i.e. extension is obstructed from both sides)

\[ A = \frac{Q_1 + Q_2}{q_{\text{net(all)}}} \]
\[ X = \frac{Q_2 L_3}{Q_1 + Q_2} \]
\[ A = \frac{B_1 + B_2}{2} L \] \hspace{1cm} (eqn. 1)
\[ X + \frac{C_1}{2} = \left( \frac{B_1 + 2B_2}{B_1 + B_2} \right) \frac{L}{3} \] \hspace{1cm} (eqn. 2)

Solving equations (1) and (2):

\[ B_1 = \text{been found} \]
\[ B_2 = \text{been found} \]
(3) **Strap footing (Cantilever footing)**

Cantilever footing construction uses a strap beam to connect an eccentrically loaded column foundation to the foundation of an interior column.

Cantilever footings may be used in place of trapezoidal or rectangular combined footings when the allowable soil bearing capacity is high and the distances between the columns are large.

\[
R = Q_1 + Q_2 = R_1 + R_2
\]
\[ X_r = \frac{Q_2 \times d}{R} \]

\[ L_1 = \text{given/assumed} \]

\[ a = X_r + \frac{W_1}{2} - \frac{L_1}{2} \]

\[ b = d - X_r \]

\[ R_1 = \frac{R \times b}{(a + b)} \]

\[ R_2 = R - R_1 \]

\[ A_1 = \frac{R_1}{q_{ali(\text{net})}} \]

\[ A_2 = \frac{R_2}{q_{ali(\text{net})}} \]

### C. Mat Foundation (Raft Foundation)

It is a combined footing that may cover the entire area under a structure supporting several columns and walls.

Mat foundations are sometimes preferred for soils that have low load-bearing capacities, but that will have to support high column or wall loads. Under some conditions, spread footings would have to cover more than half the building area, and mat foundations might be more economical.

### Common types of mat foundation

Several types of mat foundations are currently used. Some of the common ones are listed as given under:

- Flat plate: the mat is of uniform thickness (Figure a).
- Flat plate thickened under columns (Figure b).
- Beams and slab: the beams run both ways, and the columns are located at the intersection of the beams (Figure c).
- Flat plates with pedestals (Figure d).
- Slab with basement walls as a part of the mat: the walls act as stiffeners for the mat (Figure e).
Comparison of isolated foundation and mat foundation
The Figure below shows the difference between the depth $D_f$ and the width $B$ of isolated foundations and mat foundations.
Bearing capacity of mat foundation

The gross ultimate bearing capacity of a mat foundation can be determined by the same equation used for shallow foundations (see Section 4.6 in textbook), or:

\[ q_u = C N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qa} F_{qi} + \frac{1}{2} B \gamma N_{\gamma} F_{\gamma s} F_{\gamma d} F_{\gamma i} \]

The term B in the load-bearing capacity equation is the smallest dimension of the mat. The net ultimate capacity of a mat foundation is:

\[ q_u(\text{net}) = q_u - q \]

A suitable factor of safety should be used to calculate the net allowable bearing capacity:

- For mats on clay, the factor of safety should not be less than 3 under dead load or maximum live load. However, under the most extreme conditions, the factor of safety should be at least 1.75 to 2.
- For mats constructed over sand, a factor of safety of 3 should normally be used. Under most working conditions, the factor of safety against bearing capacity failure of mats on sand is very large.

For saturated clays with \( \phi = 0 \) and a vertical loading condition (\( \beta = 0 \)), the net ultimate bearing capacity \( q_u \) is:

\[ q_u = 5.14 \epsilon_u \left( 1 + 0.195 \frac{B}{L} \right) \left( 1 + 0.4 \frac{D_f}{B} \right) + q \]

Hence, the net ultimate bearing capacity \( q_{u(\text{net})} \) is:

\[ q_{u(\text{net})} = q_u - q = 5.14 \epsilon_u \left( 1 + 0.195 \frac{B}{L} \right) \left( 1 + 0.4 \frac{D_f}{B} \right) \]

\[ q_{u(\text{net})} = \frac{q_u - q}{FS} \]

Or, the load-bearing capacity can be obtained based on Standard Penetration Test (SPT) and may be expressed as:
In English units, the previous equation may be expressed as:

\[
q_{\text{net}}(\text{kN/m}^2) = \frac{N_{60}}{0.08} F_d \left( \frac{S_e}{25} \right) \\
= \frac{N_{60}}{0.08} \left[ 1 + 0.33 \left( \frac{D_f}{B} \right) \left( \frac{S_e(\text{mm})}{25} \right) \right] \\
\leq 16.63 N_{60} \left( \frac{S_e(\text{mm})}{25} \right)
\]

Recall that, generally, shallow foundations are designed for a maximum settlement of 25 mm (1 in.) and a differential settlement of three-fourths of the tolerable settlement, which is about 19 mm (0.75 in.).

However, the width of a raft foundation is larger than those of the isolated spread footings. The depth of significant stress increase in the soil below a foundation depends on the width of the foundation. Hence, for a raft foundation, the depth of the zone of influence is likely to be much larger than that of a spread footing. Thus, the loose soil pockets under a raft may be more evenly distributed, resulting in a smaller differential settlement. Accordingly, the customary assumption is that, for a maximum raft settlement of 50 mm (2 in.), the differential settlement would be 19 mm (0.75 in.).

The net pressure applied on a foundation (as shown in the Figure below) may be expressed as:

\[
q_{\text{net(all)}}(\text{kip/ft}^2) = 0.25 N_{60} \left[ 1 + 0.33 \left( \frac{D_f}{B} \right) \right] [S_e(\text{in.})] \\
\leq 0.33 N_{60} [S_e(\text{in.})]
\]
\[ q = \frac{Q}{A} - \gamma D_f \leq q_{\text{net(all)}} \]

**Example 8.3/4**
See example 8.3/4 in textbook, page 363.

**Compensated foundation**
The net pressure increase in the soil under a mat foundation can be reduced by increasing the depth \( D_f \) of the mat. This approach is generally referred to as the compensated foundation design and is extremely useful when structures are to be built on very soft clays. In this design, a deeper basement is made below the higher portion of the superstructure, so that the net pressure increase in soil at any depth is relatively uniform.

\[ q = \frac{Q}{A} - \gamma D_f \]

For a *fully compensated foundation*, the depth of foundation is:

\[ D_f = \frac{Q}{A\gamma} \]

For a *partially compensated foundation* (i.e. \( D_f < \frac{Q}{A\gamma} \)), the factor of safety against bearing capacity failure may be given as:

\[ \text{FS} = \frac{q_{\text{net(ii)}}}{q} = \frac{q_{\text{net(ii)}}}{\frac{Q}{A} - \gamma D_f} \]

**Example 8.5/6**
See example 8.5/6 in textbook, page 367.
Part B: Structural Design

The structural design of mat foundations can be carried out by two conventional methods: *the conventional rigid method* and *the approximate flexible method*. Finite-difference and finite-element methods can also be used, but this section covers only the basic concepts of the first design method.

Conventional Rigid Method

The conventional rigid method of mat foundation design can be explained step by step with reference to the Figure shown below:
Step 1: Determine the resultant load and the location of the mat centroided:

\[ Q = Q_1 + Q_2 + \cdots + Q_3 \]

\[ \bar{X} = \frac{B}{2} \]

\[ \bar{Y} = \frac{L}{2} \]

Step 2: Determine the load eccentricities, \( e_x \) and \( e_y \), in the x and y directions. They can be determined by using \((X', Y')\) coordinates:

\[ X' = \frac{Q_1X'_1 + Q_2X'_2 + Q_3X'_3 + \cdots}{Q} \]

\[ e_x = X' - \bar{X} \]

\[ Y' = \frac{Q_1Y'_1 + Q_2Y'_2 + Q_3Y'_3 + \cdots}{Q} \]

\[ e_y = Y' - \bar{Y} \]
Step 3: Determine the pressure on the soil, \( q \), below the mat at points A, B, C, D, … by using the equation:

\[
q = \frac{Q}{A} \mp \frac{M_y X}{I_x} \mp \frac{M_x Y}{I_y}
\]

where

\[
A = BL
\]

\[
I_x = BL^3/12 = \text{moment of inertia about the x-axis}
\]

\[
I_y = LB^3/12 = \text{moment of inertia about the y-axis}
\]

\[
M_x = Q X E_y = \text{moment of the column loads about the x-axis}
\]

\[
M_y = Q X E_x = \text{moment of the column loads about the y-axis}
\]

Step 4: Compare the values of the soil pressures determined in Step 3 with the net allowable soil pressure to determine whether \( q \leq q_{all(\text{net})} \).

Step 5: Divide the mat into several strips in the x and y directions. (See the previous Figure). Let the width of any strip be \( B_1 \).

Step 6: For example, the average soil pressure of the bottom strip in the x direction of the previous Figure (a) is:

\[
q_{av} \approx \frac{q_l + q_F}{2}
\]

The total soil reaction is equal to \( q_{av} B_1 B \) and the total column load on the strip is expressed as \( Q_1 + Q_2 + Q_3 + Q_4 \).

Note that: \( Q_1 + Q_2 + Q_3 + Q_4 = q_{av} B_1 B \)

because the shear between the adjacent strips has not been taken into account. For this reason, the soil reaction and the column loads need to be adjusted, or:

\[
Q_{i(\text{av})} = \frac{q_{av} B_1 B + (Q_1 + Q_2 + Q_3 + Q_4)}{2}
\]

\[
q_{av(\text{modified})} = q_{av} \left( \frac{Q_{i(\text{av})}}{q_{av} B_1 B} \right)
\]

Step 7: Calculate the modified column loads \( FQ_1, FQ_2, FQ_3, \) and \( FQ_4 \).
\[ F = \frac{Q_{i(\text{av})}}{Q_1 + Q_2 + Q_3 + Q_4} = \frac{Q_{i(\text{av})}}{Q} \]

where
F = column load modification factor

\[ FQ_i = F \times Q_i = \frac{Q_{i(\text{av})}}{Q} \times Q_i \]

**Step 8:** Draw the Shear Force Diagram (SFD) and the Bending Moment Diagram (BMD) for this strip, and the procedure is repeated in the x and y directions for all strips.

**Step 9:** Design (i.e. check punching shear and determine the required area of reinforcing steel bars) the strip for the maximum shear and the maximum moment using any adopted code version (i.e. ACI Code 318, in any version, is highly recommended).

*Example 8.7/8*
See example 8.7/8 in textbook, page 380.

**Problems**