Chapter 13: Retaining Walls

Introduction
In general, retaining walls can be divided into two major categories:

(a) conventional retaining walls and
(b) mechanically stabilized earth walls

Conventional retaining walls can generally be classified into four varieties:

(1) Gravity retaining walls
(2) Semi-gravity retaining walls
(3) Cantilever retaining walls
(4) Counterfort retaining walls

(1) Gravity retaining walls
Gravity retaining walls (Figure a) are constructed with plain concrete or stone masonry. They depend for stability on their own weight and any soil resting on the masonry. This type of construction is not economical for high walls.

(2) Semi-gravity retaining walls
In many cases, a small amount of steel may be used for the construction of gravity walls, thereby minimizing the size of wall sections. Such walls are generally referred to as semi-gravity walls (Figure b).
(3) Cantilever retaining walls
Cantilever retaining walls (Figure c) are made of reinforced concrete that consists of a thin stem and a base slab. This type of wall is economical to a height of about 8 m (25 ft).

(4) Counterfort retaining walls
Counterfort retaining walls (Figure d) are similar to cantilever walls. At regular intervals, however, they have thin vertical concrete slabs known as counterforts that tie the wall and the base slab together. The purpose of the counterforts is to reduce the shear and the bending moments.
To design retaining walls properly, an engineer must know the basic parameters – the unit weight, angle of friction, and cohesion – of the soil retained behind the wall and the soil below the base slab. Knowing the properties of the soil behind the wall enables the engineer to determine the lateral pressure distribution that has to be designed for.

There are two phases in the design of a conventional retaining wall:

- First, with the lateral earth pressure known, the structure as a whole is checked for stability. The structure is examined for possible overturning, sliding, and bearing capacity failures.
- Second, each component of the structure is checked for strength, and the steel reinforcement of each component is determined.

This chapter presents the procedures for determining the stability of the retaining wall. Checks for strength can be found in any textbook on reinforced concrete.

**Gravity and Cantilever Walls**

*Proportioning Retaining Walls*

In designing retaining walls, an engineer must assume some of their dimensions. Called proportioning, such assumptions allow the engineer to check trial sections of the walls for stability. If the stability checks yield
undesirable results, the sections can be changed and rechecked. Figure 13.3 shows the general proportions of various retaining wall components that can be used for initial checks.

Note that the top of the stem of any retaining wall should not be less than about 0.3 m (≈12 in) for proper placement of concrete. The depth, \( D \), to the bottom of the base slab should be a minimum of 0.6 m (≈2 ft). However, the bottom of the base slab should be positioned below the seasonal frost line.

For counterfort retaining walls, the general proportion of the stem and the base slab is the same as for cantilever walls. However, the counterfort slabs may be about 0.3 m (≈12 in) thick and spaced at centre-to-centre distances of 0.3H to 0.7H.

\[ \text{Figure 13.3 Approximate dimensions for various components of retaining wall for initial stability checks: (a) gravity wall; (b) cantilever wall} \]

Application of Lateral Earth Pressure Theories to Design

Rankine theory

Rankine active earth pressure equations may then be used to calculate the lateral pressure on the face AB of the wall. In the analysis of the wall’s stability, the force \( P_a \) (Rankine), the weight of soil above the heel, and the weight \( W_c \) of the concrete all should be taken into consideration.
**Case (1): vertical backface and horizontal granular backfill**

\[ P_a = P_{a(h)} = \frac{1}{2} \gamma H^2 K_a \]

\[ K_a = (45 - \frac{\phi}{2}) \]

**Case (2): vertical backface and inclined granular backfill**
\[ P_a = \frac{1}{2} \gamma H'^2 K_a \]

where:

\[ H' = H + L \tan \alpha \]

\[ K_a = \frac{1}{\cos^2 \phi'} \left\{ 2 \cos^2 \alpha + 2 \left( \frac{c'}{\gamma z} \right) \cos \phi' \sin \phi' \right\} - 1 \]

Some values of \( K'_a \) are given in Table 12.4.

<table>
<thead>
<tr>
<th>( c' ) (deg)</th>
<th>( \alpha ) (deg)</th>
<th>0.025</th>
<th>0.05</th>
<th>0.1</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0</td>
<td>0.550</td>
<td>0.512</td>
<td>0.435</td>
<td>-0.179</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.566</td>
<td>0.525</td>
<td>0.445</td>
<td>-0.184</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.621</td>
<td>0.571</td>
<td>0.477</td>
<td>-0.186</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.776</td>
<td>0.683</td>
<td>0.546</td>
<td>-0.196</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>0.455</td>
<td>0.420</td>
<td>0.350</td>
<td>-0.210</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.465</td>
<td>0.429</td>
<td>0.357</td>
<td>-0.212</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.497</td>
<td>0.456</td>
<td>0.377</td>
<td>-0.218</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.567</td>
<td>0.514</td>
<td>0.417</td>
<td>-0.229</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td>0.374</td>
<td>0.342</td>
<td>0.278</td>
<td>-0.231</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.381</td>
<td>0.348</td>
<td>0.283</td>
<td>-0.233</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.402</td>
<td>0.366</td>
<td>0.296</td>
<td>-0.239</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.443</td>
<td>0.401</td>
<td>0.321</td>
<td>-0.250</td>
</tr>
<tr>
<td>30</td>
<td>0</td>
<td>0.305</td>
<td>0.276</td>
<td>0.218</td>
<td>-0.244</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.309</td>
<td>0.280</td>
<td>0.221</td>
<td>-0.246</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.323</td>
<td>0.292</td>
<td>0.230</td>
<td>-0.252</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.350</td>
<td>0.315</td>
<td>0.246</td>
<td>-0.263</td>
</tr>
</tbody>
</table>

and then we use the horizontal and vertical components in design as will explained later:

\[ P_{a(h)} = P_a \cos(\alpha) \]

\[ P_{a(v)} = P_a \sin(\alpha) \]
Case (3): inclined backface and inclined granular backfill

![Diagram of a wall with inclined backface and granular backfill]

Note that the force $P_a$ is inclined with angle $(\alpha)$ and not depend on the inclination of the wall because the force applied on the vertical line and can be calculated as following:

$$P_a = \frac{1}{2} \gamma H'^2 K_a$$

where:

$H' = H + L \tan \alpha$

$$K_{a(R)} = \frac{\cos(\alpha - \theta) \sqrt{1 + \sin^2 \phi' - 2 \sin \phi' \cos \psi_a}}{\cos^2 \theta \left( \cos \alpha + \sqrt{\sin^2 \phi' - \sin^2 \alpha} \right)}$$

= Rankine active earth-pressure coefficient for generalized case

$$\psi_a = \sin^{-1} \left( \frac{\sin \alpha}{\sin \phi'} \right) - \alpha + 2 \theta.$$ 

$P_a(h) = P_a \cos(\alpha)$

$P_a(v) = P_a \sin(\alpha)$
Coulomb theory
Coulomb’s active earth pressure theory also may be used, as shown in Figure 13.4c. If it is used, the only forces to be considered are $P_a$ (Coulomb) and the weight of the wall, $W_c$.

$$P_a = \frac{1}{2} \gamma H'^2 K_a$$

(acts at a distance $H/3$ above the base of the wall)

where:

$$K_a = \text{Coulomb’s active earth pressure coefficient}$$

$$= \frac{\sin^2(\beta + \phi')}{\sin^2 \beta \sin(\beta - \delta') \left[ 1 + \sqrt{\frac{\sin(\phi' + \delta')\sin(\phi' - \alpha)}{\sin(\beta - \delta')\sin(\alpha + \beta)}} \right]^2}$$

The values of the active earth pressure coefficient, $K_a$, for a vertical retaining wall ($\beta = 90^\circ$) with horizontal backfill ($\alpha = 0^\circ$) are given in Table 12.5.

<table>
<thead>
<tr>
<th>$\phi'$ (deg)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>0.3610</td>
<td>0.3448</td>
<td>0.3330</td>
<td>0.3251</td>
<td>0.3203</td>
<td>0.3186</td>
</tr>
<tr>
<td>30</td>
<td>0.3333</td>
<td>0.3189</td>
<td>0.3085</td>
<td>0.3014</td>
<td>0.2973</td>
<td>0.2956</td>
</tr>
<tr>
<td>32</td>
<td>0.3073</td>
<td>0.2945</td>
<td>0.2853</td>
<td>0.2791</td>
<td>0.2755</td>
<td>0.2745</td>
</tr>
<tr>
<td>34</td>
<td>0.2827</td>
<td>0.2714</td>
<td>0.2633</td>
<td>0.2579</td>
<td>0.2549</td>
<td>0.2542</td>
</tr>
</tbody>
</table>
As shown, the force $P_a$ is inclined with angle $(\delta + \theta)$ with horizontal, so:

\[ P_{a(h)} = P_a \cos(\delta + \theta) \]
\[ P_{a(v)} = P_a \sin(\delta + \theta) \]

If Coulomb’s theory is used, it will be necessary to know the range of the wall friction angle $\delta$ with various types of backfill material. Following are some ranges of wall friction angle for masonry or mass concrete walls:

<table>
<thead>
<tr>
<th>Backfill material</th>
<th>Range of $\delta'$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel</td>
<td>27–30</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>20–28</td>
</tr>
<tr>
<td>Fine sand</td>
<td>15–25</td>
</tr>
<tr>
<td>Stiff clay</td>
<td>15–20</td>
</tr>
<tr>
<td>Silty clay</td>
<td>12–16</td>
</tr>
</tbody>
</table>

**Note:** In the case of ordinary retaining walls, water table problems and hence hydrostatic pressure are not encountered. Facilities for drainage from the soils that are retained are always provided.

**Stability of Retaining Walls**
A retaining wall may fail in any of the following ways:

- It may **overturn** about its toe. (see Figure 13.5a.)
- It may **slide** along its base. (see Figure 13.5b.)
- It may **fail due to the loss of bearing capacity** of the soil supporting the base. (see Figure 13.5c.)
- It may undergo deep-seated shear failure. (see Figure 13.5d.)
- It may go through excessive settlement.
Check for Overturning

Based on the assumption that the Rankine active pressure is acting along a vertical plane AB drawn through the heel of the structure.

\[ P_p = \frac{1}{2} K_p \gamma_2 D^2 + 2c' \sqrt{K_p D} \]

The factor of safety against overturning about the toe –that is, about point C in Figure 13.7– may be expressed as:

\[ \text{FS}_{(\text{overturning})} = \frac{\sum M_R}{\sum M_o} \]

\[ \text{FS}_{(\text{overturning})} = \frac{M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_v}{P_a \cos \alpha (H'/3)} \]

Some designers prefer to determine the factor of safety against overturning with the formula:

\[ \text{FS}_{(\text{overturning})} = \frac{M_1 + M_2 + M_3 + M_4 + M_5 + M_6}{P_a \cos \alpha (H'/3) - M_v} \]

The usual minimum desirable value of the factor of safety with respect to overturning is 2 to 3.

The overturning moment is:

\[ \sum M_o = P_h \left( \frac{H'}{3} \right) \]

where:
\[ P_{a(h)} = P_a \cos \alpha \]
\[ P_{a(v)} = P_a \sin \alpha \]

To calculate the resisting moment, \( \sum M_R \) (neglecting \( P_p \)), a table such as Table 13.1 can be prepared.

### Table 13.1 Procedure for Calculating \( \sum M_R \)

<table>
<thead>
<tr>
<th>Section (1)</th>
<th>Area (2)</th>
<th>Weight/unit length of wall (3)</th>
<th>Moment arm measured from ( C ) (4)</th>
<th>Moment about ( C ) (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( A_1 )</td>
<td>( W_1 = \gamma_1 \times A_1 )</td>
<td>( X_1 )</td>
<td>( M_1 )</td>
</tr>
<tr>
<td>2</td>
<td>( A_2 )</td>
<td>( W_2 = \gamma_1 \times A_2 )</td>
<td>( X_2 )</td>
<td>( M_2 )</td>
</tr>
<tr>
<td>3</td>
<td>( A_3 )</td>
<td>( W_3 = \gamma_c \times A_3 )</td>
<td>( X_3 )</td>
<td>( M_3 )</td>
</tr>
<tr>
<td>4</td>
<td>( A_4 )</td>
<td>( W_4 = \gamma_c \times A_4 )</td>
<td>( X_4 )</td>
<td>( M_4 )</td>
</tr>
<tr>
<td>5</td>
<td>( A_5 )</td>
<td>( W_5 = \gamma_c \times A_5 )</td>
<td>( X_5 )</td>
<td>( M_5 )</td>
</tr>
<tr>
<td>6</td>
<td>( A_6 )</td>
<td>( W_6 = \gamma_c \times A_6 )</td>
<td>( X_6 )</td>
<td>( M_v )</td>
</tr>
</tbody>
</table>

(Note: \( \gamma_1 \) = unit weight of backfill
\( \gamma_c \) = unit weight of concrete
\( X_i \) = horizontal distance between \( C \) and the centroid of the section)
Check for Sliding along the Base

The factor of safety against sliding may be expressed by the equation:

$$FS_{\text{(sliding)}} = \frac{\sum F_R'}{\sum F_d}$$
Figure 13.8 shows that the passive force $P_p$ is also a horizontal resisting force. Hence:

$$
\Sigma F_r = (\Sigma V) \tan \delta' + Bc'_a + P_p
$$

The only horizontal force that will tend to cause the wall to slide (a driving force) is the horizontal component of the active force $P_a$, so:

$$
\Sigma F_d = P_a \cos \alpha
$$

$$
FS_{\text{(sliding)}} = \frac{(\Sigma V) \tan \delta' + Bc'_a + P_p}{P_a \cos \alpha}
$$

A minimum factor of safety of $1.5$ against sliding is generally required.

In many cases, the passive force $P_p$ is ignored in calculating the factor of safety with respect to sliding. In general, we can write $\delta' = k_1 \phi'$ and $c'_a = k_2 c'_2$. In most cases, $k_1$ and $k_2$ are in the range from $\frac{1}{2}$ to $\frac{2}{3}$. Thus:

$$
FS_{\text{(sliding)}} = \frac{(\Sigma V) \tan (k_1 \phi') + Bk_2 c'_2 + P_p}{P_a \cos \alpha}
$$

If the desired value of $FS_{\text{(sliding)}}$ is not achieved, several alternatives may be investigated (see Figure 13.9):

- Increase the width of the base slab (i.e., the heel of the footing).
- Use a key to the base slab. If a key is included, the passive force per unit length of the wall becomes.

$$
P_p = \frac{1}{2} \gamma_2 D_1^2 K_p + 2c'_2 D_1 \sqrt{K_p}
$$

where $K_p = \tan^2 \left( 45 + \frac{\phi' \pi}{2} \right)$. 
Check for Bearing Capacity Failure

The nature of variation of the vertical pressure transmitted by the base slab into the soil is shown in the following Figure.
Note that $q_{toe}$ and $q_{heel}$ are the maximum and the minimum pressures occurring at the ends of the toe and heel sections, respectively. The magnitudes of $q_{toe}$ and $q_{heel}$ can be determined in the following manner:

\[
P_h = P_a \cos \alpha
\]

\[
R = \Sigma V + P_h
\]

The net moment of these forces about point C in the preceding Figure:

\[
M_{\text{net}} = \Sigma M_R - \Sigma M_o
\]

\[
\frac{CE}{X} = \frac{M_{\text{net}}}{\Sigma V}
\]

Hence, the eccentricity of the resultant R may be expressed as:

\[
e = \frac{B}{2} - \frac{CE}{X}
\]

The pressure distribution under the base slab may be determined by using simple principles from the mechanics of materials. First, we have:

\[
q = \frac{\Sigma V}{A} \pm \frac{M_{\text{net}}y}{I}
\]

where

\[
M_{\text{net}} = \text{moment} = (\Sigma V)e
\]

\[
I = \text{moment of inertia per unit length of the base section} = \frac{1}{12} (1)(B^3)
\]

For maximum and minimum pressures:

\[
q_{\text{max}} = q_{toe} = \frac{\Sigma V}{(B)(1)} + e\frac{(\Sigma V)B}{2} = \frac{\Sigma V}{B} \left( 1 + \frac{6e}{B} \right)
\]

\[
q_{\text{min}} = q_{heel} = \frac{\Sigma V}{B} \left( 1 - \frac{6e}{B} \right)
\]
Note that $\sum V$ includes the weight of the soil, as shown in Table 13.1, and that when the value of the eccentricity $e$ becomes greater than $B/6$, $q_{\text{min}}$ becomes negative. Thus, there will be some tensile stress at the end of the heel section. This stress is not desirable, because the tensile strength of soil is very small. If the analysis of a design shows that $e > B/6$, the design should be reproportioned and calculations redone.

$$q_u = c'_{2} N_c F_{cd} F_{ci} + q N_q F_{qd} F_{qi} + \frac{1}{2} \gamma_2 B' N_\gamma F_{\gamma d} F_{\gamma i}$$

where

$$q = \gamma_2 D$$

$$B' = B - 2e$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi_2'}$$

$$F_{qd} = 1 + 2 \tan \phi_2' (1 - \sin \phi_2') \frac{D}{B'}$$

$$F_{\gamma d} = 1$$

$$F_{ci} = F_{qi} = \left(1 - \frac{\psi^\circ}{90^\circ}\right)^2$$

$$F_{\gamma i} = \left(1 - \frac{\psi^\circ}{\phi_2^\circ}\right)^2$$

$$\psi^\circ = \tan^{-1}\left(\frac{P_a \cos \alpha}{\Sigma V}\right)$$

$$F_{cs} = F_{qs} = F_{\gamma s} = 1.0 \text{ (treated as a continuous foundation)}$$

Once the ultimate bearing capacity of the soil has been calculated by, the factor of safety against bearing capacity failure can be determined:

$$FS_{\text{(bearing capacity)}} = \frac{q_u}{q_{\text{max}}}$$

Generally, a factor of safety of 3 is required.
Example 12.1
See example 12.1 in textbook, page 665.

Example 12.2
See example 12.2 in textbook, page 669.

Problems
Here a bunch of different problems is to be discussed on the board.