Chapter 2: Random Variables (Cont’d)

Section 2.4: The Variance of a Random Variable

Problem (1):
Suppose that the random variable $X$ takes the values $-2, 1, 4,$ and $6$ with probability values $1/3, 1/6, 1/3,$ and $1/6,$ respectively.

(a) Find the expectation of $X$.

(b) Find the variance of $X$ using the formula:

\[
\text{Var}(X) = \text{E}((X - \text{E}(X))^2).
\]

(c) Find the variance of $X$ using the formula:

\[
\text{Var}(X) = \text{E}(X^2) - (\text{E}(X))^2.
\]

Solution:

(a) $\text{E}(X) = ?!$

\[
\text{E}(X) = \sum x_i p_i
\]

\[
\text{E}(X) = (-2 \times 1/3) + (1 \times 1/6) + (4 \times 1/3) + (6 \times 1/6) = 11/6
\]

(b) the variance of $X$ using the formula: $\text{Var}(X) = \text{E}((X - \text{E}(X))^2)$

\[
\text{Var}(X) = \sigma^2 = \text{E}((X - \text{E}(X))^2) = 1/3 \times (-2 - 11/6)^2 + 1/6 \times (1 - 11/6)^2 + 1/3 \times (4 - 11/6)^2 + 1/6 \times (6 - 11/6)^2 = 341/36
\]

(b) the variance of $X$ using the formula: $\text{Var}(X) = \text{E}(X^2) - (\text{E}(X))^2$

\[
\text{E}(X^2) = \left[(-2)^2 \times 1/3\right] + \left[(1)^2 \times 1/6\right] + \left[(4)^2 \times 1/3\right] + \left[(6)^2 \times 1/6\right] = 77/6
\]

\[
\text{Var}(X) = \sigma^2 = \text{E}(X^2) - (\text{E}(X))^2 = 77/6 - (11/6)^2 = 341/36
\]
**Problem (2):**
An office has four copying machines, and the random variable X measures how many of them are in use at a particular moment in time. Suppose that:

\[
P(X = 0) = 0.08, \quad P(X = 1) = 0.11, \quad P(X = 2) = 0.27, \quad P(X = 3) = 0.33 \quad \text{and} \quad P(X = 4) = 0.21.
\]

Calculate the variance and standard deviation of the number of copying machines in use at a particular moment.

(problem 2.4.2 in textbook)

**Solution:**

\[
E(X) = 2.48 \text{ (from Lecture 09)}
\]

\[
\text{Var}(X) = \sigma^2 = E((X - E(X))^2)
\]

\[
= 0.08 \times (0 - 2.48)^2 + 0.11 \times (1 - 2.48)^2 + 0.27 \times (2 - 2.48)^2
\]

\[
+ 0.33 \times (3 - 2.48)^2 + 0.21 \times (4 - 2.48)^2
\]

\[
= 1.3696
\]

\[
\text{Stddev}(X) = \sigma = \sqrt{\text{Var}(X)} = \sqrt{\sigma^2}
\]

\[
= \sqrt{1.3696} = 1.170299
\]

**Problem (3):**
A company has five warehouses, only two of which have a particular product in stock. A salesperson calls the five warehouses in a random order until a warehouse with the product is reached. Let the random variable X be the number of calls made by the salesperson. Calculate the variance and standard deviation of the number of warehouses called by the salesperson.

(problem 2.4.3 in textbook)

**Solution:**

From Lecture 09:

\[
\begin{array}{c|cccc}
  x_i & 1 & 2 & 3 & 4 \\
  p_i & \frac{2}{5} & \frac{3}{5} \times \frac{2}{4} = \frac{3}{10} & \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{1}{5} & \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} = \frac{1}{10} \\
\end{array}
\]
E(X) = (1 × 2/5) + (2 × 3/10) + (3 × 1/5) + (4 × 1/10) = 2.0

Var(X) = \sigma^2 = E((X - E(X))^2)
= 2/5 \times (1 - 2.0)^2 + 3/10 \times (2 - 2.0)^2 + 1/5 \times (3 - 2.0)^2
+ 1/10 \times (4 - 2.0)^2
= 1.0

\text{Stdev}(X) = \sigma = \sqrt{\text{Var}(X)} = \sqrt{\sigma^2}
= \sqrt{1.0} = 1.0

**Problem (4):**
Consider a random variable with a probability density function of:

\[ f(x) = \begin{cases} 
\frac{1}{x \ln(1.5)} & \text{for } 4 \leq x \leq 6 \\
0 & \text{elsewhere}
\end{cases} \]

(a) What is the variance of this random variable?
(b) What is the standard deviation of this random variable?
(c) Find the upper and lower quartiles of this random variable.
(d) What is the interquartile range?

(problem 2.4.5 in textbook)

**Solution:**

E(X) = 4.94 (from Lecture 09)

(a) the variance = ?!

\[ \text{Var}(X) = \sigma^2 = \int (x - E(X))^2 f(x) \, dx \]
\[ = E(X^2) - (E(X))^2 \]
\[ = \int x^2 \left( \frac{1}{x \ln(1.5)} \right) \, dx - (E(X))^2 \]
\[ = \int_4^6 x^2 \left( \frac{1}{x \ln(1.5)} \right) \, dx = (4.94)^2 \]
\[
= (24.66) - (4.94)^2 = 0.26
\]

(b) the standard deviation = ?!
\[
\text{Stddev}(X) = \sigma = \sqrt{\text{Var}(X)} = \sqrt{\sigma^2} = \sqrt{0.26} = 0.51
\]

(c) the upper and lower quartiles = ?!

upper quartile = ?!
\[
F(x) = 0.75
\int_{4}^{x} \frac{1}{y \ln(1.5)} \, dy = 0.75
X = 5.42
\]

lower quartile = ?!
\[
F(x) = 0.25
\int_{4}^{x} \frac{1}{y \ln(1.5)} \, dy = 0.25
X = 4.43
\]

(d) the interquartile range = ?!
\[
\text{interquartile range} = \text{upper quartile} - \text{lower quartile} = 5.42 - 4.43 = 0.99
\]

**Problem (5):**
Consider a random variable with a cumulative distribution function of:
\[
F(x) = \frac{x^2}{16} \quad \text{for} \quad 0 \leq x \leq 4
\]

(a) What is the variance of this random variable?
(b) What is the standard deviation of this random variable?
(c) Find the upper and lower quartiles of this random variable.
(d) What is the interquartile range?

(problem 2.4.6 in textbook)

**Solution:**

From Lecture 09:

E(\(X\)) = 2.67

\(f(x) = \frac{x}{8}\) \(\quad\text{for}\ 0 \leq x \leq 4\)

(a) the variance = ?!

\[\text{Var}(X) = \sigma^2 = \int (x - E(X))^2 f(x) \, dx\]

\[= E(X^2) - (E(X))^2\]

\[= \int x^2 \left(\frac{x}{8}\right) \, dx - (E(X))^2\]

\[= \int_0^4 x^2 \left(\frac{x}{8}\right) \, dx - (2.67)^2\]

\[= (8.0) - \left(\frac{8}{3}\right)^2 = \frac{8}{9}\]

(b) the standard deviation = ?!

\[\text{Stddev}(X) = \sigma = \sqrt{\text{Var}(X)} = \sqrt{\sigma^2} = \sqrt{8/9} = 0.9428\]

(c) the upper and lower quartiles = ?!

upper quartile = ?!

\(F(x) = 0.75\)

\[
\frac{x^2}{16} = 0.75
\]

\[X = 3.464102\]
lower quartile = ?!

\[ F(x) = 0.25 \]
\[ \frac{x^2}{16} = 0.25 \]
\[ x = 2.0 \]

(d) the interquartile range = ?!

interquartile range = upper quartile - lower quartile

\[ = 3.464102 - 2.0 \]
\[ = 1.464102 \]

**Problem (6):**
The time taken to serve a customer at a fast-food restaurant has a mean of 75.0 seconds and a standard deviation of 7.3 seconds. Use Chebyshev’s inequality to calculate time intervals that have 75% and 89% probabilities of containing a particular service time.

(problem 2.4.10 in textbook)

**Solution:**

Adding and subtracting \(2 \times \text{Stdev}\) from the mean value gives:

\[
P( (\mu - 2 \times \text{Stdev}) \leq X \leq (\mu + 2 \times \text{Stdev}) ) \geq 0.75
\]
\[
P((75.0 - 2 \times 7.3) \leq X \leq (75.0 + 2 \times 7.3)) \geq 0.75
\]
\[
P(60.4 \leq X \leq 89.6) \geq 0.75
\]

Adding and subtracting \(3 \times \text{Stdev}\) from the mean value gives:

\[
P( (\mu - 3 \times \text{Stdev}) \leq X \leq (\mu + 3 \times \text{Stdev}) ) \geq 0.89
\]
\[
P((75.0 - 3 \times 7.3) \leq X \leq (75.0 + 3 \times 7.3)) \geq 0.89
\]
\[
P(53.1 \leq X \leq 96.9) \geq 0.89
\]
Problem (7):
A machine produces iron bars whose lengths have a mean of 110.8 cm and a standard deviation of 0.5 cm. Use Chebyshev’s inequality to obtain a lower bound on the probability that an iron bar chosen at random has a length between 109.55 cm and 112.05 cm.

(problem 2.4.11 in textbook)

Solution:

\[ \mu - k \times \text{Stdev} = 109.55 \]
\[ 110.8 - k \times 0.5 = 109.55 \]
\[ k = (110.8 - 109.55) / 0.5 \]
\[ k = 2.5 \]

So that the interval (109.55, 112.05) is:

\[ (\mu - 2.5 \times \text{Stdev}, \mu + 2.5 \times \text{Stdev}) \]
and the Chebyshev’s inequality gives:

\[ P( (\mu - 2.5 \times \text{Stdev}) \leq X \leq (\mu + 2.5 \times \text{Stdev}) ) \geq 1 - \frac{1}{(2.5)^2} \]

\[ P( 110.8 - 2.5 \times 0.5 \leq X \leq (110.8 + 2.5 \times 0.5) ) \geq 1 - \frac{1}{(2.5)^2} \]

\[ P(109.55 \leq X \leq 112.05) \geq 0.84 \]

Problem (8):
A continuous random variable has a probability density function:

\[ f(x) = Ax^{2.5} \quad \text{for} \quad 2 \leq x \leq 3. \]

(a) What is the value of A?
(b) What is the expectation of the random variable?
(c) What is the standard deviation of the random variable?
(d) What is the median of the random variable?

(problem 2.4.14 in textbook)

Solution:

(a) the value of A = ?!
Area under the pdf = \( \int f(x) \, dx = 1.0 \)

\[
\int_{2}^{3} Ax^{2.5} = 1.0
\]

\[
\left[ \frac{Ax^{3.5}}{3.5} \right]_{2}^{3} = 1.0
\]

\[
\left( \frac{A(3)^{3.5}}{3.5} \right) - \left( \frac{A(2)^{3.5}}{3.5} \right) = 1.0
\]

\( A = 0.098726 \)

So that:

\( f(x) = 0.098726 \, x^{2.5} \)  \( \text{for} \ 2 \leq x \leq 3. \)

(b) \( E(X) = ?! \)

\[
E(X) = \int x f(x) \, dx = \int_{2}^{3} x (0.098726 \, x^{2.5}) \, dx = 2.58
\]

(c) \( \text{Stdev}(X) = ?! \)

\[
Var(X) = \sigma^2 = \int (x - E(X))^2 f(x) \, dx
\]

\[
= E(X^2) - (E(X))^2
\]

\[
= \int x^2 (0.098726 \, x^{2.5}) \, dx - (E(X))^2
\]

\[
= \int_{2}^{3} x^2 (0.098726 \, x^{2.5}) \, dx - (2.58)^2
\]

\[
= 6.741 - (2.58)^2 = 0.0846
\]

\[
\text{Stdev}(X) = \sigma = \sqrt{Var(X)} = \sqrt{\sigma^2} = \sqrt{0.0846} = 0.290861
\]

(d) \( \text{median} = ?! \)
\[ F(x) = 0.5 \]
\[
\int_{2}^{3} 0.098726 x^{2.5} \, dx = 0.5
\]
\[
[0.098726 x^{2.5}]_{2}^{3} = 0.5
\]
\[
x = 2.62
\]

**Problem (9):**
In a game, a player either loses $1 with a probability 0.25, wins $1 with a probability 0.4, or wins $4 with a probability 0.35. What are the expectation and the standard deviation of the winnings?

(problem 2.4.15 in textbook)

**Solution:**
\[
E(X) = (-1 \times 0.25) + (1 \times 0.4) + (4 \times 0.35) = 1.55
\]
\[
\text{Var}(X) = \sigma^2 = E((X - E(X))^2)
\]
\[
= 0.25 \times (-1 - 1.55)^2 + 0.4 \times (1 - 1.55)^2 + 0.35 \times (4 - 1.55)^2
\]
\[
= 3.8475
\]
\[
\text{Stddev}(X) = \sigma = \sqrt{\text{Var}(X)} = \sqrt{\sigma^2}
\]
\[
= \sqrt{3.8475} = 1.9615
\]

**Problem (10):**
A random variable \( X \) has a distribution given by the probability density function \( f(x) = \frac{(1 - x)}{2} \) with a state space \(-1 \leq x \leq 1\).

(a) What is the expected value of \( X \)?
(b) What is the standard deviation of \( X \)?
(c) What is the upper quartile of \( X \)?

(problem 2.4.18 in textbook)

**Solution:**

(a) \( E(X) = \) ?!
\[
E(X) = \int_{-1}^{1} x f(x) \, dx = \int_{-1}^{1} x \frac{1-x}{2} \, dx = -\frac{1}{3}
\]
(b) $\text{Stddev}(X) = \sqrt{\text{Var}(X)}$

\[
\text{Var}(X) = \sigma^2 = \int (x - E(X))^2 f(x) \, dx
\]

\[
= E(X^2) - (E(X))^2
\]

\[
= \int x^2 \left( \frac{1-x}{2} \right) \, dx - (E(X))^2
\]

\[
= \int_{-1}^{1} x^2 \left( \frac{1-x}{2} \right) \, dx - \left( \frac{1}{3} \right)^2
\]

\[
= \frac{1}{3} - \left( \frac{1}{3} \right)^2 = \frac{2}{9}
\]

$\text{Stddev}(X) = \sigma = \sqrt{\text{Var}(X)} = \sqrt{\sigma^2} = \sqrt{2/9} = 0.4714$

(c) the upper quartile = ?

upper quartile = ?

$F(x) = 0.75$

\[
\int_{-1}^{x} \frac{1-y}{2} \, dy = 0.75
\]

$X = 0.0$

**Problem (11):**

Three independent events A, B and C have probabilities $1/2$, $2/3$, $3/4$, respectively. Let $X$ be the number of these events that occur, where:

$0 \leq X \leq 3$. Find the following:

(a) $P(X = 0)$
(b) $P(X = 1)$
(c) $P(X = 2)$
(d) $P(X = 3)$
(e) $E(X)$
(f) $\text{Var}(X)$
(Question 5: in Midterm Exam 2005)

Solution:

(a) \( P(X = 0) = \?)

\[
P(X = 0) = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{24}
\]

(b) \( P(X = 1) = \?)

\[
P(X = 1) = \left( \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \right) + \left( \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} \right) + \left( \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \right)
= \frac{3}{8}
\]

(c) \( P(X = 2) = \?)

\[
P(X = 1) = \left( \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} \right) + \left( \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} \right) + \left( \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \right)
= \frac{11}{24}
\]

(d) \( P(X = 3) = \?)

\[
P(X = 3) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}
\]

(e) \( E(X) = \?)

\[
E(X) = (0 \times \frac{1}{24}) + (1 \times \frac{3}{8}) + (2 \times \frac{11}{24}) + (3 \times \frac{1}{4}) = \frac{49}{24}
\]

(f) \( Var(X) = \?)

\[
Var(X) = \sigma^2 = E((X - E(X))^2)
= \frac{1}{24} \times (0 - \frac{49}{24})^2 + \frac{3}{8} \times (1 - \frac{49}{24})^2
+ \frac{11}{24} \times (2 - \frac{49}{24})^2
= 1.001519
\]
Problem (12):
Suppose the cumulative distribution function of the random variable \( X \) is:
\[
F(X) = \begin{cases} 
0.0 & x < -2 \\
Ax + 0.5 & -2 \leq x < 2 \\
B & 2 \leq x
\end{cases}
\]
(a) Find the value of \( A \) and the expected value of \( B \)?
(b) What is the probability \( P(-1 \leq X \leq 1) \)?
(c) Calculate the variance of this random variable?
(d) What is the interquartile range?
(e) Sketch the probability density function and the cumulative distribution function?

(Question 3: (15 points) in Midterm Exam 2007)

Solution:

(a) the value of \( A \) and the expected value of \( B \) = ?!
\[
\text{Area under the pdf} = \int f(x) \, dx = 0.0 + \int_{-2}^{2} A \, dx = 1.0
\]
\[
1.0 = 0.0 + [Ax]_{-2}^{2}
\]
\[
1.0 = 0.0 + A(2 - (-2))
\]
\[
A = 0.25
\]
\[
B = 1.0 \text{ (since the maximum value of } F(x) \text{ is one)}
\]

So that:
\[
F(X) = \begin{cases} 
0.0 & x < -2 \\
0.25x + 0.5 & -2 \leq x < 2 \\
1.0 & 2 \leq x
\end{cases}
\]

(b) \( P(-1 \leq X \leq 1) = ?! \)
\[
P(-1 \leq X \leq 1) = F(1) - F(-1)
\]
\[
= (0.25 \times 1 + 0.5) - (0.25 \times -1 + 0.5)
\]
\[
= 0.50
\]
(c) \( \text{Var}(X) = ?! \)

\[
E(X) = \int_{-2}^{2} x f(x) \, dx = \int_{-2}^{2} x(0.25) \, dx = 0.00
\]

\[
\text{Var}(X) = \sigma^2 = \int (x - E(X))^2 f(x) \, dx
\]

\[
= E(X^2) - (E(X))^2
\]

\[
= \int x^2(0.25x + 0.5) \, dx - (E(X))^2
\]

\[
= \int_{-2}^{2} x^2(0.25x + 0.5) \, dx - (0.00)^2
\]

\[
= 4/3 - (0.00)^2 = 4/3
\]

(d) interquartile range = ?!

upper quartile = ?!

\( F(x) = 0.75 \)

\( 0.25x + 0.5 = 0.75 \)

\( x = 1.0 \)

lower quartile = ?!

\( F(x) = 0.25 \)

\( 0.25x + 0.5 = 0.25 \)

\( x = -1.0 \)

interquartile range = upper quartile - lower quartile

\( = (1.0) - (-1.0) = 2.0 \)

(e) Sketch the pdf and cdf = ?!

Deriving the cdf gives that:

\[
F(X) = \begin{cases} 
0.0 & x < -2 \\
0.25 & -2 \leq x < 2 \\
0.0 & 2 \leq x 
\end{cases}
\]
From the cdf given:

\[
F(X) = \begin{cases} 
0 & x < -2 \\
0.25x + 0.5 & -2 \leq x < 2 \\
1.0 & 2 \leq x 
\end{cases}
\]
Problem (13):
The pdf of a random variable $X$ is shown:

(a) Calculate the median of $X$.
(b) Calculate $E(X)$.
(c) Graph the cdf on the chart below.
(d) On the cdf graph in part “c”, locate the upper quartile and the lower quartile and find their values from the cdf graph.

(Question 3: (6 points) in Midterm Exam 2010)

Solution:

(a) median = ?!

\[ 0.50 \times a \times 2 + 2 \times a = 1.0 \]

\[ a = \frac{1}{3} \]

\[ b \times \frac{1}{3} = 0.50 \]

\[ b = 1.50 \]

\[ x = 7 - 1.50 \]

\[ x = 5.50 \]

(b) $E(X) = ?!$

\[ f(x) = c \times x + d \]

slope = \[ \frac{\frac{1}{3} - 0}{5 - 3} = \frac{1}{6} \]

\[ f(x) = \frac{1}{6} (x - 3) \]

\[ f(x) = \frac{x - 3}{6} \quad \text{for} \quad 3 \leq x \leq 5 \]
\[ f(x) = \frac{1}{3} \quad \text{for } 5 \leq x \leq 7 \]

So that:

\[
\begin{align*}
  f(x) &= \begin{cases} 
    \frac{x - 3}{6} & 3 \leq x \leq 5 \\
    \frac{1}{3} & 5 \leq x \leq 7
  \end{cases}
\end{align*}
\]

Therefore:

\[
E(X) = \int x f(x) \, dx = \int_{3}^{5} x \left(\frac{x - 3}{6}\right) \, dx + \int_{5}^{7} x \frac{1}{3} \, dx = \frac{13}{9} + 4 = 5.4444
\]

(c) Graph the cdf = ?!

\[
F(x) = \begin{cases} 
    \int_{3}^{x} \frac{y - 3}{6} \, dy = \left[ \frac{y^2}{2} - 3y \right]_3^x = \frac{x^2}{12} - \frac{x}{2} + \frac{3}{4} & 3 \leq x \leq 5 \\
    \frac{1}{3} + \int_{5}^{x} \frac{1}{3} \, dy = \left[ \frac{y}{3} \right]_5^x + \frac{1}{3} = \frac{x}{3} - \frac{4}{3} & 5 \leq x \leq 7
  \end{cases}
\]

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<th>4</th>
<th>4.5</th>
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<td>0.33</td>
<td>0.50</td>
<td>0.83</td>
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![C.D.F Graph]
Problem (14):
A random variable X takes values between 0 and ∞ with a cumulative distribution function:

\[ F(x) = A + B e^{-x} \]

(a) Find the value of both A and B.
(b) What is the probability that the random variable X takes a value between 1 and 7.
(c) Find the probability density function.
(d) Calculate the median of this random variable.
(e) Compute the interquartile range.
(f) Compute the variance.

(Question 4: (7 points) in Midterm Exam 2010)

Solution:

(a) the values of A and B = ?!

\[ F(\infty) = 1.0 \]
\[ A + B e^{-x} = 1.0 \]
\[ A = 1.0 \]
\[ F(0) = 0.0 \]
\[ A + Be^{-x} = 0.0 \]
\[ 1 + B = 0.0 \]
\[ B = -1.0 \]

Therefore:
\[ F(x) = 1 - e^{-x} \quad \text{for} \ 0 \leq x \leq \infty \]

(b) \[ P(1 \leq X \leq 7) = F(X = 7) - F(X = 1) = f(\leq 7) - f(\leq 1) \]
\[ P(1 \leq X \leq 7) = F(X = 7) - F(X = 1) \]
\[ = (1 - e^{-7}) - (1 - e^{-1}) \]
\[ = 0.9991 - 0.6321 = 0.3670 \]

(c) Construct and sketch the probability density function

\[ f(x) = \frac{dF(x)}{dx} = \frac{d(1 - e^{-x})}{dx} = e^{-x} \quad \text{for} \ 0 \leq x \leq \infty \]

Therefore:
\[ f(x) = e^{-x} \quad \text{for} \ 0 \leq x \leq \infty \]

(c) median = ?!
\[ F(x) = 0.50 \]
\[ 1 - e^{-x} = 0.50 \]
\[ x = 0.693147 \]
(c) interquartile range = ?!

upper quartile = ?!

\[ F(x) = 0.75 \]
\[ 1 - e^{-x} = 0.75 \]
\[ x = 1.386294 \]

lower quartile = ?!

\[ F(x) = 0.25 \]
\[ 1 - e^{-x} = 0.25 \]
\[ x = 0.287682 \]

interquartile range = upper quartile - lower quartile
\[ = 1.386294 - 0.287682 \]
\[ = 1.098612 \]

(f) \( \text{Var}(X) = ?! \)

\[ \text{Var}(X) = \sigma^2 = \int (x - E(X))^2 f(x)dx \]

\[ = E(X^2) - (E(X))^2 \]
\[ = \int_0^\infty x^2(e^{-x})dx - \left( \int_0^\infty x(e^{-x})dx \right)^2 \]