Chapter 6
Fundamental Principals of Traffic Flow

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Traffic flow theory involves **mathematical relationships** among the primary elements of a traffic stream: flow, density, and speed.
help the traffic engineer in:

planning, designing, and evaluating

the effectiveness such as:

- adequate lane lengths for storing left-turn
- the average delay at intersections and freeway ramp merging areas,
- the level of freeway performance
- simulation
6.1 TRAFFIC FLOW ELEMENTS

6.1.1 Time-Space Diagram

The time-space diagram is a graph that describes the relationship between:
- the location of vehicles in a traffic stream and
- the time

as the vehicles progress along the highway.
Fundamental Principles of Traffic Flow

This will be explained later.

Figure 6.1  Time-Space Diagram
6.1.2 Primary Elements of Traffic Flow

- flow,
- density (concentration),
- speed
- the gap or headway between two vehicles in a traffic stream
Flow rate:

Flow rate \( (q) \) is the equivalent hourly rate at which vehicles pass a point on a highway during a time period less than 1 hour. It can be determined by:

\[
q = \frac{n \times 3600}{T} \text{veh/h}
\]

(6.1)

Where

\( n = \text{the number of vehicles passing a point in the roadway in } T \text{ sec} \)

\( q = \text{the equivalent hourly flow} \)
6.1.2 Primary Elements of Traffic Flow

**Traffic volume:**
Traffic volume is the number of vehicles that pass a given point on the highway in a given period of time. Period of time may be year, month, day, hour, or sub-hour.

To distinct between volume and rate of flow, see the following example:

**Example:**
The following traffic counts were made during an hour-long study period:

<table>
<thead>
<tr>
<th>Time period</th>
<th>Number of vehicle (veh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5:00 –5:15</td>
<td>100</td>
</tr>
<tr>
<td>5:15 –5:30</td>
<td>120</td>
</tr>
<tr>
<td>5:30 –5:45</td>
<td>110</td>
</tr>
<tr>
<td>5:45 –6:00</td>
<td>100</td>
</tr>
</tbody>
</table>

Calculate the hourly volume and the rates of flow?
6.1.2 Primary Elements of Traffic Flow

*Answer:*

The hourly volume = 100+120+110+100 = 430 veh
The rates of low = [ No. of veh.] / [time(in hours)]

<table>
<thead>
<tr>
<th>Time period</th>
<th>Number of vehicle (veh)</th>
<th>Rate of flow vph</th>
</tr>
</thead>
<tbody>
<tr>
<td>5:00 –5:15</td>
<td>100</td>
<td>100/0.25 = 400</td>
</tr>
<tr>
<td>5:15 –5:30</td>
<td>120</td>
<td>120/0.25 = 480</td>
</tr>
<tr>
<td>5:30 –5:45</td>
<td>110</td>
<td>110/0.25 = 440</td>
</tr>
<tr>
<td>5:45 –6:00</td>
<td>100</td>
<td>100/0.25 = 400</td>
</tr>
</tbody>
</table>
Peak hour factor is:
Peak hour factor is the ratio of total hourly volume to the maximum 15-min rate of flow within the hour. It may be computed as:

\[
PHF = \frac{V}{(4 \times V_{15})_{max}}
\]

where:
- \(PHF\) = the peak hour factor;
- \(V\) = hourly volume, in vph; and
- \((V_{15})_{max}\) = volume during the peak 15 min of the peak hour, in veh/15min
Density:

- *Density* ($k$)
- *sometimes referred to as* concentration,
- the number of vehicles traveling over a unit length of highway at an instant in time.
- The unit length is usually 1 kilometer (km)
- the unit of density is *vehicles per km* (veh/km).
**Speed:**

*Speed (u) is the distance traveled by a vehicle during a unit of time.*

*It can be expressed in:*

- miles per hour (mi/h),
- kilometers per hour (km/h),
- feet per second (ft/sec), or
- meter/second (m/sec).

*Note:*

- $1 \text{ km/h} = 1/3.6 \text{ m/sec}$
- $1 \text{ m/sec} = 3.6 \text{ km/h}$
**Time mean speed** $(\bar{u}_t)$:

is the arithmetic mean of the speeds of vehicles passing a point on a highway during an interval of time.

The time mean speed is found by:

$$\bar{u}_t = \frac{1}{n} \sum_{i=1}^{n} u_i$$  \hspace{2cm} (6.2)

where

$n = \text{number of vehicles passing a point on the highway}$

$U_i = \text{speed of the } i^{th} \text{ vehicle (m/sec)}$
Space mean speed ($\bar{u}_s$):

- is the harmonic mean of the speeds of vehicles passing a point on a highway during an interval of time.
- the total distance traveled by two or more vehicles on a section of highway divided by the total time required by these vehicles to travel that distance.
- This is the speed that is involved in flow-density relationships.
The space mean speed is found by:

\[
\overline{u}_s = \frac{n}{\sum_{i=1}^{n} \left(\frac{1}{u_i}\right)}
\]

Where:
- \(\overline{u}_s\) = space mean speed (m/sec)
- \(n\) = number of vehicles
- \(u_i\) = speed of the \(i^{th}\) vehicle (m/sec)
- \(t_i\) = the time it takes the \(i^{th}\) vehicle to travel across a section of highway (sec)
- \(L\) = length of section of highway (m)

The time mean speed is always higher than the space mean speed.
**Time Headway**

- *Time headway* ($h$) is the difference between the time the front of a vehicle arrives at a point on the highway and the time the front of the next vehicle arrives at that same point.

- Time headway is usually expressed in seconds.

- For example, in the time space diagram (Figure 6.1), the time headway between vehicles 3 and 4 at $d1$ is $h3 - 4$. 
Space Headways

*Space headway* \((d)\) is the distance between the front of a vehicle and the front of the following vehicle and is usually expressed in meter (feet).

The space headway between vehicles 3 and 4 at time \(t_5\) is \(d_{3 – 4}\) (see Figure 6.1).
Example 6.1 Determining Flow, Density, Time Mean Speed, and Space Mean Speed

Figure 6.3 shows vehicles traveling at constant speeds on a two-lane highway between sections $X$ and $Y$ with their positions and speeds obtained at an instant of time by photography.
Example 6.1 ....

- An observer located at point $X$ observes the four vehicles passing point $X$ during a period of $T$ sec.
- The velocities of the vehicles are measured as 45, 45, 40, and 30 km/h, respectively.
- Calculate the flow, density, time mean speed, and space mean speed.
Example 6.1 ....

Figure 6.3 Locations and Speeds of Four Vehicles on a Two-Lane Highway at an Instant of Time
Solution: The flow is calculated by

\[
q = \frac{n \times 3600}{T}
= \frac{4 \times 3600}{T} = \frac{14,400}{T} \text{ veh/h}
\]  

With \( L \) equal to the distance between \( X \) and \( Y \) (m), density is obtained by

\[
k = \frac{n}{L}
= \frac{4}{90} \times 1000 = 44.4 \text{ veh/km}
\]
The time mean speed is found by

$$u_t = \frac{1}{n} \sum_{i=1}^{n} u_i$$

$$= \frac{30 + 40 + 45 + 45}{4} = 40 \text{ km/h}$$

The space mean speed is found by

$$\bar{u}_s = \frac{n}{\sum_{i=1}^{n} (1/u_i)}$$

$$= \frac{Ln}{\sum_{i=1}^{n} t_i}$$

$$= \frac{90n}{\sum_{i=1}^{n} t_i}$$
where \( t_i \) is the time it takes the \( i \)th vehicle to travel from \( X \) to \( Y \) at speed \( u_i \), and \( L \) (ft) is the distance between \( X \) and \( Y \).

\[
t_i = \frac{L}{0.278 \times u_i} \text{ sec}
\]

\[
t_A = \frac{90}{0.278 \times 45} = 7.2 \text{ sec}
\]

\[
t_B = \frac{90}{0.278 \times 45} = 7.2 \text{ sec}
\]

\[
t_C = \frac{90}{0.278 \times 40} = 8.1 \text{ sec}
\]

\[
t_D = \frac{90}{0.278 \times 30} = 10.8 \text{ sec}
\]

\[
\bar{u}_s = \frac{4 \times 90}{7.2 + 7.2 + 8.1 + 10.8} = 10.8 \text{ m/sec}
\]

\[
= 39.0 \text{ mk/h}
\]

\[
1/3.6 = 0.278
\]
6.2 FLOW-DENSITY RELATIONSHIPS

The general equation relating flow, density, and space mean speed is given as:

Flow = density $\times$ space mean speed

$q = k \bar{u}_s$
6.2 FLOW-DENSITY RELATIONSHIPS

\[ q = k \bar{u}_s \]

\[ k = \frac{q}{\bar{u}_s} \quad \bar{u}_s = \frac{q}{k} \]
\[ q = k \bar{u}_s \]

\[ \overline{h} = \frac{1}{q} \]

\[ \overline{d} = \frac{1}{k} \]
Space mean speed = (flow) × (space headway)

\[ \bar{u}_s = q\bar{d} \]

Where:

\[ \bar{d} = \frac{1}{k} = \text{average space headway} \]

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\[
K = 5 \text{ veh/Km} \\
\bar{d} = \frac{d_1 + d_2 + d_3 + d_4 + d_5}{5} \\
= 1\text{Km}/K = 1/K
\]
Density = (flow) × (travel time for unit distance)

\[
k = q \overline{t}
\]

\[
\overline{t} = \frac{1}{\bar{u}_s}
\]

Where:

\( \overline{t} \) is the average travel time for unit distance.

\( k = \frac{q}{\bar{u}_s} \)
**Fundamental Principals of Traffic Flow**

- \( t = 1 / u_s \)

Time taken by vehicle 1 to travel unit length (1Km) = \( t_1 \)

Time taken by vehicle \( n \) to travel unit length (1Km) = \( t_n \)

- \( t = (t_1 + t_2 + t_3 + \cdots + t_n) / n = \sum t / n \)
- \( \bar{u_s} = (n \ L) / \sum t, \)
- \( \bar{u_s} = n / \sum t, \) where \( L = 1 \) Km
- \( 1 / \bar{u_s} = \sum t / n, \)
- \( 1 / \bar{u_s} = t \)
Assume:
\[ q = 5 \text{ veh / minute} = 5 \text{ veh / 60 sec} \]
\[ 1/q = 60 \text{ sec / 5} = 12 \text{ sec per vehicle} = h \]

\[ h = 1/q \]

\[ d = 1/K \]
Average space headway = (space mean speed) × (average time headway)

\( d = u_s h \)

Average time headway = (average travel time for unit distance) × (average space headway)

\( h = t d \)
6.2.1 Fundamental Diagram of Traffic Flow
Fundamental Principals of Traffic Flow

Figure 6.4 Fundamental Diagrams of Traffic Flow
Figure 6.4 Fundamental Diagrams of Traffic Flow
6.2.2 Mathematical Relationships Describing Traffic Flow

Mathematical relationships describing traffic flow can be classified into two general categories: **macroscopic** and **microscopic** depending on the approach used in the development of these relationships.
6.2.2 Mathematical Relationships Describing Traffic Flow

- The **macroscopic** approach considers flow density relationships whereas
- the **microscopic** approach considers spacings between vehicles and speeds of individual vehicles.
6.2.2 Mathematical Relationships

**Greenshields Model**

**Linear relationship**

\[
\bar{u}_s = u_f - \frac{u_f}{k_j} k
\]  \hspace{1cm} (6.13)

Corresponding relationships for flow and density and for flow and speed can be developed. Since \( q = \bar{u}_s k \), substituting \( q/\bar{u}_s \) for \( k \) in Eq. 6.13 gives

\[
\bar{u}_s^2 = u_f \bar{u}_s - \frac{u_f}{k_j} q
\]  \hspace{1cm} (6.14)

Also substituting \( q/k \) for \( \bar{u}_s \) in Eq. 6.13 gives

\[
q = u_f k - \frac{u_f}{k_j} k^2
\]  \hspace{1cm} (6.15)
6.2.2 Mathematical Relationships

**Greenshields Model**

Eqs. 6.14 and 6.15 indicate that if a linear relationship in the form of Eq. 6.13 is assumed for speed and density, then parabolic relationships are obtained between flow and density and between flow and speed. The shape of the curve shown in Figure 6.4a will therefore be a parabola. Also, Eqs. 6.14 and 6.15 can be used to determine the corresponding speed and the corresponding density for maximum flow.

Consider Eq. 6.14:

\[ \overline{u_s}^2 = u_f \overline{u_s} - \frac{u_f}{k} q \]

Differentiating \( q \) with respect to \( \overline{u_s} \), we obtain

\[ 2\overline{u_s} = u_f - \frac{u_f}{k} \frac{dq}{du_s} \]
6.2.2 Mathematical Relationships

**Greenshields Model**

that is

\[
\frac{dq}{d\bar{u}_s} = u_f \frac{k_j}{u_f} - 2\bar{u}_s \frac{k_j}{u_f} = k_j - 2\bar{u}_s \frac{k_j}{u_f}
\]

For maximum flow,

\[
\frac{dq}{d\bar{u}_s} = 0 \quad k_j = 2\bar{u}_s \frac{k_j}{u_f} \quad u_o = \frac{u_f}{2}
\]  \hspace{1cm} (6.16)

Thus, the space mean speed \( u_o \) at which the volume is maximum is equal to half the free mean speed.

Consider Eq. 6.15:

\[
q = u_f k - \frac{u_f}{k_j} k^2
\]
Fundamental Principals of Traffic Flow

Differentiating $q$ with respect to $k$, we obtain

\[
\frac{dq}{dk} = u_f - 2k \frac{u_f}{k_j}
\]

For maximum flow,

\[
\frac{dq}{dk} = 0
\]

\[
u_f = 2k \frac{u_f}{k_j}
\]

\[
\frac{k_j}{2} = k_o
\]

(6.17)

Thus, at the maximum flow, the density $k_o$ is half the jam density. The maximum flow for the Greenshields relationship can therefore be obtained from Eqs. 6.7, 6.16, and 6.17, as shown in Eq. 6.18:

\[
q_{\text{max}} = \frac{k_j u_f}{4}
\]

(6.18)
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Example 6.2 Fitting Speed and Density Data to the Greenshields Model

Let us now use the data shown in Table 6.1 (columns 1 and 2) to demonstrate the use of the method of regression analysis in fitting speed and density data to the macroscopic models discussed earlier.

Table 6.1 Speed and Density Observations at a Rural Road

<table>
<thead>
<tr>
<th>Speed, $u_s$ (km/h) $y_i$</th>
<th>Density, $k$ (veh/km) $x_i$</th>
<th>$x_iy_i$</th>
<th>$x_i^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>53.2</td>
<td>20</td>
<td>1064.0</td>
<td>400</td>
</tr>
<tr>
<td>48.1</td>
<td>27</td>
<td>1298.7</td>
<td>729</td>
</tr>
<tr>
<td>44.8</td>
<td>35</td>
<td>1568.0</td>
<td>1,225</td>
</tr>
<tr>
<td>40.1</td>
<td>44</td>
<td>1764.4</td>
<td>1,936</td>
</tr>
<tr>
<td>37.3</td>
<td>52</td>
<td>1939.6</td>
<td>2,704</td>
</tr>
<tr>
<td>35.2</td>
<td>58</td>
<td>2041.6</td>
<td>3,364</td>
</tr>
<tr>
<td>34.1</td>
<td>60</td>
<td>2046.0</td>
<td>3,600</td>
</tr>
<tr>
<td>27.2</td>
<td>64</td>
<td>1740.8</td>
<td>4,096</td>
</tr>
<tr>
<td>20.4</td>
<td>70</td>
<td>1428.0</td>
<td>4,900</td>
</tr>
<tr>
<td>17.5</td>
<td>75</td>
<td>1312.5</td>
<td>5,625</td>
</tr>
<tr>
<td>14.6</td>
<td>82</td>
<td>1197.2</td>
<td>6,724</td>
</tr>
<tr>
<td>13.1</td>
<td>90</td>
<td>1179.0</td>
<td>8,100</td>
</tr>
<tr>
<td>11.2</td>
<td>100</td>
<td>1120.0</td>
<td>10,000</td>
</tr>
<tr>
<td>8.0</td>
<td>115</td>
<td>920.0</td>
<td>13,225</td>
</tr>
</tbody>
</table>

$\Sigma = 404.8$  \quad $\Sigma = 892$  \quad $\Sigma = 20,619.8$  \quad $\Sigma = 66,628.0$

$\bar{y} = 28.91$  \quad $\bar{x} = 63.71$
If a dependent variable \( y \) and an independent variable \( x \) are related by an estimated regression function, then

\[
y = a + bx
\]  

(6.21)

The constants \( a \) and \( b \) could be determined from Eqs. 6.22 and 6.23. (For development of these equations, see Appendix B.)

\[
a = \frac{1}{n} \sum_{i=1}^{n} y_i - \frac{b}{n} \sum_{i=1}^{n} x_i = \bar{y} - b \bar{x}
\]  

(6.22)

and

\[
b = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n} \left( \sum_{i=1}^{n} x_i \right) \left( \sum_{i=1}^{n} y_i \right)}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n} \left( \sum_{i=1}^{n} x_i \right)^2}
\]  

(6.23)

where

- \( n \) = number of sets of observations
- \( x_i \) = \( i \)th observation for \( x \)
- \( y_i \) = \( i \)th observation for \( y \)
Solution: Let us first consider the Greenshields expression

$$\bar{u}_s = u_f - \frac{u_f}{k_j} k$$

Comparing this expression with our estimated regression function, Eq. 6.21, we see that the speed $\bar{u}_s$ in the Greenshields expression is represented by $y$ in the estimated regression function, the mean free speed $u_f$ is represented by $a$, and the value of the mean free speed $u_f$ divided by the jam density $k_j$ is represented by $-b$. We therefore obtain

$$\sum y_i = 404.8 \quad \sum x_i = 892 \quad \bar{y} = 28.91$$

$$\sum x_i y_i = 20619.8 \quad \sum x_i^2 = 66,628 \quad \bar{x} = 63.71$$

- Using Eqs. 6.22 and 6.23, we obtain

$$a = 28.91 - 63.71b$$

$$b = \frac{20,619.8 - \frac{(892)(4048)}{14}}{\frac{66,628 - \frac{(892)^2}{14}}{14}} = -0.53$$
or

\[ a = 28.91 - 63.71(-0.53) = 62.68 \]

Since \( a = 62.68 \) and \( b = -0.53 \), then \( u_f = 62.68 \text{ km/h} \), \( u_f/k_j = 0.53 \), and so \( k_j = 118 \text{ veh/km} \), and \( \bar{u}_s = 62.68 - 0.53k \).

- Using Eq. 6.24 to determine the value of \( R^2 \), we obtain \( R^2 = 0.95 \).
- Using the above estimated values for \( u_f \) and \( k_j \), we can determine the maximum flow from Eq. 6.18 as

\[ q_{\text{max}} = \frac{k_j u_f}{4} = \frac{118 \times 62.68}{4} = 1849 \text{ veh/h} \]

- Using Eq. 6.16 we also obtain the velocity at which flow is maximum, that is, \((62.68/2) = 31.3 \text{ km/h}\), and Eq. 6.17, the density at which flow is maximum, or \((118/2) = 59 \text{ Veh./km}\).
Thus: 
\[ a = 62.68 \text{ and } b = -0.53 \]

\[ U_s = 62.68 - 0.53 \, K \]

\[ U_f = 62.68 \text{ km/h} \]

\( K_j \) can be found at \( U_s = 0 \)

\[ K_j = \frac{62.68}{0.53} = 118 \text{ veh/km} \]

\[ q_{\text{max}} = K_j \times U_f / 4 \]
\[ = \frac{(118 \times 62.68)}{4} \]
\[ q_{\text{max}} = 1849 \text{ veh/h} \]

\( q_{\text{max}} \) occur at \( K_j / 2 = \frac{118}{2} = 59 \text{ veh/km} \)

\( q_{\text{max}} \) occur at \( U_f / 2 = \frac{62.68}{2} = 31.3 \text{ km/h} \)
Two sets of students are collecting traffic data at two sections, $xx$ and $yy$, of a highway 450 m apart. Observations at $xx$ show that five vehicles passed that section at intervals of 3, 4, 3, and 5 s, respectively. If the speeds of the vehicles were 50, 45, 40, 35, and 30 km/h, respectively.

1) Draw a schematic showing the locations of the vehicles 20 s after the first vehicle passed section $xx$.

2) Determine the time mean speed, the space mean speed, and the density on the highway.
Two sets of students are collecting traffic data at two sections, $xx$ and $yy$, of a highway 450 m apart. Observations at $xx$ show that five vehicles passed that section at intervals of 3, 4, 3, and 5 s, respectively. If the speeds of the vehicles were 50, 45, 40, 35, and 30 km/h, respectively.

1) draw a schematic showing the locations of the vehicles 20 s after the first vehicle passed section $xx$

2) Determine the time mean speed, the space mean speed, and the density on the highway.
Use the following relationship between speed and density to answer the following questions:

• Derive the speed-density and density-flow relationships.
• Draw the relationship between flow and density using the above chart.
• Find the density at maximum flow.
• Calculate the speed at maximum flow.
• Find the capacity
Example 2 (continued)