Circular Shift

\[ x(n) \]
\[ x_d(n) \]
\[ x_d(n-2) \]
Circular Shift

Doing normal shift on $x_p(n)$ is equivalent to do circular shift on $x(n)$.

In previous example, the samples from $x_p(n-2)$ to $N-1$ result in a circular shifted version of $x(n)$ by 2.

Circular Shift

This can be expressed by using the following formula:

$$x'(n) = x(n - k, \text{mod ule } N)$$

$$= x((n - k))_N$$

For our example:

$x'(0) = x((-2))_4 = x(2)$

$x'(1) = x((-1))_4 = x(3)$

$x'(2) = x((0))_4 = x(0)$

$x'(3) = x((1))_4 = x(1)$

Green arrow indicates positive direction.
Circular Reflection

The time reversal of an N-point sequence is attained by reversing it samples about the point zero on the circle.

\[ x((-n))_N = x(N - n) \]

**Example** \( y(n) = x((-n))_N \)

![Diagram showing circular reflection](image)

Circular Symmetry

An N-point sequence is called **circularly even** if

\[ x(N - n) = x(n) \quad 1 \leq n \leq N - 1 \]

**Example** : \( x(n) = \{1, 2, 3, 3, 2\} \)

An N-point sequence is called **circularly odd** if

\[ x(N - n) = -x(n) \quad 1 \leq n \leq N - 1 \]

**Example** : \( x(n) = \{0, 2, 3, -3, 2\} \)

For circularly odd signal \( x(0) = 0 \) and if N is even \( x(N/2) = 0 \).
Circular Convolution

The circular convolution is very similar to normal convolution apart from that the signal is shifted using circular shift.

If $x_1(n)$ and $x_2(n)$ is two discrete signals, then the result of the circular convolution $x_3(n)$ can be realised by

$$x_3(m) = x_1(n) \otimes x_2(n) = \sum_{n=0}^{N-1} x_1(n)x_2((m-n))_N$$

Example: Perform the circular convolution of the following two sequences

$x_1(n) = \{2,1,2,1\}$ and $x_2(n) = \{1,2,3,4\}$

$$x_3(0) = 14$$
$x_1(n) = \{2,1,2,1\}$ and $x_2(n) = \{1,2,3,4\}$

$x_1(n) = \{2,1,2,1\}$ and $x_2(n) = \{1,2,3,4\}$

$x_3(1) = 16$

$x_3(2) = 14$
Digital Signal Processing

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\[ x_1(n) = \{2,1,2,1\} \text{ and } x_2(n) = \{1,2,3,4\} \]

\[ x_1(n) \uparrow \quad x_2(n) \uparrow \]

\[ x_1((3-n))_4 = \{3,2,4,1\} \quad x_2((3-n))_4 = \{4,3,1,2\} \]

\[ x_1(n) \otimes x_2((3-n))_4 = \{1,14,16,14\} \]

\[ x_3(3) = 16 \]

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\[ x_1(n) = \{2,1,2,1\} \text{ and } x_2(n) = \{1,2,3,4\} \]

\[ x_1(n) \uparrow \quad x_2(n) \uparrow \]

The final solution is

\[ x_3(m) = x_1(n) \otimes x_2(n) = \{14,16,14,16\} \]
Self Study

Students are encouraged to solve the following questions from the textbook:

5.8.